CHAPTER 2
MOTION ALONG A STRAIGHT LINE

Discussion Questions

Q2.1 The speedometer measures the magnitude of the instantaneous velocity, the speed. It does not measure velocity because it does not measure direction.

Q2.2 Graph (d) . The dots represent the insect's position as a function of time. If the photographs are taken at equal spaced time intervals, then the displacement in successive intervals is increasing and this means the speed is increasing. Therefore, graphs (a) and (e) can be ruled out. Graph (b) shows decreasing acceleration so would correspond to the speed approaching a constant value, which is not what the photographs show. Graph (c) shows motion in the negative x-direction, which is not the case. This leaves graph (d). This graph shows velocity in the positive x-direction and increasing speed. This is consistent with the photographs.

Q2.3 The answer to the first question is yes. If the object is initially moving and the acceleration direction is opposite to the velocity direction, then the object slows down, stops for an instant and then starts to move in the opposite direction with increasing speed. An example is an object thrown straight up into the air. Gravity gives the object a constant downward acceleration. The object travels upward, stops at its maximum height and then moves downward. The answer to the second question is no. After the first reversal of the direction of travel the velocity and acceleration are then in the same direction. The object continues moving in the second direction with increasing speed.

Q2.4 Average velocity equals instantaneous velocity when the speed is constant and motion is in a straight line.

Q2.5 a) Yes. For an object to be slowing down, all that is required is that the acceleration be nonzero and for the velocity and acceleration to be in opposite directions. The magnitude of the acceleration determines the rate at which the speed is changing. b) Yes. For an object to be speeding up, all that is required is that the acceleration be nonzero and for the velocity and acceleration to be in the same direction. The magnitude of the acceleration determines the rate at which the speed is changing. But for any nonzero acceleration the speed is increasing when the velocity and acceleration are in the same direction.

Q2.6 Average velocity is the magnitude of the displacement divided by the time interval. Average speed is the distance traveled divided by the time interval. Displacement equals the distance traveled when the motion is in the same direction for the entire time interval, and therefore this is when average velocity equals average speed.

Q2.7 For the same time interval they have displacements of equal magnitude but opposite directions, so their average velocities are in opposite directions. One average velocity vector is the negative of the other.

Q2.8 If in the next time interval the second car had pulled ahead of the first, then the speed of the second car was greater. The second car could also be observed to be alongside a pedestrian standing at the curb, but that does not mean the pedestrian was speeding.

Q2.9 The answer to the first question is no. Average velocity is displacement divided by the time interval. If the displacement is zero, then the average velocity must be zero. The answer to the second question is yes. Zero displacement means the object has returned to its starting point, but its speed at that point need not be zero. See Fig. DQ2.9.

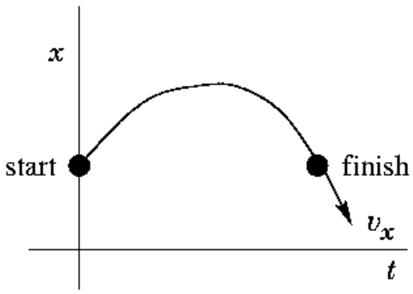


Figure DQ2.9

Q2.10 Zero acceleration means constant velocity, so the velocity could be constant but not zero. See Fig. DQ2.10. An example is a car traveling at constant speed in a straight line.

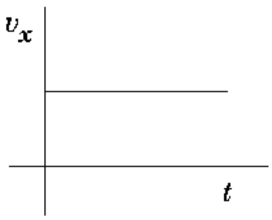


Figure DQ2.10

Q2.11 No. Average acceleration refers to an interval of time and if the velocity is zero throughout that interval, the average acceleration for that time interval is zero. But yes, you can have zero velocity and nonzero acceleration at one instant of time. For example, in Fig. DQ2.11, $v_x = 0$ when the graph crosses the time axis but the acceleration is the nonzero slope of the line. An example is an object thrown straight up into the air. At its maximum height its velocity is zero but its acceleration is g downward.

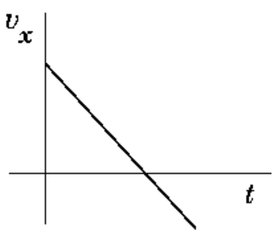


Figure DQ2.11

Q2.12 Yes. When the velocity and acceleration are in opposite directions the object is slowing down.

Q2.13 (a) Two possible *x*-*t* graphs for the motion of the truck are sketched in Fig. DQ2.13.
(b) Yes, the displacement is -258 m and the time interval is 9.0 s, no matter what path the truck takes between x_1 and x_2. The average velocity is the displacement divided by the time interval.

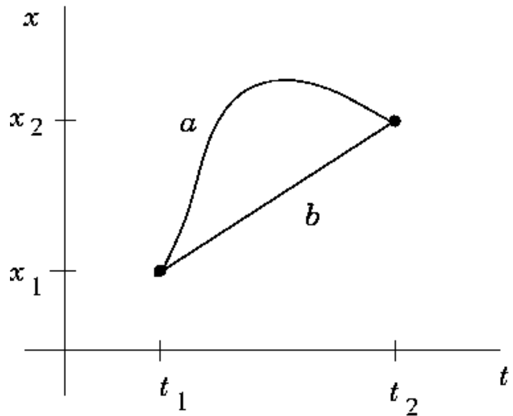


Figure DQ2.13

Q2.14 This is true only when the acceleration is constant. The average velocity is defined to be the displacement divided by the time interval. If the acceleration is not constant, objects can have the same initial and final velocities but different displacements and therefore different average velocities.

Q2.15 It is greater while the ball is being thrown. While being thrown, the ball accelerates from rest to velocity v_{0y} while traveling a distance less than your height. After it leaves your hand, it slows from v_{0y} to zero at the maximum height, while traveling a distance much greater than your height. Eq.(2.13) says that $a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y-y_0)}$. Larger $y - y_0$ means smaller a_y.

Q2.16 (a) Eq.(2.13): $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$. When an object returns to the release point, $y - y_0 = 0$. Eq.(2.13) then gives $v_y^2 = v_{0y}^2$ and $v_y = \pm v_{0y}$.
(b) $v_y = v_{0y} + a_y t$. At the highest point $v_y = 0$, so $t_{\text{up}} = -v_{0y}/a_y$. At the end of the motion, when the object has returned to the release point, we have shown in (i) that $v_y = -v_{0y}$,
so $t_{\text{total}} = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-2v_{0y}}{a_y}$ and $t_{\text{total}} = 2t_{\text{up}}$.

Q2.17 The distance between adjacent drops will increase. The drops have the downward acceleration $g = 9.8\ \text{m/s}^2$ of a free-falling object. Therefore, their speed is continually increasing and the distance one drop travels in each successive 1.0 s time interval increases. A given drop has fallen for 1.0 s longer than the next drop released after it, so the additional distance it has fallen increases as they fall. Mathematically, let t be the time the second drop has fallen, so the first drop has fallen for time $t + 1.0\ \text{s}$. The distance between these two drops then is
$\Delta y = \frac{1}{2}g(t+1.0\ \text{s})^2 - \frac{1}{2}gt^2 = \frac{1}{2}g\left[(2.0\ \text{s})t + 1.0\ \text{s}^2\right]$. The separation Δy increases as t increases.

Q2.18 Yes. Consider very small time intervals during which the acceleration doesn't have time to change very much, so can be assumed to be constant. Calculate $\Delta v_x = a_x\,\Delta t_1$, for a very small time interval, starting at $t = 0$. Then $v_{1x} = v_{0x} + \Delta v_x$. Since the acceleration is assumed constant for the small time interval, $v_{\text{av},x} = (v_{0x} + v_{1x})/2$ and $\Delta x_1 = v_{\text{av},x}\,\Delta t_1$. Then the position at the end of the interval is $x_1 = x_0 + \Delta x_1$. Repeat the calculation for the next small time interval Δt_2: $\Delta v_x = a_x\,\Delta t_2$, $v_{2x} = v_{1x} + \Delta v_x$, $v_{\text{av},x} = (v_{1x} + v_{2x})/2$, $\Delta x_2 = v_{\text{av},x}\,\Delta t_2$. Repeat for successive small time intervals.

Q2.19 In the absence of air resistance, the first ball rises to its maximum height and then returns to the level of the top of the building. When it returns to the height from which it was thrown, at the top of the building, it is moving downward with speed v_0. The rest of its motion is the same as for the second ball. (a) Since the last part of the motion of the first ball starts with it moving downward with speed v_0 from the top of the building, the two balls have the same speed just before they reach the ground. (b) The second ball reaches the ground first, since the first ball has to move up and then down before repeating the motion of the second ball. (c) Displacement is final position minus initial position. Both balls start at the top of the building and end up at the ground. So they have the same displacement. (d) The first ball has traveled a greater distance.

Q2.20 Let the $+x$-direction be east. The average velocity is the displacement divided by the time interval. The first 120.0 m displacement requires a time of $(120.0\text{ m/s})/(3.00\text{ m/s}) = 40.0\text{ s}$. The second 120.0 m displacement requires a time of $(120.0\text{ m/s})/(5.00\text{ m/s}) = 24.0\text{ s}$. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{240.0\text{ m}}{64.0\text{ s}} = 3.75\text{ m/s}$. This is less than 4.00 m/s since you spend more time running at 3.00 m/s than at 5.00 m/s.

Q2.21 At the highest point the object instantaneously has zero speed. But its velocity is continually changing, at a constant rate. Acceleration measures the rate of change of velocity. For example, in Fig. 2.25b when the graph crosses the time axis it still has a constant slope that corresponds to the acceleration. Also note the comments in part (d) of the solution to Example 2.7.

Q2.22 For an object released from rest and then moving downward in free-fall, its downward displacement from its initial position of $y_0 = 0$ is given by $y = \frac{1}{2}gt^2$. To increase y by a factor of 3, increase t by a factor of $\sqrt{3}$. You can also see this by letting Y be the original height, so $Y = \frac{1}{2}gT^2$. Let the new height be Y' and the corresponding time be T', so $Y' = \frac{1}{2}g(T')^2$. But $Y' = 3Y = 3\left(\frac{1}{2}gT^2\right)$, so $3\left(\frac{1}{2}gT^2\right) = \frac{1}{2}g(T')^2$ and $T' = \sqrt{3}T$.

2

MOTION ALONG A STRAIGHT LINE

2.1. **IDENTIFY:** $\Delta x = v_{\text{av-}x}\Delta t$

SET UP: We know the average velocity is 6.25 m/s.

EXECUTE: $\Delta x = v_{\text{av-}x}\Delta t = 25.0\ \text{m}$

EVALUATE: In round numbers, 6 m/s × 4 s = 24 m ≈ 25 m, so the answer is reasonable.

2.2. **IDENTIFY:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$

SET UP: $13.5\ \text{days} = 1.166\times10^6\ \text{s}$. At the release point, $x = +5.150\times10^6\ \text{m}$.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{x_2 - x_1}{\Delta t} = \dfrac{-5.150\times10^6\ \text{m}}{1.166\times10^6\ \text{s}} = -4.42\ \text{m/s}$.

(b) For the round trip, $x_2 = x_1$ and $\Delta x = 0$. The average velocity is zero.

EVALUATE: The average velocity for the trip from the nest to the release point is positive.

2.3. **IDENTIFY:** Target variable is the time Δt it takes to make the trip in heavy traffic. Use Eq. (2.2) that relates the average velocity to the displacement and average time.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so $\Delta x = v_{\text{av-}x}\Delta t$ and $\Delta t = \dfrac{\Delta x}{v_{\text{av-}x}}$.

EXECUTE: Use the information given for normal driving conditions to calculate the distance between the two cities, where the time is 1 h and 50 min, which is 110 min:

$$\Delta x = v_{\text{av-}x}\Delta t = (105\ \text{km/h})(1\ \text{h}/60\ \text{min})(110\ \text{min}) = 192.5\ \text{km}.$$

Now use $v_{\text{av-}x}$ for heavy traffic to calculate Δt; Δx is the same as before:

$$\Delta t = \frac{\Delta x}{v_{\text{av-}x}} = \frac{192.5\ \text{km}}{70\ \text{km/h}} = 2.75\ \text{h} = 2\ \text{h}\ \text{and 45 min}.$$

The additional time is (2 h and 45 min) – (1 h and 50 min) = (1 h and 105 min) – (1 h and 50 min) = 55 min.

EVALUATE: At the normal speed of 105 km/s the trip takes 110 min, but at the reduced speed of 70 km/h it takes 165 min. So decreasing your average speed by about 30% adds 55 min to the time, which is 50% of 110 min. Thus a 30% reduction in speed leads to a 50% increase in travel time. This result (perhaps surprising) occurs because the time interval is inversely proportional to the average speed, not directly proportional to it.

2.4. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use the average speed for each segment to find the time traveled in that segment. The average speed is the distance traveled divided by the time.

SET UP: The post is 80 m west of the pillar. The total distance traveled is $200\ \text{m} + 280\ \text{m} = 480\ \text{m}$.

EXECUTE: **(a)** The eastward run takes time $\dfrac{200\ \text{m}}{5.0\ \text{m/s}} = 40.0\ \text{s}$ and the westward run takes $\dfrac{280\ \text{m}}{4.0\ \text{m/s}} = 70.0\ \text{s}$. The average speed for the entire trip is $\dfrac{480\ \text{m}}{110.0\ \text{s}} = 4.4\ \text{m/s}$.

(b) $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{-80\ \text{m}}{110.0\ \text{s}} = -0.73\ \text{m/s}$. The average velocity is directed westward.

EVALUATE: The displacement is much less than the distance traveled, and the magnitude of the average velocity is much less than the average speed. The average speed for the entire trip has a value that lies between the average speed for the two segments.

2.5. **IDENTIFY:** Given two displacements, we want the average velocity and the average speed.

SET UP: The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ and the average speed is just the total distance walked divided by the total time to walk this distance.

EXECUTE: **(a)** Let $+x$ be east. $\Delta x = 60.0 \text{ m} - 40.0 \text{ m} = 20.0 \text{ m}$ and $\Delta t = 28.0 \text{ s} + 36.0 \text{ s} = 64.0 \text{ s}$. So $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{20.0 \text{ m}}{64.0 \text{ s}} = 0.312 \text{ m/s}$.

(b) $\text{average speed} = \dfrac{60.0 \text{ m} + 40.0 \text{ m}}{64.0 \text{ s}} = 1.56 \text{ m/s}$

EVALUATE: The average speed is much greater than the average velocity because the total distance walked is much greater than the magnitude of the displacement vector.

2.6. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use $x(t)$ to find x for each t.

SET UP: $x(0) = 0$, $x(2.00 \text{ s}) = 5.60 \text{ m}$, and $x(4.00 \text{ s}) = 20.8 \text{ m}$

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{5.60 \text{ m} - 0}{2.00 \text{ s}} = +2.80 \text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{20.8 \text{ m} - 0}{4.00 \text{ s}} = +5.20 \text{ m/s}$

(c) $v_{\text{av-}x} = \dfrac{20.8 \text{ m} - 5.60 \text{ m}}{2.00 \text{ s}} = +7.60 \text{ m/s}$

EVALUATE: The average velocity depends on the time interval being considered.

2.7. **(a) IDENTIFY:** Calculate the average velocity using $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement Δx for this time interval.

EXECUTE: $t = 0$: $x = 0$

$t = 10.0 \text{ s}$: $x = (2.40 \text{ m/s}^2)(10.0 \text{ s})^2 - (0.120 \text{ m/s}^3)(10.0 \text{ s})^3 = 240 \text{ m} - 120 \text{ m} = 120 \text{ m}$.

Then $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{120 \text{ m}}{10.0 \text{ s}} = 12.0 \text{ m/s}$.

(b) IDENTIFY: Use $v_x = \dfrac{dx}{dt}$ to calculate $v_x(t)$ and evaluate this expression at each specified t.

SET UP: $v_x = \dfrac{dx}{dt} = 2bt - 3ct^2$.

EXECUTE: (i) $t = 0$: $v_x = 0$

(ii) $t = 5.0 \text{ s}$: $v_x = 2(2.40 \text{ m/s}^2)(5.0 \text{ s}) - 3(0.120 \text{ m/s}^3)(5.0 \text{ s})^2 = 24.0 \text{ m/s} - 9.0 \text{ m/s} = 15.0 \text{ m/s}$.

(iii) $t = 10.0 \text{ s}$: $v_x = 2(2.40 \text{ m/s}^2)(10.0 \text{ s}) - 3(0.120 \text{ m/s}^3)(10.0 \text{ s})^2 = 48.0 \text{ m/s} - 36.0 \text{ m/s} = 12.0 \text{ m/s}$.

(c) IDENTIFY: Find the value of t when $v_x(t)$ from part (b) is zero.

SET UP: $v_x = 2bt - 3ct^2$

$v_x = 0$ at $t = 0$.

$v_x = 0$ next when $2bt - 3ct^2 = 0$

EXECUTE: $2b = 3ct$ so $t = \dfrac{2b}{3c} = \dfrac{2(2.40 \text{ m/s}^2)}{3(0.120 \text{ m/s}^3)} = 13.3 \text{ s}$

EVALUATE: $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

2.8. **IDENTIFY:** We know the position $x(t)$ of the bird as a function of time and want to find its instantaneous velocity at a particular time.

SET UP: The instantaneous velocity is $v_x(t) = \dfrac{dx}{dt} = \dfrac{d\left[28.0\text{ m} + (12.4\text{ m/s})t - (0.0450\text{ m/s}^3)t^3\right]}{dt}$.

EXECUTE: $v_x(t) = \dfrac{dx}{dt} = 12.4\text{ m/s} - (0.135\text{ m/s}^3)t^2$. Evaluating this at $t = 8.0\text{ s}$ gives $v_x = 3.76\text{ m/s}$.

EVALUATE: The acceleration is not constant in this case.

2.9. **IDENTIFY:** The average velocity is given by $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. We can find the displacement Δt for each constant velocity time interval. The average speed is the distance traveled divided by the time.

SET UP: For $t = 0$ to $t = 2.0\text{ s}$, $v_x = 2.0\text{ m/s}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $v_x = 3.0\text{ m/s}$. In part (b), $v_x = -3.0\text{ m/s}$ for $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$. When the velocity is constant, $\Delta x = v_x \Delta t$.

EXECUTE: **(a)** For $t = 0$ to $t = 2.0\text{ s}$, $\Delta x = (2.0\text{ m/s})(2.0\text{ s}) = 4.0\text{ m}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $\Delta x = (3.0\text{ m/s})(1.0\text{ s}) = 3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + 3.0\text{ m} = 7.0\text{ m}$. The distance traveled is also 7.0 m. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{7.0\text{ m}}{3.0\text{ s}} = 2.33\text{ m/s}$. The average speed is also 2.33 m/s.

(b) For $t = 2.0\text{ s to }3.0\text{ s}$, $\Delta x = (-3.0\text{ m/s})(1.0\text{ s}) = -3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + (-3.0\text{ m}) = +1.0\text{ m}$. The ball travels 4.0 m in the $+x$-direction and then 3.0 m in the $-x$-direction, so the distance traveled is still 7.0 m. $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{1.0\text{ m}}{3.0\text{ s}} = 0.33\text{ m/s}$. The average speed is $\dfrac{7.00\text{ m}}{3.00\text{ s}} = 2.33\text{ m/s}$.

EVALUATE: When the motion is always in the same direction, the displacement and the distance traveled are equal and the average velocity has the same magnitude as the average speed. When the motion changes direction during the time interval, those quantities are different.

2.10. **IDENTIFY** and **SET UP:** The instantaneous velocity is the slope of the tangent to the x versus t graph.

EXECUTE: **(a)** The velocity is zero where the graph is horizontal; point IV.

(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.

(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.

(d) The slope is positive and increasing at point II.

(e) The slope is positive and decreasing at point III.

EVALUATE: The sign of the velocity indicates its direction.

2.11. **IDENTIFY:** Find the instantaneous velocity of a car using a graph of its position as a function of time.

SET UP: The instantaneous velocity at any point is the slope of the x versus t graph at that point. Estimate the slope from the graph.

EXECUTE: A: $v_x = 6.7\text{ m/s}$; B: $v_x = 6.7\text{ m/s}$; C: $v_x = 0$; D: $v_x = -40.0\text{ m/s}$; E: $v_x = -40.0\text{ m/s}$; F: $v_x = -40.0\text{ m/s}$; G: $v_x = 0$.

EVALUATE: The sign of v_x shows the direction the car is moving. v_x is constant when x versus t is a straight line.

2.12. **IDENTIFY:** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. $a_x(t)$ is the slope of the v_x versus t graph.

SET UP: $60\text{ km/h} = 16.7\text{ m/s}$

EXECUTE: **(a)** (i) $a_{\text{av-}x} = \dfrac{16.7\text{ m/s} - 0}{10\text{ s}} = 1.7\text{ m/s}^2$. (ii) $a_{\text{av-}x} = \dfrac{0 - 16.7\text{ m/s}}{10\text{ s}} = -1.7\text{ m/s}^2$.

(iii) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$. (iv) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$.

(b) At $t = 20\text{ s}$, v_x is constant and $a_x = 0$. At $t = 35\text{ s}$, the graph of v_x versus t is a straight line and $a_x = a_{\text{av-}x} = -1.7\text{ m/s}^2$.

EVALUATE: When $a_{\text{av-}x}$ and v_x have the same sign the speed is increasing. When they have opposite signs, the speed is decreasing.

2.13. **IDENTIFY:** The average acceleration for a time interval Δt is given by $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$.

SET UP: Assume the car is moving in the $+x$ direction. $1\text{ mi/h} = 0.447\text{ m/s}$, so $60\text{ mi/h} = 26.82\text{ m/s}$, $200\text{ mi/h} = 89.40\text{ m/s}$ and $253\text{ mi/h} = 113.1\text{ m/s}$.

EXECUTE: **(a)** The graph of v_x versus t is sketched in Figure 2.13. The graph is not a straight line, so the acceleration is not constant.

(b) (i) $a_{\text{av-}x} = \dfrac{26.82\text{ m/s} - 0}{2.1\text{ s}} = 12.8\text{ m/s}^2$ (ii) $a_{\text{av-}x} = \dfrac{89.40\text{ m/s} - 26.82\text{ m/s}}{20.0\text{ s} - 2.1\text{ s}} = 3.50\text{ m/s}^2$

(iii) $a_{\text{av-}x} = \dfrac{113.1\text{ m/s} - 89.40\text{ m/s}}{53\text{ s} - 20.0\text{ s}} = 0.718\text{ m/s}^2$. The slope of the graph of v_x versus t decreases as t increases. This is consistent with an average acceleration that decreases in magnitude during each successive time interval.

EVALUATE: The average acceleration depends on the chosen time interval. For the interval between 0 and 53 s, $a_{\text{av-}x} = \dfrac{113.1\text{ m/s} - 0}{53\text{ s}} = 2.13\text{ m/s}^2$.

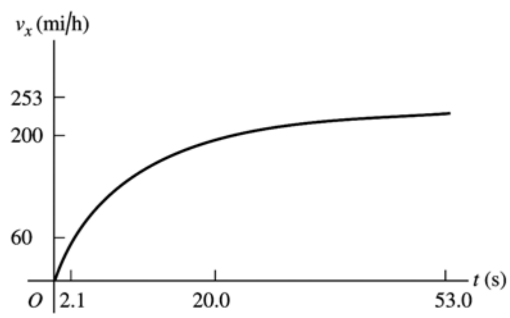


Figure 2.13

2.14. **IDENTIFY:** We know the velocity $v(t)$ of the car as a function of time and want to find its acceleration at the instant that its velocity is 12.0 m/s.

SET UP: We know that $v_x(t) = (0.860\text{ m/s}^3)t^2$ and that $a_x(t) = \dfrac{dv_x}{dt} = \dfrac{d\left[(0.860\text{ m/s}^3)t^2\right]}{dt}$.

EXECUTE: $a_x(t) = \dfrac{dv_x}{dt} = (1.72\text{ m/s}^3)t$. When $v_x = 12.0\text{ m/s}$, $(0.860\text{ m/s}^3)t^2 = 12.0\text{ m/s}$, which gives $t = 3.735\text{ s}$. At this time, $a_x = 6.42\text{ m/s}^2$.

EVALUATE: The acceleration of this car is not constant.

2.15. **IDENTIFY** and **SET UP:** Use $v_x = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

EXECUTE: $v_x = \dfrac{dx}{dt} = 2.00\text{ cm/s} - (0.125\text{ cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125\text{ cm/s}^2$

(a) At $t = 0$, $x = 50.0\text{ cm}$, $v_x = 2.00\text{ cm/s}$, $a_x = -0.125\text{ cm/s}^2$.

(b) Set $v_x = 0$ and solve for t: $t = 16.0\text{ s}$.

(c) Set $x = 50.0\text{ cm}$ and solve for t. This gives $t = 0$ and $t = 32.0\text{ s}$. The turtle returns to the starting point after 32.0 s.

(d) The turtle is 10.0 cm from starting point when $x = 60.0\text{ cm}$ or $x = 40.0\text{ cm}$.

Set $x = 60.0$ cm and solve for t: $t = 6.20$ s and $t = 25.8$ s.
At $t = 6.20$ s, $v_x = +1.23$ cm/s.
At $t = 25.8$ s, $v_x = -1.23$ cm/s.
Set $x = 40.0$ cm and solve for t: $t = 36.4$ s (other root to the quadratic equation is negative and hence nonphysical).
At $t = 36.4$ s, $v_x = -2.55$ cm/s.
(e) The graphs are sketched in Figure 2.15.

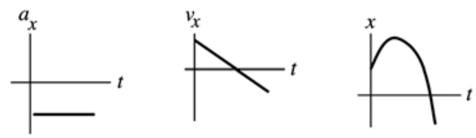


Figure 2.15

EVALUATE: The acceleration is constant and negative. v_x is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$-direction.

2.16. **IDENTIFY:** Use $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$, with $\Delta t = 10$ s in all cases.

SET UP: v_x is negative if the motion is to the left.

EXECUTE: **(a)** $[(5.0\text{ m/s}) - (15.0\text{ m/s})]/(10\text{ s}) = -1.0\text{ m/s}^2$

(b) $[(-15.0\text{ m/s}) - (-5.0\text{ m/s})]/(10\text{ s}) = -1.0\text{ m/s}^2$

(c) $[(-15.0\text{ m/s}) - (+15.0\text{ m/s})]/(10\text{ s}) = -3.0\text{ m/s}^2$

EVALUATE: In all cases, the negative acceleration indicates an acceleration to the left.

2.17. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. Use $v_x(t)$ to find v_x at each t. The instantaneous acceleration is $a_x = \dfrac{dv_x}{dt}$.

SET UP: $v_x(0) = 3.00$ m/s and $v_x(5.00\text{ s}) = 5.50$ m/s.

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{5.50\text{ m/s} - 3.00\text{ m/s}}{5.00\text{ s}} = 0.500\text{ m/s}^2$

(b) $a_x = \dfrac{dv_x}{dt} = (0.100\text{ m/s}^3)(2t) = (0.200\text{ m/s}^3)t$. At $t = 0$, $a_x = 0$. At $t = 5.00$ s, $a_x = 1.00\text{ m/s}^2$.

(c) Graphs of $v_x(t)$ and $a_x(t)$ are given in Figure 2.17 (next page).

EVALUATE: $a_x(t)$ is the slope of $v_x(t)$ and increases as t increases. The average acceleration for $t = 0$ to $t = 5.00$ s equals the instantaneous acceleration at the midpoint of the time interval, $t = 2.50$ s, since $a_x(t)$ is a linear function of t.

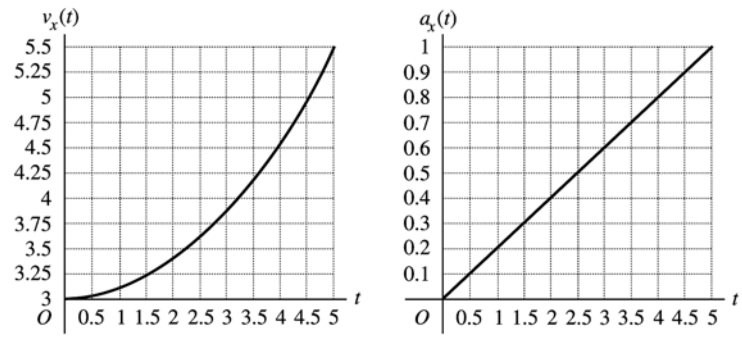


Figure 2.17

2.18. **IDENTIFY:** $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$ for $n \geq 1$.

EXECUTE: **(a)** $v_x(t) = (9.60 \text{ m/s}^2)t - (0.600 \text{ m/s}^6)t^5$ and $a_x(t) = 9.60 \text{ m/s}^2 - (3.00 \text{ m/s}^6)t^4$. Setting $v_x = 0$ gives $t = 0$ and $t = 2.00$ s. At $t = 0$, $x = 2.17$ m and $a_x = 9.60 \text{ m/s}^2$. At $t = 2.00$ s, $x = 15.0$ m and $a_x = -38.4 \text{ m/s}^2$.

(b) The graphs are given in Figure 2.18.

EVALUATE: For the entire time interval from $t = 0$ to $t = 2.00$ s, the velocity v_x is positive and x increases. While a_x is also positive the speed increases and while a_x is negative the speed decreases.

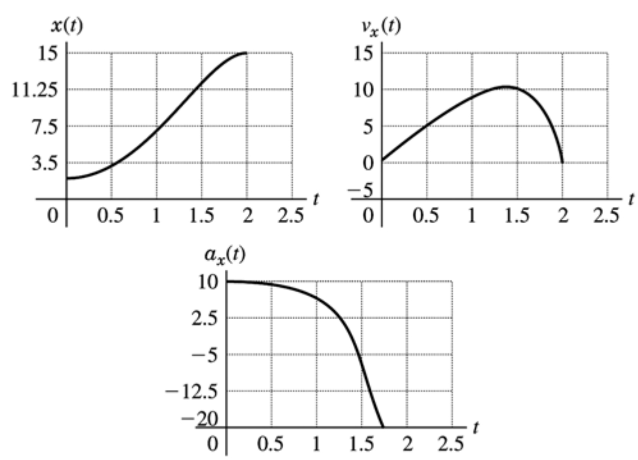
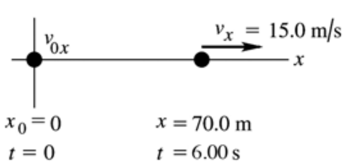


Figure 2.18

2.19. **IDENTIFY:** Use the constant acceleration equations to find v_{0x} and a_x.

(a) SET UP: The situation is sketched in Figure 2.19.



$x - x_0 = 70.0$ m
$t = 6.00$ s
$v_x = 15.0$ m/s
$v_{0x} = ?$

Figure 2.19

EXECUTE: Use $x-x_0=\left(\dfrac{v_{0x}+v_x}{2}\right)t,$ so $v_{0x}=\dfrac{2(x-x_0)}{t}-v_x=\dfrac{2(70.0\text{ m})}{6.00\text{ s}}-15.0\text{ m/s}=8.33\text{ m/s}.$

(b) Use $v_x=v_{0x}+a_xt,$ so $a_x=\dfrac{v_x-v_{0x}}{t}=\dfrac{15.0\text{ m/s}-5.0\text{ m/s}}{6.00\text{ s}}=1.11\text{ m/s}^2.$

EVALUATE: The average velocity is $(70.0\text{ m})/(6.00\text{ s})=11.7\text{ m/s}.$ The final velocity is larger than this, so the antelope must be speeding up during the time interval; $v_{0x}<v_x$ and $a_x>0.$

2.20. **IDENTIFY:** In (a) find the time to reach the speed of sound with an acceleration of 5*g*, and in (b) find his speed at the end of 5.0 s if he has an acceleration of 5*g*.

SET UP: Let $+x$ be in his direction of motion and assume constant acceleration of 5*g* so the standard kinematics equations apply so $v_x=v_{0x}+a_xt.$ **(a)** $v_x=3(331\text{ m/s})=993\text{ m/s},$ $v_{0x}=0,$ and $a_x=5g=49.0\text{ m/s}^2.$ **(b)** $t=5.0\text{ s}$

EXECUTE: **(a)** $v_x=v_{0x}+a_xt$ and $t=\dfrac{v_x-v_{0x}}{a_x}=\dfrac{993\text{ m/s}-0}{49.0\text{ m/s}^2}=20.3\text{ s}.$ Yes, the time required is larger than 5.0 s.

(b) $v_x=v_{0x}+a_xt=0+(49.0\text{ m/s}^2)(5.0\text{ s})=245\text{ m/s}.$

EVALUATE: In 5.0 s he can only reach about 2/3 the speed of sound without blacking out.

2.21. **IDENTIFY:** For constant acceleration, the standard kinematics equations apply.

SET UP: Assume the ball starts from rest and moves in the $+x$-direction.

EXECUTE: **(a)** $x-x_0=1.50\text{ m},$ $v_x=45.0\text{ m/s}$ and $v_{0x}=0.$ $v_x^2=v_{0x}^2+2a_x(x-x_0)$ gives

$$a_x=\frac{v_x^2-v_{0x}^2}{2(x-x_0)}=\frac{(45.0\text{ m/s})^2}{2(1.50\text{ m})}=675\text{ m/s}^2.$$

(b) $x-x_0=\left(\dfrac{v_{0x}+v_x}{2}\right)t$ gives $t=\dfrac{2(x-x_0)}{v_{0x}+v_x}=\dfrac{2(1.50\text{ m})}{45.0\text{ m/s}}=0.0667\text{ s}$

EVALUATE: We could also use $v_x=v_{0x}+a_xt$ to find $t=\dfrac{v_x}{a_x}=\dfrac{45.0\text{ m/s}}{675\text{ m/s}^2}=0.0667\text{ s}$ which agrees with our previous result. The acceleration of the ball is very large.

2.22. **IDENTIFY:** For constant acceleration, the standard kinematics equations apply.

SET UP: Assume the ball moves in the $+x$ direction.

EXECUTE: **(a)** $v_x=73.14\text{ m/s},$ $v_{0x}=0$ and $t=30.0\text{ ms}.$ $v_x=v_{0x}+a_xt$ gives

$$a_x=\frac{v_x-v_{0x}}{t}=\frac{73.14\text{ m/s}-0}{30.0\times10^{-3}\text{ s}}=2440\text{ m/s}^2.$$

(b) $x-x_0=\left(\dfrac{v_{0x}+v_x}{2}\right)t=\left(\dfrac{0+73.14\text{ m/s}}{2}\right)(30.0\times10^{-3}\text{ s})=1.10\text{ m}.$

EVALUATE: We could also use $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ to calculate $x-x_0$:

$x-x_0=\frac{1}{2}(2440\text{ m/s}^2)(30.0\times10^{-3}\text{ s})^2=1.10\text{ m},$ which agrees with our previous result. The acceleration of the ball is very large.

2.23. **IDENTIFY:** Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

SET UP: Let $+x$ be the direction of the initial velocity of the car. $a_x=-250\text{ m/s}^2.$

$105\text{ km/h}=29.17\text{ m/s}.$

EXECUTE: $v_{0x}=29.17\text{ m/s}.$ $v_x=0.$ $v_x^2=v_{0x}^2+2a_x(x-x_0)$ gives

$$x-x_0=\frac{v_x^2-v_{0x}^2}{2a_x}=\frac{0-(29.17\text{ m/s})^2}{2(-250\text{ m/s}^2)}=1.70\text{ m}.$$

EVALUATE: The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

2.24. **IDENTIFY:** In (a) we want the time to reach Mach 4 with an acceleration of 4*g*, and in (b) we want to know how far he can travel if he maintains this acceleration during this time.

SET UP: Let $+x$ be the direction the jet travels and take $x_0 = 0$. With constant acceleration, the equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply. $a_x = 4g = 39.2\ \text{m/s}^2$, $v_x = 4(331\ \text{m/s}) = 1324\ \text{m/s}$, and $v_{0x} = 0$.

EXECUTE: **(a)** Solving $v_x = v_{0x} + a_x t$ for t gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{1324\ \text{m/s} - 0}{39.2\ \text{m/s}^2} = 33.8\ \text{s}$.

(b) $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(39.2\ \text{m/s}^2)(33.8\ \text{s})^2 = 2.24\times 10^4\ \text{m} = 22.4\ \text{km}$.

EVALUATE: The answer in (a) is about ½ min, so if he wanted to reach Mach 4 any sooner than that, he would be in danger of blacking out.

2.25. **IDENTIFY:** If a person comes to a stop in 36 ms while slowing down with an acceleration of 60*g*, how far does he travel during this time?

SET UP: Let $+x$ be the direction the person travels. $v_x = 0$ (he stops), a_x is negative since it is opposite to the direction of the motion, and $t = 36\ \text{ms} = 3.6\times 10^{-2}\ \text{s}$. The equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply since the acceleration is constant.

EXECUTE: Solving $v_x = v_{0x} + a_x t$ for v_{0x} gives $v_{0x} = -a_x t$. Then $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $x = -\frac{1}{2}a_x t^2 = -\frac{1}{2}(-588\ \text{m/s}^2)(3.6\times 10^{-2}\ \text{s})^2 = 38\ \text{cm}$.

EVALUATE: Notice that we were not given the initial speed, but we could find it:

$$v_{0x} = -a_x t = -(-588\ \text{m/s}^2)(36\times 10^{-3}\ \text{s}) = 21\ \text{m/s} = 47\ \text{mph}.$$

2.26. **IDENTIFY:** In (a) the hip pad must reduce the person's speed from 2.0 m/s to 1.3 m/s over a distance of 2.0 cm, and we want the acceleration over this distance, assuming constant acceleration. In (b) we want to find out how long the acceleration in (a) lasts.

SET UP: Let $+y$ be downward. $v_{0y} = 2.0\ \text{m/s}$, $v_y = 1.3\ \text{m/s}$, and $y - y_0 = 0.020\ \text{m}$. The equations $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ apply for constant acceleration.

EXECUTE: **(a)** Solving $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for a_y gives

$a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{(1.3\ \text{m/s})^2 - (2.0\ \text{m/s})^2}{2(0.020\ \text{m})} = -58\ \text{m/s}^2 = -5.9g$.

(b) $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y} + v_y} = \dfrac{2(0.020\ \text{m})}{2.0\ \text{m/s} + 1.3\ \text{m/s}} = 12\ \text{ms}$.

EVALUATE: The acceleration is very large, but it only lasts for 12 ms so it produces a small velocity change.

2.27. **IDENTIFY:** We know the initial and final velocities of the object, and the distance over which the velocity change occurs. From this we want to find the magnitude and duration of the acceleration of the object.

SET UP: The constant-acceleration kinematics formulas apply. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, where $v_{0x} = 0$, $v_x = 5.0\times 10^3\ \text{m/s}$, and $x - x_0 = 4.0\ \text{m}$.

EXECUTE: **(a)** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{(5.0\times 10^3\ \text{m/s})^2}{2(4.0\ \text{m})} = 3.1\times 10^6\ \text{m/s}^2 = 3.2\times 10^5\ g$.

(b) $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{5.0\times 10^3\ \text{m/s}}{3.1\times 10^6\ \text{m/s}^2} = 1.6\ \text{ms}$.

EVALUATE: **(c)** The calculated a is less than 450,000 g so the acceleration required doesn't rule out this hypothesis.

2.28. **IDENTIFY:** Apply constant acceleration equations to the motion of the car.

SET UP: Let $+x$ be the direction the car is moving.

EXECUTE: **(a)** From $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, with $v_{0x} = 0$, $a_x = \dfrac{v_x^2}{2(x - x_0)} = \dfrac{(20\ \text{m/s})^2}{2(120\ \text{m})} = 1.67\ \text{m/s}^2$.

(b) Using Eq. (2.14), $t = 2(x - x_0)/v_x = 2(120\text{ m})/(20\text{ m/s}) = 12\text{ s}$.

(c) $(12\text{ s})(20\text{ m/s}) = 240\text{ m}$.

EVALUATE: The average velocity of the car is half the constant speed of the traffic, so the traffic travels twice as far.

2.29. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. For constant acceleration, the standard kinematics equations apply.

SET UP: Assume the rocket ship travels in the $+x$ direction. $161\text{ km/h} = 44.72\text{ m/s}$ and $1610\text{ km/h} = 447.2\text{ m/s}$. $1.00\text{ min} = 60.0\text{ s}$

EXECUTE: **(a)** (i) $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{44.72\text{ m/s} - 0}{8.00\text{ s}} = 5.59\text{ m/s}^2$

(ii) $a_{\text{av-}x} = \dfrac{447.2\text{ m/s} - 44.72\text{ m/s}}{60.0\text{ s} - 8.00\text{ s}} = 7.74\text{ m/s}^2$

(b) (i) $t = 8.00\text{ s}$, $v_{0x} = 0$, and $v_x = 44.72\text{ m/s}$. $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{0 + 44.72\text{ m/s}}{2}\right)(8.00\text{ s}) = 179\text{ m}$.

(ii) $\Delta t = 60.0\text{ s} - 8.00\text{ s} = 52.0\text{ s}$, $v_{0x} = 44.72\text{ m/s}$, and $v_x = 447.2\text{ m/s}$.

$x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{44.72\text{ m/s} + 447.2\text{ m/s}}{2}\right)(52.0\text{ s}) = 1.28 \times 10^4\text{ m}$.

EVALUATE: When the acceleration is constant the instantaneous acceleration throughout the time interval equals the average acceleration for that time interval. We could have calculated the distance in part (a) as $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(5.59\text{ m/s}^2)(8.00\text{ s})^2 = 179\text{ m}$, which agrees with our previous calculation.

2.30. **IDENTIFY:** The acceleration a_x is the slope of the graph of v_x versus t.

SET UP: The signs of v_x and of a_x indicate their directions.

EXECUTE: **(a)** Reading from the graph, at $t = 4.0\text{ s}$, $v_x = 2.7\text{ cm/s}$, to the right and at $t = 7.0\text{ s}$, $v_x = 1.3\text{ cm/s}$, to the left.

(b) v_x versus t is a straight line with slope $-\dfrac{8.0\text{ cm/s}}{6.0\text{ s}} = -1.3\text{ cm/s}^2$. The acceleration is constant and equal to 1.3 cm/s^2, to the left. It has this value at all times.

(c) Since the acceleration is constant, $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$. For $t = 0$ to 4.5 s, $x - x_0 = (8.0\text{ cm/s})(4.5\text{ s}) + \tfrac{1}{2}(-1.3\text{ cm/s}^2)(4.5\text{ s})^2 = 22.8\text{ cm}$. For $t = 0$ to 7.5 s, $x - x_0 = (8.0\text{ cm/s})(7.5\text{ s}) + \tfrac{1}{2}(-1.3\text{ cm/s}^2)(7.5\text{ s})^2 = 23.4\text{ cm}$

(d) The graphs of a_x and x versus t are given in Figure 2.30.

EVALUATE: In part (c) we could have instead used $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$.

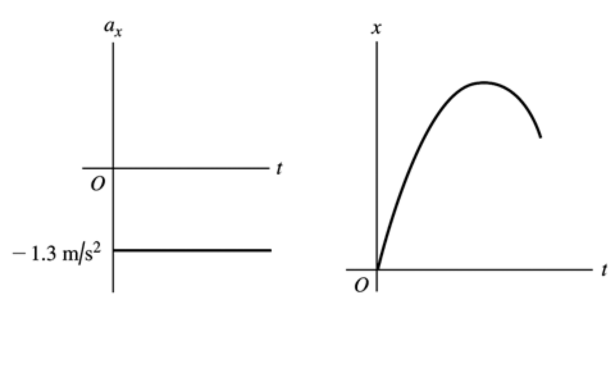


Figure 2.30

2.31. **(a) IDENTIFY** and **SET UP:** The acceleration a_x at time *t* is the slope of the tangent to the v_x versus *t* curve at time *t*.

EXECUTE: At $t = 3$ s, the v_x versus *t* curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7$ s, the v_x versus *t* curve is a straight-line segment with slope $\dfrac{45\text{ m/s} - 20\text{ m/s}}{9\text{ s} - 5\text{ s}} = 6.3\text{ m/s}^2$.

Thus $a_x = 6.3\text{ m/s}^2$.

At $t = 11$ s the curve is again a straight-line segment, now with slope $\dfrac{-0 - 45\text{ m/s}}{13\text{ s} - 9\text{ s}} = -11.2\text{ m/s}^2$.

Thus $a_x = -11.2\text{ m/s}^2$.

EVALUATE: $a_x = 0$ when v_x is constant, $a_x > 0$ when v_x is positive and the speed is increasing, and $a_x < 0$ when v_x is positive and the speed is decreasing.

(b) IDENTIFY: Calculate the displacement during the specified time interval.

SET UP: We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval $t = 0$ to $t = 5$ s the acceleration is constant and equal to zero. For the time interval $t = 5$ s to $t = 9$ s the acceleration is constant and equal to 6.25 m/s^2. For the interval $t = 9$ s to $t = 13$ s the acceleration is constant and equal to -11.2 m/s^2.

EXECUTE: During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

$v_{0x} = 20\text{ m/s}$ $a_x = 0$ $t = 5\text{ s}$ $x - x_0 = ?$

$x - x_0 = v_{0x}t$ ($a_x = 0$ so no $\frac{1}{2}a_x t^2$ term)

$x - x_0 = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$; this is the distance the officer travels in the first 5 seconds.

During the interval $t = 5$ s to 9 s the acceleration is again constant. The constant acceleration formulas can be applied to this 4-second interval. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time $t = 4$ s. (Note that the acceleration is *not* constant over the entire $t = 0$ to $t = 9$ s interval.)

$v_{0x} = 20\text{ m/s}$ $a_x = 6.25\text{ m/s}^2$ $t = 4\text{ s}$ $x_0 = 100\text{ m}$ $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (20\text{ m/s})(4\text{ s}) + \frac{1}{2}(6.25\text{ m/s}^2)(4\text{ s})^2 = 80\text{ m} + 50\text{ m} = 130\text{ m}.$

Thus $x - x_0 + 130\text{ m} = 100\text{ m} + 130\text{ m} = 230\text{ m}.$

At $t = 9$ s the officer is at $x = 230$ m, so she has traveled 230 m in the first 9 seconds.

During the interval $t = 9$ s to $t = 13$ s the acceleration is again constant. The constant acceleration formulas can be applied for this 4-second interval but *not* for the whole $t = 0$ to $t = 13$ s interval. To use the equations restart our clock so this interval begins at time $t = 0$ and ends at time $t = 4$ s.

$v_{0x} = 45\text{ m/s}$ (at the start of this time interval)

$a_x = -11.2\text{ m/s}^2$ $t = 4\text{ s}$ $x_0 = 230\text{ m}$ $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (45\text{ m/s})(4\text{ s}) + \frac{1}{2}(-11.2\text{ m/s}^2)(4\text{ s})^2 = 180\text{ m} - 89.6\text{ m} = 90.4\text{ m}.$

Thus $x = x_0 + 90.4\text{ m} = 230\text{ m} + 90.4\text{ m} = 320\text{ m}.$

At $t = 13$ s the officer is at $x = 320$ m, so she has traveled 320 m in the first 13 seconds.

EVALUATE: The velocity v_x is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval Δt is $v_{\text{av-}x} = \Delta x/\Delta t$. For $t = 0$ to 5 s, $v_{\text{av-}x} = 20\text{ m/s}$. For $t = 0$ to 9 s, $v_{\text{av-}x} = 26\text{ m/s}$. For $t = 0$ to 13 s, $v_{\text{av-}x} = 25\text{ m/s}$. These results are consistent with the figure in the textbook.

2.32. **IDENTIFY:** $v_x(t)$ is the slope of the *x* versus *t* graph. Car *B* moves with constant speed and zero acceleration. Car *A* moves with positive acceleration; assume the acceleration is constant.

SET UP: For car B, v_x is positive and $a_x = 0$. For car A, a_x is positive and v_x increases with t.

EXECUTE: **(a)** The motion diagrams for the cars are given in Figure 2.32a.

(b) The two cars have the same position at times when their x-t graphs cross. The figure in the problem shows this occurs at approximately $t = 1$ s and $t = 3$ s.

(c) The graphs of v_x versus t for each car are sketched in Figure 2.32b.

(d) The cars have the same velocity when their x-t graphs have the same slope. This occurs at approximately $t = 2$ s.

(e) Car A passes car B when x_A moves above x_B in the x-t graph. This happens at $t = 3$ s.

(f) Car B passes car A when x_B moves above x_A in the x-t graph. This happens at $t = 1$ s.

EVALUATE: When $a_x = 0$, the graph of v_x versus t is a horizontal line. When a_x is positive, the graph of v_x versus t is a straight line with positive slope.

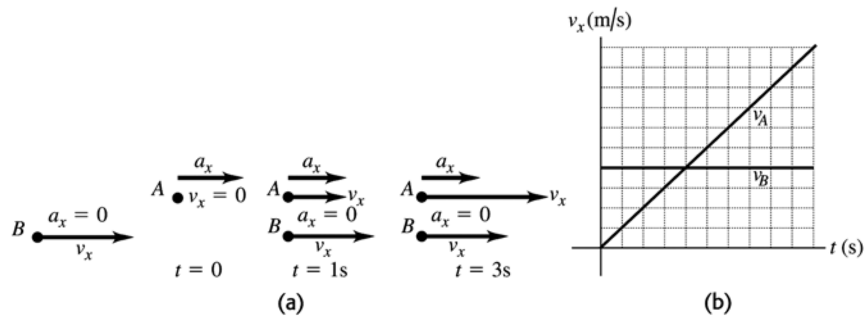


Figure 2.32

2.33. **IDENTIFY:** For constant acceleration, the kinematics formulas apply. We can use the total displacement ande final velocity to calculate the acceleration and then use the acceleration and shorter distance to find the speed.

SET UP: Take $+x$ to be down the incline, so the motion is in the $+x$ direction. The formula $v_x^2 = v_{0x}^2 + 2a(x - x_0)$ applies.

EXECUTE: First look at the motion over 6.80 m. We use the following numbers: $v_{0x} = 0$, $x - x_0 = 6.80$ m, and $v_x = 3.80$ /s. Solving the above equation for a_x gives $a_x = 1.062$ m/s^2. Now look at the motion over the 3.40 m using $v_{0x} = 0$, $a_x = 1.062$ m/s^2 and $x - x_0 = 3.40$ m. Solving the same equation, but this time for v_x, gives $v_x = 2.69$ m/s.

EVALUATE: Even though the block has traveled half way down the incline, its speed is not half of its speed at the bottom.

2.34. **IDENTIFY:** Apply the constant acceleration equations to the motion of each vehicle. The truck passes the car when they are at the same x at the same $t > 0$.

SET UP: The truck has $a_x = 0$. The car has $v_{0x} = 0$. Let $+x$ be in the direction of motion of the vehicles. Both vehicles start at $x_0 = 0$. The car has $a_C = 2.80\ \text{m/s}^2$. The truck has $v_x = 20.0$ m/s.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. Setting $x_T = x_C$ gives $t = 0$ and $v_{0T} = \frac{1}{2}a_C t$, so $t = \dfrac{2v_{0T}}{a_C} = \dfrac{2(20.0\ \text{m/s})}{2.80\ \text{m/s}^2} = 14.29$ s. At this t, $x_T = (20.0\ \text{m/s})(14.29\ \text{s}) = 286$ m and $x = \frac{1}{2}(3.20\ \text{m/s}^2)(14.29\ \text{s})^2 = 286$ m. The car and truck have each traveled 286 m.

(b) At $t = 14.29$ s, the car has $v_x = v_{0x} + a_x t = (2.80\ \text{m/s}^2)(14.29\ \text{s}) = 40$ m/s.

(c) $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. The x-t graph of the motion for each vehicle is sketched in Figure 2.34a.

(d) $v_T = v_{0T}$. $v_C = a_C t$. The v_x-t graph for each vehicle is sketched in Figure 2.34b (next page).

EVALUATE: When the car overtakes the truck its speed is twice that of the truck.

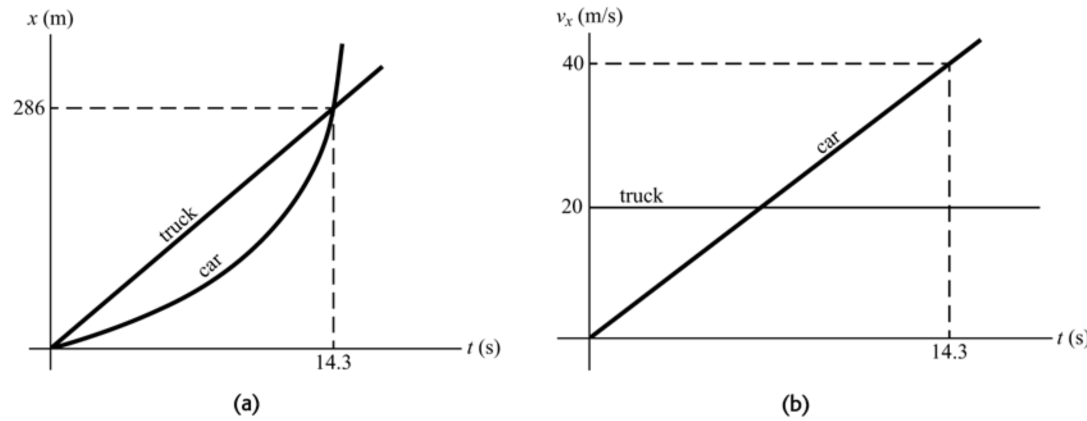


Figure 2.34

2.35. **IDENTIFY:** Apply the constant acceleration equations to the motion of the flea. After the flea leaves the ground, $a_y = g$, downward. Take the origin at the ground and the positive direction to be upward.

(a) SET UP: At the maximum height $v_y = 0$.

$v_y = 0 \quad y - y_0 = 0.440\text{ m} \quad a_y = -9.80\text{ m/s}^2 \quad v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.440\text{ m})} = 2.94\text{ m/s}$

(b) SET UP: When the flea has returned to the ground $y - y_0 = 0$.

$y - y_0 = 0 \quad v_{0y} = +2.94\text{ m/s} \quad a_y = -9.80\text{ m/s}^2 \quad t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(2.94\text{ m/s})}{-9.80\text{ m/s}^2} = 0.600\text{ s}.$

EVALUATE: We can use $v_y = v_{0y} + a_y t$ to show that with $v_{0y} = 2.94$ m/s, $v_y = 0$ after 0.300 s.

2.36. **IDENTIFY:** The rock has a constant downward acceleration of 9.80 m/s^2. We know its initial velocity and position and its final position.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: **(a)** $y - y_0 = -30$ m, $v_{0y} = 22.0$ m/s, $a_y = -9.80\text{ m/s}^2$. The kinematics formulas give

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(22.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-30\text{ m})} = -32.74\text{ m/s}$, so the speed is 32.7 m/s.

(b) $v_y = v_{0y} + a_y t$ and $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-32.74\text{ m/s} - 22.0\text{ m/s}}{-9.80\text{ m/s}^2} = 5.59\text{ s}.$

EVALUATE: The vertical velocity in part (a) is negative because the rock is moving downward, but the speed is always positive. The 5.59 s is the total time in the air.

2.37. **IDENTIFY:** The pin has a constant downward acceleration of 9.80 m/s^2 and returns to its initial position.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: The kinematics formulas give $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$. We know that $y - y_0 = 0$, so

$t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(8.20\text{ m/s})}{-9.80\text{ m/s}^2} = +1.67\text{ s}.$

EVALUATE: It takes the pin half this time to reach its highest point and the remainder of the time to return.

2.38. **IDENTIFY:** The putty has a constant downward acceleration of 9.80 m/s^2. We know the initial velocity of the putty and the distance it travels.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: **(a)** $v_{0y} = 9.50$ m/s and $y - y_0 = 3.60$ m, which gives

$$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(9.50\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(3.60\text{ m})} = 4.44\text{ m/s}$$

(b) $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{4.44\text{ m/s} - 9.50\text{ m/s}}{-9.8\text{ m/s}^2} = 0.517\text{ s}$

EVALUATE: The putty is stopped by the ceiling, not by gravity.

2.39. **IDENTIFY:** A ball on Mars that is hit directly upward returns to the same level in 8.5 s with a constant downward acceleration of $0.379g$. How high did it go and how fast was it initially traveling upward?

SET UP: Take $+y$ upward. $v_y = 0$ at the maximum height. $a_y = -0.379g = -3.71\text{ m/s}^2$. The constant-acceleration formulas $v_y = v_{0y} + a_y t$ and $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ both apply.

EXECUTE: Consider the motion from the maximum height back to the initial level. For this motion $v_{0y} = 0$ and $t = 4.25$ s. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.71\text{ m/s}^2)(4.25\text{ s})^2 = -33.5\text{ m}$. The ball went 33.5 m above its original position.

(b) Consider the motion from just after it was hit to the maximum height. For this motion $v_y = 0$ and $t = 4.25$ s. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = -(-3.71\text{ m/s}^2)(4.25\text{ s}) = 15.8\text{ m/s}$.

(c) The graphs are sketched in Figure 2.39.

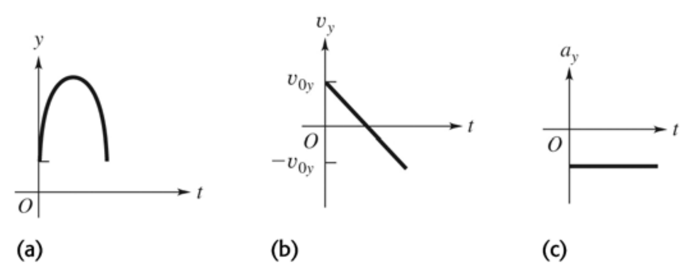


Figure 2.39

EVALUATE: The answers can be checked several ways. For example, $v_y = 0$, $v_{0y} = 15.8\text{ m/s}$, and $a_y = -3.71\text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (15.8\text{ m/s})^2}{2(-3.71\text{ m/s}^2)} = 33.6\text{ m}$, which agrees with the height calculated in (a).

2.40. **IDENTIFY:** Apply constant acceleration equations to the motion of the lander.

SET UP: Let $+y$ be downward. Since the lander is in free-fall, $a_y = +1.6\text{ m/s}^2$.

EXECUTE: $v_{0y} = 0.8\text{ m/s}$, $y - y_0 = 5.0\text{ m}$, $a_y = +1.6\text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8\text{ m/s})^2 + 2(1.6\text{ m/s}^2)(5.0\text{ m})} = 4.1\text{ m/s}.$$

EVALUATE: The same descent on earth would result in a final speed of 9.9 m/s, since the acceleration due to gravity on earth is much larger than on the moon.

2.41. **IDENTIFY:** Apply constant acceleration equations to the motion of the meterstick. The time the meterstick falls is your reaction time.

SET UP: Let $+y$ be downward. The meter stick has $v_{0y} = 0$ and $a_y = 9.80\text{ m/s}^2$. Let d be the distance the meterstick falls.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $d = (4.90\text{ m/s}^2)t^2$ and $t = \sqrt{\dfrac{d}{4.90\text{ m/s}^2}}$.

(b) $t = \sqrt{\dfrac{0.176\text{ m}}{4.90\text{ m/s}^2}} = 0.190\text{ s}$

EVALUATE: The reaction time is proportional to the square of the distance the stick falls.

2.42. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the brick.

SET UP: Let $+y$ be downward. $a_y = 9.80\ \text{m/s}^2$

EXECUTE: **(a)** $v_{0y} = 0, t = 1.90\ \text{s}, a_y = 9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\ \text{m/s}^2)(1.90\ \text{s})^2 = 17.7\ \text{m}$. The building is 17.7 m tall.

(b) $v_y = v_{0y} + a_y t = 0 + (9.80\ \text{m/s}^2)(1.90\ \text{s}) = 18.6\ \text{m/s}$

(c) The graphs of a_y, v_y and y versus t are given in Figure 2.42. Take $y = 0$ at the ground.

EVALUATE: We could use either $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to check our results.

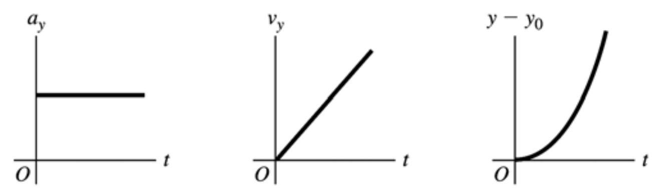


Figure 2.42

2.43. **IDENTIFY:** When the only force is gravity the acceleration is $9.80\ \text{m/s}^2$, downward. There are two intervals of constant acceleration and the constant acceleration equations apply during each of these intervals.

SET UP: Let $+y$ be upward. Let $y = 0$ at the launch pad. The final velocity for the first phase of the motion is the initial velocity for the free-fall phase.

EXECUTE: **(a)** Find the velocity when the engines cut off. $y - y_0 = 525\ \text{m}$, $a_y = 2.25\ \text{m/s}^2$, $v_{0y} = 0$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(2.25\ \text{m/s}^2)(525\ \text{m})} = 48.6\ \text{m/s}$.

Now consider the motion from engine cut-off to maximum height: $y_0 = 525\ \text{m}$, $v_{0y} = +48.6\ \text{m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80\ \text{m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (48.6\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 121\ \text{m} \text{ and } y = 121\ \text{m} + 525\ \text{m} = 646\ \text{m}.$$

(b) Consider the motion from engine failure until just before the rocket strikes the ground: $y - y_0 = -525\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +48.6\ \text{m/s}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(48.6\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-525\ \text{m})} = -112\ \text{m/s}$. Then $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-112\ \text{m/s} - 48.6\ \text{m/s}}{-9.80\ \text{m/s}^2} = 16.4\ \text{s}.$$

(c) Find the time from blast-off until engine failure: $y - y_0 = 525\ \text{m}$, $v_{0y} = 0$, $a_y = +2.25\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(525\ \text{m})}{2.25\ \text{m/s}^2}} = 21.6\ \text{s}$. The rocket strikes the launch pad $21.6\ \text{s} + 16.4\ \text{s} = 38.0\ \text{s}$ after blast-off. The acceleration a_y is $+2.25\ \text{m/s}^2$ from $t = 0$ to $t = 21.6\ \text{s}$. It is $-9.80\ \text{m/s}^2$ from $t = 21.6\ \text{s}$ to $38.0\ \text{s}$. $v_y = v_{0y} + a_y t$ applies during each constant acceleration segment, so the graph of v_y versus t is a straight line with positive slope of $2.25\ \text{m/s}^2$ during the blast-off phase and with negative slope of $-9.80\ \text{m/s}^2$ after engine failure. During each phase $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. The sign of a_y determines the curvature of $y(t)$. At $t = 38.0\ \text{s}$ the rocket has returned to $y = 0$. The graphs are sketched in Figure 2.43.

EVALUATE: In part (b) we could have found the time from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, finding v_y first allows us to avoid solving for *t* from a quadratic equation.

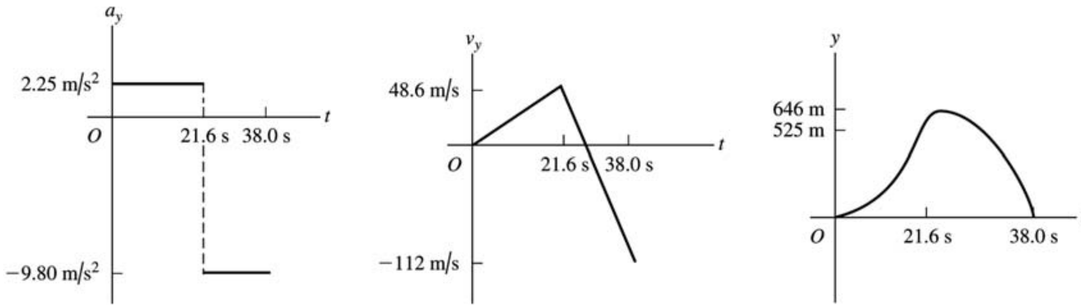


Figure 2.43

2.44. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the sandbag.

SET UP: Take $+y$ upward. $a_y = -9.80\ \text{m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y} = +5.00\ \text{m/s}$. When the balloon reaches the ground, $y - y_0 = -40.0\ \text{m}$. At its maximum height the sandbag has $v_y = 0$.

EXECUTE: (a)

$t = 0.250\ \text{s}$: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (5.00\ \text{m/s})(0.250\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(0.250\ \text{s})^2 = 0.94\ \text{m}$. The sandbag is 40.9 m above the ground. $v_y = v_{0y} + a_y t = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(0.250\ \text{s}) = 2.55\ \text{m/s}$.

$t = 1.00\ \text{s}$: $y - y_0 = (5.00\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = 0.10\ \text{m}$. The sandbag is 40.1 m above the ground. $v_y = v_{0y} + a_y t = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(1.00\ \text{s}) = -4.80\ \text{m/s}$.

(b) $y - y_0 = -40.0\ \text{m}$, $v_{0y} = 5.00\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-40.0\ \text{m} = (5.00\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$. $(4.90\ \text{m/s}^2)t^2 - (5.00\ \text{m/s})t - 40.0\ \text{m} = 0$ and

$t = \dfrac{1}{9.80}\left(5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)}\right)\ \text{s} = (0.51 \pm 2.90)\ \text{s}$. *t* must be positive, so $t = 3.41\ \text{s}$.

(c) $v_y = v_{0y} + a_y t = +5.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(3.41\ \text{s}) = -28.4\ \text{m/s}$

(d) $v_{0y} = 5.00\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (5.00\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 1.28\ \text{m}$. The maximum height is 41.3 m above the ground.

(e) The graphs of a_y, v_y, and *y* versus *t* are given in Figure 2.44. Take $y = 0$ at the ground.

EVALUATE: The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

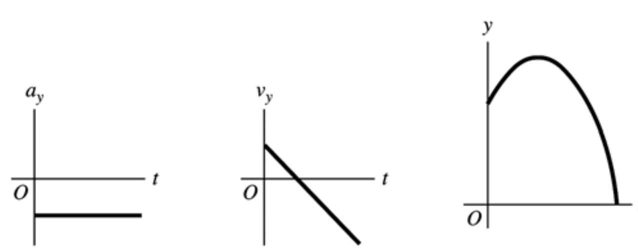


Figure 2.44

2.45. **IDENTIFY:** Use the constant acceleration equations to calculate a_x and $x - x_0$.

(a) SET UP: $v_x = 224\ \text{m/s},\ v_{0x} = 0,\ t = 0.900\ \text{s},\ a_x = ?$

$v_x = v_{0x} + a_x t$

EXECUTE: $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{224\ \text{m/s} - 0}{0.900\ \text{s}} = 249\ \text{m/s}^2$

(b) $a_x/g = (249\ \text{m/s}^2)/(9.80\ \text{m/s}^2) = 25.4$

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(249\ \text{m/s}^2)(0.900\ \text{s})^2 = 101\ \text{m}$

(d) SET UP: Calculate the acceleration, assuming it is constant:

$t = 1.40\ \text{s},\ v_{0x} = 283\ \text{m/s},\ v_x = 0\ \text{(stops)},\ a_x = ?$

$v_x = v_{0x} + a_x t$

EXECUTE: $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{0 - 283\ \text{m/s}}{1.40\,\text{s}} = -202\ \text{m/s}^2$

$a_x/g = (-202\ \text{m/s}^2)/(9.80\ \text{m/s}^2) = -20.6;\ a_x = -20.6g$

If the acceleration while the sled is stopping is constant then the magnitude of the acceleration is only 20.6*g*. But if the acceleration is not constant it is certainly possible that at some point the instantaneous acceleration could be as large as 40*g*.

EVALUATE: It is reasonable that for this motion the acceleration is much larger than *g*.

2.46. **IDENTIFY:** Since air resistance is ignored, the egg is in free-fall and has a constant downward acceleration of magnitude $9.80\ \text{m/s}^2$. Apply the constant acceleration equations to the motion of the egg.

SET UP: Take $+y$ to be upward. At the maximum height, $v_y = 0$.

EXECUTE: **(a)** $y - y_0 = -30.0\ \text{m},\ t = 5.00\ \text{s},\ a_y = -9.80\ \text{m/s}^2.\ y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \tfrac{1}{2}a_y t = \frac{-30.0\ \text{m}}{5.00\ \text{s}} - \tfrac{1}{2}(-9.80\ \text{m/s}^2)(5.00\ \text{s}) = +18.5\ \text{m/s}.$$

(b) $v_{0y} = +18.5\ \text{m/s},\ v_y = 0$ (at the maximum height), $a_y = -9.80\ \text{m/s}^2.\ v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (18.5\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 17.5\ \text{m}.$$

(c) At the maximum height $v_y = 0$.

(d) The acceleration is constant and equal to $9.80\ \text{m/s}^2$, downward, at all points in the motion, including at the maximum height.

(e) The graphs are sketched in Figure 2.46.

EVALUATE: The time for the egg to reach its maximum height is $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-18.5\ \text{m/s}}{-9.8\ \text{m/s}^2} = 1.89\ \text{s}$. The egg has returned to the level of the cornice after 3.78 s and after 5.00 s it has traveled downward from the cornice for 1.22 s.

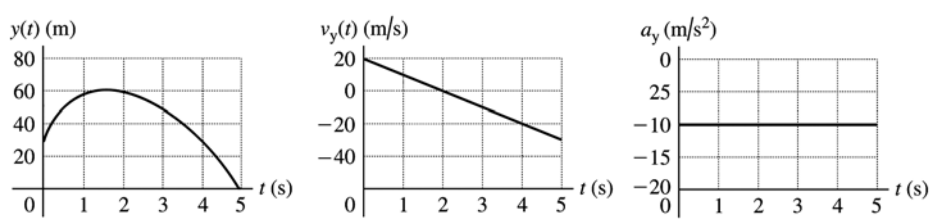


Figure 2.46

2.47. **IDENTIFY:** We can avoid solving for the common height by considering the relation between height, time of fall, and acceleration due to gravity, and setting up a ratio involving time of fall and acceleration due to gravity.

SET UP: Let g_{En} be the acceleration due to gravity on Enceladus and let g be this quantity on earth. Let h be the common height from which the object is dropped. Let $+y$ be downward, so $y - y_0 = h$. $v_{0y} = 0$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}gt_{\text{E}}^2$ and $h = \frac{1}{2}g_{\text{En}}t_{\text{En}}^2$. Combining these two equations gives $gt_{\text{E}}^2 = g_{\text{En}}t_{\text{En}}^2$ and $g_{\text{En}} = g\left(\frac{t_{\text{E}}}{t_{\text{En}}}\right)^2 = (9.80\ \text{m/s}^2)\left(\frac{1.75\ \text{s}}{18.6\ \text{s}}\right)^2 = 0.0868\ \text{m/s}^2$.

EVALUATE: The acceleration due to gravity is inversely proportional to the square of the time of fall.

2.48. **IDENTIFY:** Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude $9.80\ \text{m/s}^2$. Apply the constant acceleration equations to the motion of the boulder.

SET UP: Take $+y$ to be upward.

EXECUTE: **(a)** $v_{0y} = +40.0\ \text{m/s}$, $v_y = +20.0\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$. $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{20.0\ \text{m/s} - 40.0\ \text{m/s}}{-9.80\ \text{m/s}^2} = +2.04\ \text{s}.$$

(b) $v_y = -20.0\ \text{m/s}$. $t = \frac{v_y - v_{0y}}{a_y} = \frac{-20.0\ \text{m/s} - 40.0\ \text{m/s}}{-9.80\ \text{m/s}^2} = +6.12\ \text{s}.$

(c) $y - y_0 = 0$, $v_{0y} = +40.0\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 0$ and

$$t = -\frac{2v_{0y}}{a_y} = -\frac{2(40.0\ \text{m/s})}{-9.80\ \text{m/s}^2} = +8.16\ \text{s}.$$

(d) $v_y = 0$, $v_{0y} = +40.0\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$. $v_y = v_{0y} + a_y t$ gives $t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 40.0\ \text{m/s}}{-9.80\ \text{m/s}^2} = 4.08\ \text{s}.$

(e) The acceleration is $9.80\ \text{m/s}^2$, downward, at all points in the motion.

(f) The graphs are sketched in Figure 2.48.

EVALUATE: $v_y = 0$ at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that $2.04\ \text{s} < 4.08\ \text{s} < 6.12\ \text{s}$. The boulder is going upward until it reaches its maximum height and after the maximum height it is traveling downward.

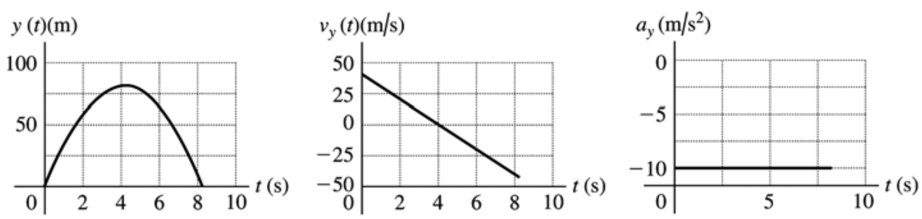


Figure 2.48

2.49. **IDENTIFY:** The rock has a constant downward acceleration of $9.80\ \text{m/s}^2$. The constant-acceleration kinematics formulas apply.

SET UP: The formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply. Call $+y$ upward. First find the initial velocity and then the final speed.

EXECUTE: **(a)** 6.00 s after it is thrown, the rock is back at its original height, so $y = y_0$ at that instant. Using $a_y = -9.80\ \text{m/s}^2$ and $t = 6.00$ s, the equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = 29.4$ m/s. When the rock reaches the water, $y - y_0 = -28.0$ m. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = -37.6$ m/s, so its speed is 37.6 m/s.

EVALUATE: The final speed is greater than the initial speed because the rock accelerated on its way down below the bridge.

2.50. **IDENTIFY:** The acceleration is not constant, so we must use calculus instead of the standard kinematics formulas.

SET UP: The general calculus formulas are $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$. First integrate a_x to find $v(t)$, and then integrate that to find $x(t)$.

EXECUTE: Find $v(t)$: $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -(0.0320\text{ m/s}^3)(15.0\text{ s} - t)dt$. Carrying out the integral and putting in the numbers gives $v_x(t) = 8.00\text{ m/s} - (0.0320\text{ m/s}^3)[(15.0\text{ s})t - t^2/2]$. Now use this result to find $x(t)$.

$x = x_0 + \int_0^t v_x dt = x_0 + \int_0^t \left[8.00\text{ m/s} - (0.0320\text{ m/s}^3)((15.0\text{ s})t - \tfrac{t^2}{2})\right]dt$, which gives

$x = x_0 + (8.00\text{ m/s})t - (0.0320\text{ m/s}^3)[(7.50\text{ s})t^2 - t^3/6)]$. Using $x_0 = -14.0$ m and $t = 10.0$ s, we get $x = 47.3$ m.

EVALUATE: The standard kinematics formulas apply only when the acceleration is constant.

2.51. **IDENTIFY:** The acceleration is not constant, but we know how it varies with time. We can use the definitions of instantaneous velocity and position to find the rocket's position and speed.

SET UP: The basic definitions of velocity and position are $v_y(t) = v_{0y} + \int_0^t a_y dt$ and $y - y_0 = \int_0^t v_y dt$.

EXECUTE: **(a)** $v_y(t) = \int_0^t a_y dt = \int_0^t (2.80\text{ m/s}^3)t dt = (1.40\text{ m/s}^3)t^2$

$y - y_0 = \int_0^t v_y dt = \int_0^t (1.40\text{ m/s}^3)t^2 dt = (0.4667\text{ m/s}^3)t^3$. For $t = 10.0$ s, $y - y_0 = 467$ m.

(b) $y - y_0 = 325$ m so $(0.4667\text{ m/s}^3)t^3 = 325$ m and $t = 8.864$ s. At this time $v_y = (1.40\text{ m/s}^3)(8.864\text{ s})^2 = 110$ m/s.

EVALUATE: The time in part (b) is less than 10.0 s, so the given formulas are valid.

2.52. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$. Use the values of v_x and of x at $t = 1.0$ s to evaluate v_{0x} and x_0.

SET UP: $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$, for $n \geq 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \int_0^t \alpha t dt = v_{0x} + \tfrac{1}{2}\alpha t^2 = v_{0x} + (0.60\text{ m/s}^3)t^2$. $v_x = 5.0$ m/s when $t = 1.0$ s gives $v_{0x} = 4.4$ m/s. Then, at $t = 2.0$ s, $v_x = 4.4\text{ m/s} + (0.60\text{ m/s}^3)(2.0\text{ s})^2 = 6.8$ m/s.

(b) $x = x_0 + \int_0^t (v_{0x} + \tfrac{1}{2}\alpha t^2)dt = x_0 + v_{0x}t + \tfrac{1}{6}\alpha t^3$. $x = 6.0$ m at $t = 1.0$ s gives $x_0 = 1.4$ m. Then, at $t = 2.0$ s, $x = 1.4\text{ m} + (4.4\text{ m/s})(2.0\text{ s}) + \tfrac{1}{6}(1.2\text{ m/s}^3)(2.0\text{ s})^3 = 11.8$ m.

(c) $x(t) = 1.4\text{ m} + (4.4\text{ m/s})t + (0.20\text{ m/s}^3)t^3$. $v_x(t) = 4.4\text{ m/s} + (0.60\text{ m/s}^3)t^2$. $a_x(t) = (1.20\text{ m/s}^3)t$. The graphs are sketched in Figure 2.52.

EVALUATE: We can verify that $a_x = \dfrac{dv_x}{dt}$ and $v_x = \dfrac{dx}{dt}$.

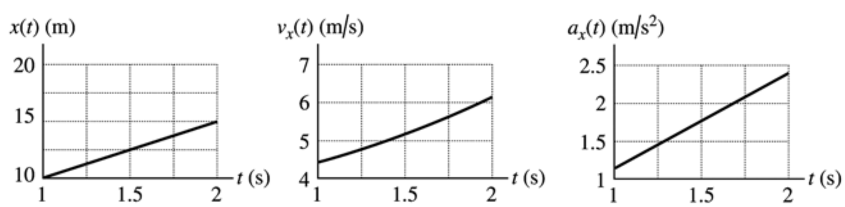


Figure 2.52

2.53. **(a) IDENTIFY:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

SET UP: $v_x = v_{0x} + \int_0^t a_x \, dt$, $a_x = At - Bt^2$ with $A = 1.50 \text{ m/s}^3$ and $B = 0.120 \text{ m/s}^4$.

EXECUTE: $v_x = v_{0x} + \int_0^t (At - Bt^2)\, dt = v_{0x} + \frac{1}{2}At^2 - \frac{1}{3}Bt^3$

At rest at $t = 0$ says that $v_{0x} = 0$, so

$v_x = \frac{1}{2}At^2 - \frac{1}{3}Bt^3 = \frac{1}{2}(1.50 \text{ m/s}^3)t^2 - \frac{1}{3}(0.120 \text{ m/s}^4)t^3$

$v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$

SET UP: $x - x_0 + \int_0^t v_x \, dt$

EXECUTE: $x = x_0 + \int_0^t (\frac{1}{2}At^2 - \frac{1}{3}Bt^3)\, dt = x_0 + \frac{1}{6}At^3 - \frac{1}{12}Bt^4$

At the origin at $t = 0$ says that $x_0 = 0$, so

$x = \frac{1}{6}At^3 - \frac{1}{12}Bt^4 = \frac{1}{6}(1.50 \text{ m/s}^3)t^3 - \frac{1}{12}(0.120 \text{ m/s}^4)t^4$

$x = (0.25 \text{ m/s}^3)t^3 - (0.010 \text{ m/s}^4)t^4$

EVALUATE: We can check our results by using them to verify that $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$.

(b) IDENTIFY and **SET UP:** At time *t*, when v_x is a maximum, $\dfrac{dv_x}{dt} = 0$. (Since $a_x = \dfrac{dv_x}{dt}$, the maximum velocity is when $a_x = 0$. For earlier times a_x is positive so v_x is still increasing. For later times a_x is negative and v_x is decreasing.)

EXECUTE: $a_x = \dfrac{dv_x}{dt} = 0$ so $At - Bt^2 = 0$

One root is $t = 0$, but at this time $v_x = 0$ and not a maximum.

The other root is $t = \dfrac{A}{B} = \dfrac{1.50 \text{ m/s}^3}{0.120 \text{ m/s}^4} = 12.5 \text{ s}$

At this time $v_x = (0.75 \text{ m/s}^3)t^2 - (0.040 \text{ m/s}^4)t^3$ gives

$v_x = (0.75 \text{ m/s}^3)(12.5 \text{ s})^2 - (0.040 \text{ m/s}^4)(12.5 \text{ s})^3 = 117.2 \text{ m/s} - 78.1 \text{ m/s} = 39.1 \text{ m/s}.$

EVALUATE: For $t < 12.5$ s, $a_x > 0$ and v_x is increasing. For $t > 12.5$ s, $a_x < 0$ and v_x is decreasing.

2.54. **IDENTIFY:** $a(t)$ is the slope of the *v* versus *t* graph and the distance traveled is the area under the *v* versus *t* graph.

SET UP: The *v* versus *t* graph can be approximated by the graph sketched in Figure 2.54 (next page).

EXECUTE: **(a)** $\text{Slope} = a = 0$ for $t \geq 1.3$ ms.

(b) $h_{\text{max}} = \text{Area under } v\text{-}t \text{ graph} \approx A_{\text{Triangle}} + A_{\text{Rectangle}} \approx \frac{1}{2}(1.3 \text{ ms})(133 \text{ cm/s}) + (2.5 \text{ ms} - 1.3 \text{ ms})(133 \text{ cm/s}) \approx$ 0.25 cm

(c) $a = \text{slope of } v\text{-}t$ graph. $a(0.5 \text{ ms}) \approx a(1.0 \text{ ms}) \approx \dfrac{133 \text{ cm/s}}{1.3 \text{ ms}} = 1.0 \times 10^5 \text{ cm/s}^2.$

$a(1.5 \text{ ms}) = 0$ because the slope is zero.

(d) $h = \text{area under } v\text{-}t$ graph. $h(0.5 \text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(0.5 \text{ ms})(33 \text{ cm/s}) = 8.3 \times 10^{-3} \text{ cm}.$

$h(1.0 \text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(1.0 \text{ ms})(100 \text{ cm/s}) = 5.0 \times 10^{-2} \text{ cm}.$

$h(1.5 \text{ ms}) \approx A_{\text{Triangle}} + A_{\text{Rectangle}} = \frac{1}{2}(1.3 \text{ ms})(133 \text{ cm/s}) + (0.2 \text{ ms})(133 \text{ cm/s}) = 0.11 \text{ cm}.$

EVALUATE: The acceleration is constant until $t = 1.3$ ms, and then it is zero. $g = 980\ \text{cm/s}^2$. The acceleration during the first 1.3 ms is much larger than this and gravity can be neglected for the portion of the jump that we are considering.

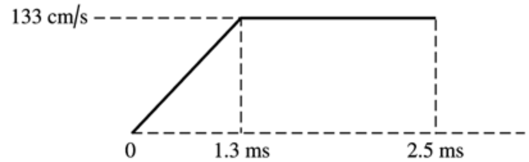


Figure 2.54

2.55. **IDENTIFY:** The sprinter's acceleration is constant for the first 2.0 s but zero after that, so it is not constant over the entire race. We need to break up the race into segments.

SET UP: When the acceleration is constant, the formula $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ applies. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

EXECUTE: **(a)** $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{0 + 10.0\ \text{m/s}}{2}\right)(2.0\ \text{s}) = 10.0\ \text{m}.$

(b) (i) 40.0 m at 10.0 m/s so time at constant speed is 4.0 s. The total time is 6.0 s, so $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{50.0\ \text{m}}{6.0\ \text{s}} = 8.33\ \text{m/s}.$

(ii) He runs 90.0 m at 10.0 m/s so the time at constant speed is 9.0 s. The total time is 11.0 s, so $v_{\text{av-}x} = \dfrac{100\ \text{m}}{11.0\ \text{s}} = 9.09\ \text{m/s}.$

(iii) He runs 190 m at 10.0 m/s so time at constant speed is 19.0 s. His total time is 21.0 s, so $v_{\text{av-}x} = \dfrac{200\ \text{m}}{21.0\ \text{s}} = 9.52\ \text{m/s}.$

EVALUATE: His average velocity keeps increasing because he is running more and more of the race at his top speed.

2.56. **IDENTIFY:** We know the vertical position of the lander as a function of time and want to use this to find its velocity initially and just before it hits the lunar surface.

SET UP: By definition, $v_y(t) = \dfrac{dy}{dt}$, so we can find v_y as a function of time and then evaluate it for the desired cases.

EXECUTE: **(a)** $v_y(t) = \dfrac{dy}{dt} = -c + 2dt$. At $t = 0$, $v_y(t) = -c = -60.0\ \text{m/s}$. The initial velocity is 60.0 m/s downward.

(b) $y(t) = 0$ says $b - ct + dt^2 = 0$. The quadratic formula says $t = 28.57\ \text{s}\ \pm 7.38\ \text{s}$. It reaches the surface at $t = 21.19$ s. At this time, $v_y = -60.0\ \text{m/s} + 2(1.05\ \text{m/s}^2)(21.19\ \text{s}) = -15.5\ \text{m/s}.$

EVALUATE: The given formula for $y(t)$ is of the form $y = y_0 + v_{0y}t + \frac{1}{2}at^2$. For part (a), $v_{0y} = -c = -60$ m/s.

2.57. **IDENTIFY:** In time t_S the S-waves travel a distance $d = v_S t_S$ and in time t_P the P-waves travel a distance $d = v_P t_P$.

SET UP: $t_S = t_P + 33\ \text{s}$

EXECUTE: $\dfrac{d}{v_S} = \dfrac{d}{v_P} + 33\ \text{s}.\ d\left(\dfrac{1}{3.5\ \text{km/s}} - \dfrac{1}{6.5\ \text{km/s}}\right) = 33\ \text{s}$ and $d = 250$ km.

EVALUATE: The times of travel for each wave are $t_S = 71\text{ s}$ and $t_P = 38\text{ s}$.

2.58. **IDENTIFY:** The brick has a constant downward acceleration, so we can use the usual kinematics formulas. We know that it falls 40.0 m in 1.00 s, but we do not know which second that is. We want to find out how far it falls in the next 1.00-s interval.

SET UP: Let the $+y$ direction be downward. The final velocity at the end of the first 1.00-s interval will be the initial velocity for the second 1.00-s interval. $a_y = 9.80\text{ m/s}^2$ and the formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applies.

EXECUTE: **(a)** First find the initial speed at the beginning of the first 1.00-s interval. Applying the above formula with $a_y = 9.80\text{ m/s}^2$, $t = 1.00$ s, and $y - y_0 = 40.0$ m, we get $v_{0y} = 35.1$ m/s. At the end of this 1.00-s interval, the velocity is $v_y = 35.1\text{ m/s} + (9.80\text{ m/s}^2)(1.00\text{ s}) = 44.9\text{ m/s}$. This is v_{0y} for the next 1.00-s interval. Using $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with this initial velocity gives $y - y_0 = 49.8$ m.

EVALUATE: The distance the brick falls during the second 1.00-s interval is greater than during the first 1.00-s interval, which it must be since the brick is accelerating downward.

2.59. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be upward.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{1000\text{ m} - 63\text{ m}}{4.75\text{ s}} = 197\text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{1000\text{ m} - 0}{5.90\text{ s}} = 169\text{ m/s}$

EVALUATE: For the first 1.15 s of the flight, $v_{\text{av-}x} = \dfrac{63\text{ m} - 0}{1.15\text{ s}} = 54.8\text{ m/s}$. When the velocity isn't constant the average velocity depends on the time interval chosen. In this motion the velocity is increasing.

2.60. **IDENTIFY:** Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

SET UP: Let $+x$ be the direction the train is traveling.

EXECUTE: $t = 0$ to 14.0 s: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60\text{ m/s}^2)(14.0\text{ s})^2 = 157\text{ m}$.

At $t = 14.0\text{ s}$, the speed is $v_x = v_{0x} + a_x t = (1.60\text{ m/s}^2)(14.0\text{ s}) = 22.4\text{ m/s}$. In the next 70.0 s, $a_x = 0$ and $x - x_0 = v_{0x}t = (22.4\text{ m/s})(70.0\text{ s}) = 1568\text{ m}$.

For the interval during which the train is slowing down, $v_{0x} = 22.4\text{ m/s}$, $a_x = -3.50\text{ m/s}^2$ and $v_x = 0$.

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (22.4\text{ m/s})^2}{2(-3.50\text{ m/s}^2)} = 72\text{ m}$.

The total distance traveled is $157\text{ m} + 1568\text{ m} + 72\text{ m} = 1800\text{ m}$.

EVALUATE: The acceleration is not constant for the entire motion, but it does consist of constant acceleration segments, and we can use constant acceleration equations for each segment.

2.61. **IDENTIFY:** When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. Since v_x is always positive the motion is always in the $+x$ direction and the total distance moved equals the magnitude of the displacement. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 10.0\text{ s}$ segment, $v_{0x} = 4.00\text{ m/s}$ and $v_x = 12.0\text{ m/s}$. For the $t = 10.0\text{ s}$ to 12.0 s segment, $v_{0x} = 12.0\text{ m/s}$ and $v_x = 0$.

EXECUTE: **(a)** For $t = 0$ to $t = 10.0\text{ s}$, $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{4.00\text{ m/s} + 12.0\text{ m/s}}{2}\right)(10.0\text{ s}) = 80.0\text{ m}$.

For $t = 10.0\text{ s}$ to $t = 12.0\text{ s}$, $x - x_0 = \left(\dfrac{12.0\text{ m/s} + 0}{2}\right)(2.00\text{ s}) = 12.0\text{ m}$. The total distance traveled is 92.0 m.

(b) $x - x_0 = 80.0\text{ m} + 12.0\text{ m} = 92.0\text{ m}$

(c) For $t = 0$ to 10.0 s, $a_x = \dfrac{12.0\text{ m/s} - 4.00\text{ m/s}}{10.0\text{ s}} = 0.800\text{ m/s}^2$. For $t = 10.0\text{ s}$ to 12.0 s, $a_x = \dfrac{0 - 12.0\text{ m/s}}{2.00\text{ s}} = -6.00\text{ m/s}^2$. The graph of a_x versus t is given in Figure 2.61.

EVALUATE: When v_x and a_x are both positive, the speed increases. When v_x is positive and a_x is negative, the speed decreases.

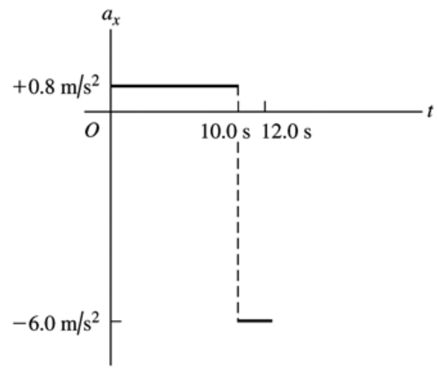


Figure 2.61

2.62. **IDENTIFY:** Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to the motion of each train. A collision means the front of the passenger train is at the same location as the caboose of the freight train at some common time.

SET UP: Let P be the passenger train and F be the freight train. For the front of the passenger train $x_0 = 0$ and for the caboose of the freight train $x_0 = 200\text{ m}$. For the freight train $v_F = 15.0\text{ m/s}$ and $a_F = 0$. For the passenger train $v_P = 25.0\text{ m/s}$ and $a_P = -0.100\text{ m/s}^2$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ for each object gives $x_P = v_P t + \frac{1}{2}a_P t^2$ and $x_F = 200\text{ m} + v_F t$. Setting $x_P = x_F$ gives $v_P t + \frac{1}{2}a_P t^2 = 200\text{ m} + v_F t$. $(0.0500\text{ m/s}^2)t^2 - (10.0\text{ m/s})t + 200\text{ m} = 0$. The quadratic formula gives $t = \dfrac{1}{0.100}\left(+10.0 \pm \sqrt{(10.0)^2 - 4(0.0500)(200)}\right)\text{ s} = (100 \pm 77.5)\text{ s}$. The collision occurs at $t = 100\text{ s} - 77.5\text{ s} = 22.5\text{ s}$. The equations that specify a collision have a physical solution (real, positive *t*), so a collision does occur.

(b) $x_P = (25.0\text{ m/s})(22.5\text{ s}) + \frac{1}{2}(-0.100\text{ m/s}^2)(22.5\text{ s})^2 = 537\text{ m}$. The passenger train moves 537 m before the collision. The freight train moves $(15.0\text{ m/s})(22.5\text{ s}) = 337\text{ m}$.

(c) The graphs of x_F and x_P versus *t* are sketched in Figure 2.62.

EVALUATE: The second root for the equation for *t*, $t = 177.5\text{ s}$ is the time the trains would meet again if they were on parallel tracks and continued their motion after the first meeting.

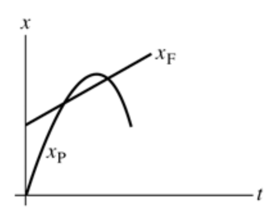


Figure 2.62

2.63. **IDENTIFY** and **SET UP:** Apply constant acceleration kinematics equations.
Find the velocity at the start of the second 5.0 s; this is the velocity at the end of the first 5.0 s. Then find $x - x_0$ for the first 5.0 s.

EXECUTE: For the first 5.0 s of the motion, $v_{0x} = 0$, $t = 5.0$ s.

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0\text{ s})$.

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$v_{0x} = a_x(5.0\text{ s})$, $t = 5.0$ s, $x - x_0 = 200$ m.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $200\text{ m} = (25\text{ s}^2)a_x + (12.5\text{ s}^2)a_x$ so $a_x = 5.333\text{ m/s}^2$.

Use this a_x and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(5.333\text{ m/s}^2)(5.0\text{ s})^2 = 67$ m.

EVALUATE: The ball is speeding up so it travels farther in the second 5.0 s interval than in the first.

2.64. **IDENTIFY:** The insect has constant speed 15 m/s during the time it takes the cars to come together.

SET UP: Each car has moved 100 m when they hit.

EXECUTE: The time until the cars hit is $\dfrac{100\text{ m}}{10\text{ m/s}} = 10$ s. During this time the grasshopper travels a distance of $(15\text{ m/s})(10\text{ s}) = 150$ m.

EVALUATE: The grasshopper ends up 100 m from where it started, so the magnitude of his final displacement is 100 m. This is less than the total distance he travels since he spends part of the time moving in the opposite direction.

2.65. **IDENTIFY:** Apply constant acceleration equations to each object.

Take the origin of coordinates to be at the initial position of the truck, as shown in Figure 2.65a.

Let *d* be the distance that the car initially is behind the truck, so $x_0(\text{car}) = -d$ and $x_0(\text{truck}) = 0$. Let *T* be the time it takes the car to catch the truck. Thus at time *T* the truck has undergone a displacement $x - x_0 = 60.0$ m, so is at $x = x_0 + 60.0\text{ m} = 60.0$ m. The car has caught the truck so at time *T* is also at $x = 60.0$ m.

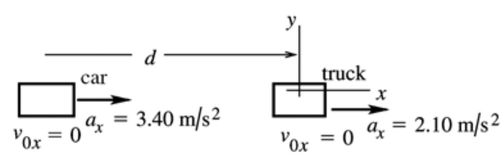


Figure 2.65a

(a) SET UP: Use the motion of the truck to calculate *T*:

$x - x_0 = 60.0$ m, $v_{0x} = 0$ (starts from rest), $a_x = 2.10\text{ m/s}^2$, $t = T$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

Since $v_{0x} = 0$, this gives $t = \sqrt{\dfrac{2(x - x_0)}{a_x}}$

EXECUTE: $T = \sqrt{\dfrac{2(60.0\text{ m})}{2.10\text{ m/s}^2}} = 7.56$ s

(b) SET UP: Use the motion of the car to calculate *d*:

$x - x_0 = 60.0\text{ m} + d$, $v_{0x} = 0$, $a_x = 3.40\text{ m/s}^2$, $t = 7.56$ s

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

EXECUTE: $d + 60.0\text{ m} = \frac{1}{2}(3.40\text{ m/s}^2)(7.56\text{ s})^2$

$d = 97.16\text{ m} - 60.0\text{ m} = 37.2$ m.

(c) car: $v_x = v_{0x} + a_x t = 0 + (3.40\text{ m/s}^2)(7.56\text{ s}) = 25.7$ m/s

truck: $v_x = v_{0x} + a_x t = 0 + (2.10\text{ m/s}^2)(7.56\text{ s}) = 15.9$ m/s

(d) The graph is sketched in Figure 2.65b.

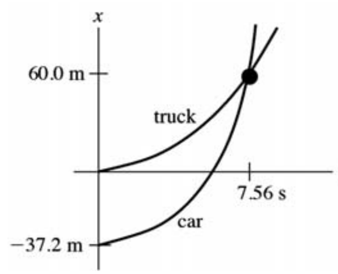


Figure 2.65b

EVALUATE: In part (c) we found that the auto was traveling faster than the truck when they came abreast. The graph in part (d) agrees with this: at the intersection of the two curves the slope of the *x-t* curve for the auto is greater than that of the truck. The auto must have an average velocity greater than that of the truck since it must travel farther in the same time interval.

2.66. **IDENTIFY:** The bus has a constant velocity but you have a constant acceleration, starting from rest.
SET UP: When you catch the bus, you and the bus have been traveling for the same time, but you have traveled an extra 12.0 m during that time interval. The constant-acceleration kinematics formula $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ applies.
EXECUTE: Call d the distance the bus travels after you start running and t the time until you catch the bus. For the bus we have $d = (5.00\text{ m/s})t$, and for you we have $d + 12.0\text{ m} = (1/2)(0.960\text{ m/s}^2)t^2$. Solving these two equations simultaneously, and using the positive root, gives $t = 12.43$ s and $d = 62.14$ m. The distance you must run is 12.0 m + 62.14 m = 74.1 m. Your final speed just as you reach the bus is $v_x = (0.960\text{ m/s}^2)(12.43\text{s}) = 11.9$ m/s. This might be possible for a college runner for a brief time, but it would be highly demanding!
EVALUATE: Note that when you catch the bus, you are moving much faster than it is.

2.67. **IDENTIFY:** Apply constant acceleration equations to each vehicle.
SET UP: **(a)** It is very convenient to work in coordinates attached to the truck.
Note that these coordinates move at constant velocity relative to the earth. In these coordinates the truck is at rest, and the initial velocity of the car is $v_{0x} = 0$. Also, the car's acceleration in these coordinates is the same as in coordinates fixed to the earth.
EXECUTE: First, let's calculate how far the car must travel relative to the truck: The situation is sketched in Figure 2.67.

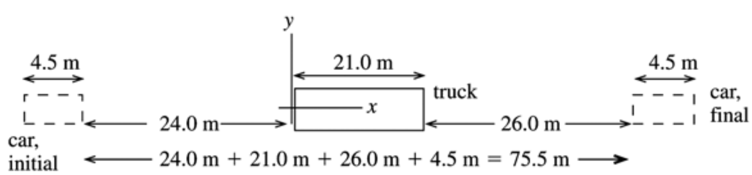


Figure 2.67

The car goes from $x_0 = -24.0$ m to $x = 51.5$ m. So $x - x_0 = 75.5$ m for the car.

Calculate the time it takes the car to travel this distance:

$a_x = 0.600\text{ m/s}^2,\ v_{0x} = 0,\ x - x_0 = 75.5\text{ m},\ t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$$t = \sqrt{\frac{2(x - x_0)}{a_x}} = \sqrt{\frac{2(75.5\text{ m})}{0.600\text{ m/s}^2}} = 15.86\text{ s}$$

It takes the car 15.9 s to pass the truck.

(b) Need how far the car travels relative to the earth, so go now to coordinates fixed to the earth. In these coordinates $v_{0x} = 20.0$ m/s for the car. Take the origin to be at the initial position of the car.

$v_{0x} = 20.0 \text{ m/s},\ a_x = 0.600 \text{ m/s}^2,\ t = 15.86 \text{ s},\ x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (20.0 \text{ m/s})(15.86 \text{ s}) + \frac{1}{2}(0.600 \text{ m/s}^2)(15.86 \text{ s})^2$

$x - x_0 = 317.2 \text{ m} + 75.5 \text{ m} = 393 \text{ m}.$

(c) In coordinates fixed to the earth:

$v_x = v_{0x} + a_x t = 20.0 \text{ m/s} + (0.600 \text{ m/s}^2)(15.86 \text{ s}) = 29.5 \text{ m/s}$

EVALUATE: In 15.86 s the truck travels $x - x_0 = (20.0 \text{ m/s})(15.86 \text{ s}) = 317.2 \text{ m}.$ The car travels $392.7 \text{ m} - 317.2 \text{ m} = 75 \text{ m}$ farther than the truck, which checks with part (a). In coordinates attached to the truck, for the car $v_{0x} = 0,\ v_x = 9.5 \text{ m/s}$ and in 15.86 s the car travels $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 75 \text{ m},$ which checks with part (a).

2.68. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $a_x(t) = \frac{dv_x}{dt}$ and $x = x_0 + \int_0^t v_x(t)dt.$

SET UP: $\int t^n dt = \frac{1}{n+1}t^{n+1}$ for $n \ge 0.$

EXECUTE: **(a)** $x(t) = x_0 + \int_0^t [\alpha - \beta t^2]dt = x_0 + \alpha t - \frac{1}{3}\beta t^3.$ $x = 0$ at $t = 0$ gives $x_0 = 0$ and

$x(t) = \alpha t - \frac{1}{3}\beta t^3 = (4.00 \text{ m/s})t - (0.667 \text{ m/s}^3)t^3.$ $a_x(t) = \frac{dv_x}{dt} = -2\beta t = -(4.00 \text{ m/s}^3)t.$

(b) The maximum positive x is when $v_x = 0$ and $a_x < 0.$ $v_x = 0$ gives $\alpha - \beta t^2 = 0$ and

$t = \sqrt{\frac{\alpha}{\beta}} = \sqrt{\frac{4.00 \text{ m/s}}{2.00 \text{ m/s}^3}} = 1.41 \text{ s}.$ At this t, a_x is negative. For $t = 1.41 \text{ s},$

$x = (4.00 \text{ m/s})(1.41 \text{ s}) - (0.667 \text{ m/s}^3)(1.41 \text{ s})^3 = 3.77 \text{ m}.$

EVALUATE: After $t = 1.41$ s the object starts to move in the $-x$ direction and goes to $x = -\infty$ as $t \to \infty.$

2.69. **(a) IDENTIFY** and **SET UP:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t).$ We know $a_x(t) = \alpha + \beta t,$ with $\alpha = -2.00 \text{ m/s}^2$ and $\beta = 3.00 \text{ m/s}^3.$

EXECUTE: $v_x = v_{0x} + \int_0^t a_x\, dt = v_{0x} + \int_0^t (\alpha + \beta t)\, dt = v_{0x} + \alpha t + \frac{1}{2}\beta t^2$

$x = x_0 + \int_0^t v_x\, dt = x_0 + \int_0^t (v_{0x} + \alpha t + \frac{1}{2}\beta t^2)\, dt = x_0 + v_{0x}t + \frac{1}{2}\alpha t^2 + \frac{1}{6}\beta t^3$

At $t = 0,\ x = x_0.$

To have $x = x_0$ at $t_1 = 4.00$ s requires that $v_{0x}t_1 + \frac{1}{2}\alpha t_1^2 + \frac{1}{6}\beta t_1^3 = 0.$

Thus $v_{0x} = -\frac{1}{6}\beta t_1^2 - \frac{1}{2}\alpha t_1 = -\frac{1}{6}(3.00 \text{ m/s}^3)(4.00 \text{ s})^2 - \frac{1}{2}(-2.00 \text{ m/s}^2)(4.00 \text{ s}) = -4.00 \text{ m/s}.$

(b) With v_{0x} as calculated in part (a) and $t = 4.00$ s,

$v_x = v_{0x} + \alpha t + \frac{1}{2}\beta t^2 = -4.00 \text{ m/s} + (-2.00 \text{ m/s}^2)(4.00 \text{ s}) + \frac{1}{2}(3.00 \text{ m/s}^3)(4.00 \text{ s})^2 = +12.0 \text{ m/s}.$

EVALUATE: $a_x = 0$ at $t = 0.67$ s. For $t > 0.67$ s, $a_x > 0.$ At $t = 0,$ the particle is moving in the $-x$-direction and is speeding up. After $t = 0.67$ s, when the acceleration is positive, the object slows down and then starts to move in the $+x$-direction with increasing speed.

2.70. **IDENTIFY:** Find the distance the professor walks during the time t it takes the egg to fall to the height of his head.

SET UP: Let $+y$ be downward. The egg has $v_{0y} = 0$ and $a_y = 9.80 \text{ m/s}^2.$ At the height of the professor's head, the egg has $y - y_0 = 44.2$ m.

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(44.2 \text{ m})}{9.80 \text{ m/s}^2}} = 3.00 \text{ s}.$ The professor walks a distance $x - x_0 = v_{0x}t = (1.20 \text{ m/s})(3.00 \text{ s}) = 3.60 \text{ m}.$ Release the egg when your professor is 3.60 m from the point directly below you.

EVALUATE: Just before the egg lands its speed is $(9.80\ \text{m/s}^2)(3.00\ \text{s}) = 29.4\ \text{m/s}$. It is traveling much faster than the professor.

2.71. IDENTIFY: Use the constant acceleration equations to establish a relationship between maximum height and acceleration due to gravity and between time in the air and acceleration due to gravity.

SET UP: Let $+y$ be upward. At the maximum height, $v_y = 0$. When the rock returns to the surface, $y - y_0 = 0$.

EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y H = -\frac{1}{2}v_{0y}^2$, which is constant, so $a_\text{E} H_\text{E} = a_\text{M} H_\text{M}$.

$$H_\text{M} = H_\text{E}\left(\frac{a_\text{E}}{a_\text{M}}\right) = H\left(\frac{9.80\ \text{m/s}^2}{3.71\ \text{m/s}^2}\right) = 2.64H.$$

(b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 0$ gives $a_y t = -2v_{0y}$, which is constant, so $a_\text{E} T_\text{E} = a_\text{M} T_\text{M}$.

$$T_\text{M} = T_\text{E}\left[\frac{a_\text{E}}{a_\text{M}}\right] = 2.64T.$$

EVALUATE: On Mars, where the acceleration due to gravity is smaller, the rocks reach a greater height and are in the air for a longer time.

2.72. IDENTIFY: Calculate the time it takes her to run to the table and return. This is the time in the air for the thrown ball. The thrown ball is in free-fall after it is thrown. Assume air resistance can be neglected.

SET UP: For the thrown ball, let $+y$ be upward. $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = 0$ when the ball returns to its original position. The constant-acceleration kinematics formulas apply.

EXECUTE: **(a)** It takes her $\dfrac{5.50\ \text{m}}{3.00\ \text{m/s}} = 1.833\ \text{s}$ to reach the table and an equal time to return, so the total time ball is in the air is 3.667 s. For the ball, $y - y_0 = 0$, $t = 3.667\ \text{s}$ and $a_y = -9.80\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = -\frac{1}{2}a_y t = -\frac{1}{2}(-9.80\ \text{m/s}^2)(3.667\ \text{s}) = 18.0\ \text{m/s}$.

(b) Find $y - y_0$ when $t = 1.833\ \text{s}$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (18.0\ \text{m/s})(1.833\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.833\ \text{s})^2 = 16.5\ \text{m}$.

EVALUATE: It takes the ball the same amount of time to reach its maximum height as to return from its maximum height, so when she is at the table the ball is at its maximum height. Note that this large maximum height requires that the act either be done outdoors, or in a building with a very high ceiling.

2.73. (a) IDENTIFY: Consider the motion from when he applies the acceleration to when the shot leaves his hand.

SET UP: Take positive y to be upward. $v_{0y} = 0$, $v_y = ?$, $a_y = 35.0\ \text{m/s}^2$, $y - y_0 = 0.640\ \text{m}$, $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(35.0\ \text{m/s}^2)(0.640\ \text{m})} = 6.69\ \text{m/s}$

(b) IDENTIFY: Consider the motion of the shot from the point where he releases it to its maximum height, where $v = 0$. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20\ \text{m}$, $y = ?$, $a_y = -9.80\ \text{m/s}^2$ (free fall), $v_{0y} = 6.69\ \text{m/s}$ (from part (a), $v_y = 0$ at maximum height), $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (6.69\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 2.29\ \text{m}$, $y = 2.20\ \text{m} + 2.29\ \text{m} = 4.49\ \text{m}$.

(c) IDENTIFY: Consider the motion of the shot from the point where he releases it to when it returns to the height of his head. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20\ \text{m}$, $y = 1.83\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +6.69\ \text{m/s}$, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $1.83\ \text{m} - 2.20\ \text{m} = (6.69\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2 = (6.69\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$,

$4.90t^2 - 6.69t - 0.37 = 0$, with t in seconds. Use the quadratic formula to solve for t:

$t = \dfrac{1}{9.80}\left(6.69 \pm \sqrt{(6.69)^2 - 4(4.90)(-0.37)}\right) = 0.6830 \pm 0.7362$. Since t must be positive, $t = 0.6830\text{ s} + 0.7362\text{ s} = 1.42\text{ s}$.

EVALUATE: Calculate the time to the maximum height: $v_y = v_{0y} + a_y t$, so $t = (v_y - v_{0y})/a_y = -(6.69\text{ m/s})/(-9.80\text{ m/s}^2) = 0.68\text{ s}$. It also takes 0.68 s to return to 2.2 m above the ground, for a total time of 1.36 s. His head is a little lower than 2.20 m, so it is reasonable for the shot to reach the level of his head a little later than 1.36 s after being thrown; the answer of 1.42 s in part (c) makes sense.

2.74. **IDENTIFY:** The flowerpot is in free-fall. Apply the constant acceleration equations. Use the motion past the window to find the speed of the flowerpot as it reaches the top of the window. Then consider the motion from the windowsill to the top of the window.

SET UP: Let $+y$ be downward. Throughout the motion $a_y = +9.80\text{ m/s}^2$. The constant-acceleration kinematics formulas all apply.

EXECUTE: Motion past the window: $y - y_0 = 1.90\text{ m}$, $t = 0.380\text{ s}$, $a_y = +9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = \dfrac{y - y_0}{t} - \frac{1}{2}a_y t = \dfrac{1.90\text{ m}}{0.380\text{ s}} - \frac{1}{2}(9.80\text{ m/s}^2)(0.380\text{ s}) = 3.138\text{ m/s}$. This is the velocity of the flowerpot when it is at the top of the window.

Motion from the windowsill to the top of the window: $v_{0y} = 0$, $v_y = 2.466\text{ m/s}$, $a_y = +9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{(3.138\text{ m/s})^2 - 0}{2(9.80\text{ m/s}^2)} = 0.502\text{ m}$. The top of the window is 0.502 m below the windowsill.

EVALUATE: It takes the flowerpot $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{3.138\text{ m/s}}{9.80\text{ m/s}^2} = 0.320\text{ s}$ to fall from the sill to the top of the window. Our result says that from the windowsill the pot falls $0.502\text{ m} + 1.90\text{ m} = 2.4\text{ m}$ in $0.320\text{ s} + 0.380\text{ s} = 0.700\text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(0.700\text{ s})^2 = 2.4\text{ m}$, which checks.

2.75. **IDENTIFY:** Two stones are thrown up with different speeds. (a) Knowing how soon the faster one returns to the ground, how long it will take the slow one to return? (b) Knowing how high the slower stone went, how high did the faster stone go?

SET UP: Use subscripts f and s to refer to the faster and slower stones, respectively. Take $+y$ to be upward and $y_0 = 0$ for both stones. $v_{0\text{f}} = 3v_{0\text{s}}$. When a stone reaches the ground, $y = 0$. The constant-acceleration formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply.

EXECUTE: **(a)** $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = -\dfrac{2v_{0y}}{t}$. Since both stones have the same a_y, $\dfrac{v_{0\text{f}}}{t_\text{f}} = \dfrac{v_{0\text{s}}}{t_\text{s}}$ and

$t_\text{s} = t_\text{f}\left(\dfrac{v_{0\text{s}}}{v_{0\text{f}}}\right) = \left(\frac{1}{3}\right)(10\text{ s}) = 3.3\text{ s}$.

(b) Since $v_y = 0$ at the maximum height, then $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = -\dfrac{v_{0y}^2}{2y}$. Since both have the same a_y, $\dfrac{v_{0\text{f}}^2}{y_\text{f}} = \dfrac{v_{0\text{s}}^2}{y_\text{s}}$ and $y_\text{f} = y_\text{s}\left(\dfrac{v_{0\text{f}}}{v_{0\text{s}}}\right)^2 = 9H$.

EVALUATE: The faster stone reaches a greater height so it travels a greater distance than the slower stone and takes more time to return to the ground.

2.76. **IDENTIFY:** The motion of the rocket can be broken into 3 stages, each of which has constant acceleration, so in each stage we can use the standard kinematics formulas for constant acceleration. But the acceleration is not the same throughout all 3 stages.

SET UP: The formulas $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$, $y - y_0 = v_{0y}t + \dfrac{1}{2}a_y t^2$, and $v_y = v_{0y} + a_y t$ apply.

EXECUTE: **(a)** Let $+y$ be upward. At $t = 25.0\ \text{s}$, $y - y_0 = 1094\ \text{m}$ and $v_y = 87.5\ \text{m/s}$. During the next 10.0 s the rocket travels upward an additional distance $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t = \left(\frac{87.5\ \text{m/s} + 132.5\ \text{m/s}}{2}\right)(10.0\ \text{s}) =$ $1100\ \text{m}$. The height above the launch pad when the second stage quits therefore is $1094\ \text{m} + 1100\ \text{m} =$ $2194\ \text{m}$. For the free-fall motion after the second stage quits: $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (132.5\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} = 896\ \text{m}$.

The maximum height above the launch pad that the rocket reaches is $2194\ \text{m} + 896\ \text{m} = 3090\ \text{m}$.

(b) $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $-2194\ \text{m} = (132.5\ \text{m/s})t - (4.9\ \text{m/s}^2)t^2$. From the quadratic formula the positive root is $t = 38.6\ \text{s}$.

(c) $v_y = v_{0y} + a_yt = 132.5\ \text{m/s} + (-9.8\ \text{m/s}^2)(38.6\ \text{s}) = -246\ \text{m/s}$. The rocket's speed will be 246 m/s just before it hits the ground.

EVALUATE: We cannot solve this problem in a single step because the acceleration, while constant in each stage, is not constant over the entire motion. The standard kinematics equations apply to each stage but not to the motion as a whole.

2.77. **IDENTIFY:** The rocket accelerates uniformly upward at 16.0 m/s^2 with the engines on. After the engines are off, it moves upward but accelerates downward at 9.80 m/s^2.

SET UP: The formulas $y - y_0 = v_{0_y} t = \tfrac{1}{2}a_yt^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply to both parts of the motion since the accelerations are both constant, but the accelerations are different in both cases. Let $+y$ be upward.

EXECUTE: With the engines on, $v_{0y} = 0$, $a_y = 16.0$ m/s^2 upward, and $t = T$ at the instant the engines just shut off. Using these quantities, we get

$y - y_0 = v_{0_y} t + \tfrac{1}{2}a_yt^2 = (8.00\ \text{m/s}^2)T^2$ and $v_y = v_{0y} + a_y\, t = (16.0\ \text{m/s}^2)T$.

With the engines off (free fall), the formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for the highest point gives $y - y_0 = (13.06\ \text{m/s}^2)T^2$, using $v_{0y} = (16.0\ \text{m/s}^2)T$, $v_y = 0$, and $a_y = -9.80$ m/s^2.

The total height reached is 960 m, so (distance in free-fall) + (distance with engines on) = 960 m.

Therefore $(13.06\ \text{m/s}^2)\,T^2 + (8.00\ \text{m/s}^2)\,T^2 = 960$ m, which gives $T = 6.75$ s.

EVALUATE: It we put in 6.75 s for T, we see that the rocket travels considerably farther during free fall than with the engines on.

2.78. **IDENTIFY:** The teacher is in free-fall and falls with constant acceleration $9.80\ \text{m/s}^2$, downward. The sound from her shout travels at constant speed. The sound travels from the top of the cliff, reflects from the ground and then travels upward to her present location. If the height of the cliff is h and she falls a distance y in 3.0 s, the sound must travel a distance $h + (h - y)$ in 3.0 s.

SET UP: Let $+y$ be downward, so for the teacher $a_y = 9.80\ \text{m/s}^2$ and $v_{0y} = 0$. Let $y = 0$ at the top of the cliff.

EXECUTE: **(a)** For the teacher, $y = \tfrac{1}{2}(9.80\ \text{m/s}^2)(3.0\ \text{s})^2 = 44.1\ \text{m}$. For the sound, $h + (h - y) = v_st$.

$h = \tfrac{1}{2}(v_st + y) = \tfrac{1}{2}\big([340\ \text{m/s}][3.0\ \text{s}] + 44.1\ \text{m}\big) = 532\ \text{m}$, which rounds to 530 m.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(9.80\ \text{m/s}^2)(532\ \text{m})} = 102\ \text{m/s}$.

EVALUATE: She is in the air for $t = \frac{v_y - v_{0y}}{a_y} = \frac{102\ \text{m/s}}{9.80\ \text{m/s}^2} = 10.4\ \text{s}$ and strikes the ground at high speed.

2.79. **IDENTIFY:** The helicopter has two segments of motion with constant acceleration: upward acceleration for 10.0 s and then free-fall until it returns to the ground. Powers has three segments of motion with constant acceleration: upward acceleration for 10.0 s, free-fall for 7.0 s and then downward acceleration of $2.0\ \text{m/s}^2$.

SET UP: Let $+y$ be upward. Let $y = 0$ at the ground.

EXECUTE: **(a)** When the engine shuts off both objects have upward velocity $v_y = v_{0y} + a_y t =$ $(5.0\text{ m/s}^2)(10.0\text{ s}) = 50.0\text{ m/s}$ and are at $y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(5.0\text{ m/s}^2)(10.0\text{ s})^2 = 250\text{ m}$.

For the helicopter, $v_y = 0$ (at the maximum height), $v_{0y} = +50.0\text{ m/s}$, $y_0 = 250\text{ m}$, and $a_y = -9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y = \dfrac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \dfrac{0-(50.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} + 250\text{ m} = 378\text{ m}$, which rounds to 380 m.

(b) The time for the helicopter to crash from the height of 250 m where the engines shut off can be found using $v_{0y} = +50.0\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, and $y - y_0 = -250\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-250\text{ m} = (50.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. $(4.90\text{ m/s}^2)t^2 - (50.0\text{ m/s})t - 250\text{ m} = 0$. The quadratic formula gives $t = \dfrac{1}{9.80}\left(50.0 \pm \sqrt{(50.0)^2 + 4(4.90)(250)}\right)\text{ s}$. Only the positive solution is physical, so $t = 13.9\text{ s}$.

Powers therefore has free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2 for $13.9\text{ s} - 7.0\text{ s} = 6.9\text{ s}$. After 7.0 s of free-fall he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 250\text{ m} + (50.0\text{ m/s})(7.0\text{ s}) +$ $\frac{1}{2}(-9.80\text{ m/s}^2)(7.0\text{ s})^2 = 360\text{ m}$ and has velocity $v_x = v_{0x} + a_x t = 50.0\text{ m/s} + (-9.80\text{ m/s}^2)(7.0\text{ s}) =$ -18.6 m/s. After the next 6.9 s he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 360\text{ m} + (-18.6\text{ m/s})(6.9\text{ s}) +$ $\frac{1}{2}(-2.00\text{ m/s}^2)(6.9\text{ s})^2 = 184\text{ m}$. Powers is 184 m above the ground when the helicopter crashes.

EVALUATE: When Powers steps out of the helicopter he retains the initial velocity he had in the helicopter but his acceleration changes abruptly from 5.0 m/s^2 upward to 9.80 m/s^2 downward. Without the jet pack he would have crashed into the ground at the same time as the helicopter. The jet pack slows his descent so he is above the ground when the helicopter crashes.

2.80. **IDENTIFY:** Apply constant acceleration equations to the motion of the rock. Sound travels at constant speed.

SET UP: Let t_f be the time for the rock to fall to the ground and let t_s be the time it takes the sound to travel from the impact point back to you. $t_\text{f} + t_\text{s} = 8.00\text{ s}$. Both the rock and sound travel a distance *h* that is equal to the height of the cliff. Take $+y$ downward for the motion of the rock. The rock has $v_{0y} = 0$ and $a_y = g = 9.80\text{ m/s}^2$.

EXECUTE: **(a)** For the falling rock, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}gt_\text{f}^2$. For the sound, $h = v_\text{s}t_\text{s}$. Equating these two equations for *h* and using the fact that $t_\text{f} + t_\text{s} = 8.00\text{ s}$, we get $\frac{1}{2}gt_\text{f}^2 = v_\text{s}t_\text{s} = v_\text{s}(8.00\text{ s} - t_\text{f})$. Using $v_\text{s} = 330\text{ m/s}$ and $g = 9.80\text{ m/s}^2$, we get a quadratic equation. Solving it using the quadratic formula and using the positive square root, we get $t_\text{f} = 7.225\text{ s}$. Therefore $h = \frac{1}{2}gt_\text{f}^2 = (1/2)(9.80\text{ m/s}^2)(7.225\text{ s})^2 = 256\text{ m}$.

(b) Ignoring sound you would calculate $d = \frac{1}{2}(9.80\text{ m/s}^2)(8.00\text{ s})^2 = 314\text{ m}$, which is greater than the actual distance. So you would have overestimated the height of the cliff. It actually takes the rock less time than 8.00 s to fall to the ground.

EVALUATE: Once we know *h* we can calculate that $t_\text{f} = 7.225\text{ s}$ and $t_\text{s} = 0.775\text{ s}$. The time for the sound of impact to travel back to you is 6% of the total time and should not be neglected for best precision.

2.81. **(a) IDENTIFY:** We have nonconstant acceleration, so we must use calculus instead of the standard kinematics formulas.

SET UP: We know the acceleration as a function of time is $a_x(t) = -Ct$, so we can integrate to find the velocity and then the *x*-coordinate of the object. We know that $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x(t)dt$.

EXECUTE: **(a)** We have information about the velocity, so we need to find that by integrating the acceleration. $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -Ctdt = v_{0x} - \frac{1}{2}Ct^2$. Using the facts that the initial velocity is 20.0 m/s and $v_x = 0$ when $t = 8.00\text{ s}$, we have $0 = 20.0\text{ m/s} - C(8.00\text{ s})^2/2$, which gives $C = 0.625\text{ m/s}^3$.

(b) We need the change in position during the first 8.00 s. Using $x(t) = x_0 + \int_0^t v_x(t)dt$ gives

$$x - x_0 = \int_0^t \left(-\tfrac{1}{2}Ct^2 + (20.0\text{ m/s})\right)dt = -Ct^3/6 + (20.0\text{ m/s})t$$

Putting in $C = 0.625\ \text{m/s}^3$ and $t = 8.00$ s gives an answer of 107 m.

EVALUATE: The standard kinematics formulas are of no use in this problem since the acceleration varies with time.

2.82. **IDENTIFY:** Both objects are in free-fall and move with constant acceleration $9.80\ \text{m/s}^2$, downward. The two balls collide when they are at the same height at the same time.

SET UP: Let $+y$ be upward, so $a_y = -9.80\ \text{m/s}^2$ for each ball. Let $y = 0$ at the ground. Let ball A be the one thrown straight up and ball B be the one dropped from rest at height H. $y_{0A} = 0$, $y_{0B} = H$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applied to each ball gives $y_A = v_0 t - \frac{1}{2}gt^2$ and $y_B = H - \frac{1}{2}gt^2$.

$y_A = y_B$ gives $v_0 t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$ and $t = \dfrac{H}{v_0}$.

(b) For ball A at its highest point, $v_{yA} = 0$ and $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_0}{g}$. Setting this equal to the time in part (a) gives $\dfrac{H}{v_0} = \dfrac{v_0}{g}$ and $H = \dfrac{v_0^2}{g}$.

EVALUATE: In part (a), using $t = \dfrac{H}{v_0}$ in the expressions for y_A and y_B gives $y_A = y_B = H\left(1 - \dfrac{gH}{2v_0^2}\right)$.

H must be less than $\dfrac{2v_0^2}{g}$ in order for the balls to collide before ball A returns to the ground. This is because it takes ball A time $t = \dfrac{2v_0}{g}$ to return to the ground and ball B falls a distance $\frac{1}{2}gt^2 = \dfrac{2v_0^2}{g}$ during this time. When $H = \dfrac{2v_0^2}{g}$ the two balls collide just as ball A reaches the ground and for H greater than this ball A reaches the ground before they collide.

2.83. **IDENTIFY** and **SET UP:** Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

EXECUTE: $x_A = \alpha t + \beta t^2$, $v_{Ax} = \dfrac{dx_A}{dt} = \alpha + 2\beta t$, $a_{Ax} = \dfrac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3$, $v_{Bx} = \dfrac{dx_B}{dt} = 2\gamma t - 3\delta t^2$, $a_{Bx} = \dfrac{dv_{Bx}}{dt} = 2\gamma - 6\delta t$

(a) IDENTIFY and **SET UP:** The car that initially moves ahead is the one that has the larger v_{0x}.

EXECUTE: At $t = 0$, $v_{Ax} = \alpha$ and $v_{Bx} = 0$. So initially car A moves ahead.

(b) IDENTIFY and **SET UP:** Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

EXECUTE: One solution is $t = 0$, which says that they start from the same point. To find the other solutions, divide by t: $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{2\delta}\left(-(\beta - \gamma) \pm \sqrt{(\beta - \gamma)^2 - 4\delta\alpha}\right) = \frac{1}{0.40}\left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)}\right) = 4.00\ \text{s} \pm 1.73\ \text{s}$$

So $x_A = x_B$ for $t = 0$, $t = 2.27$ s and $t = 5.73$ s.

EVALUATE: Car A has constant, positive a_x. Its v_x is positive and increasing. Car B has $v_{0x} = 0$ and a_x that is initially positive but then becomes negative. Car B initially moves in the $+x$-direction but then slows down and finally reverses direction. At $t = 2.27$ s car B has overtaken car A and then passes it. At $t = 5.73$ s, car B is moving in the $-x$-direction as it passes car A again.

(c) IDENTIFY: The distance from A to B is $x_B - x_A$. The rate of change of this distance is $\dfrac{d(x_B - x_A)}{dt}$. If this distance is not changing, $\dfrac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between A and B is neither decreasing nor increasing at the instant when they have the same velocity.)

SET UP: $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

EXECUTE: $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{6\delta}\left(-2(\beta-\gamma) \pm \sqrt{4(\beta-\gamma)^2 - 12\delta\alpha}\right) = \frac{1}{1.20}\left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)}\right)$$

$t = 2.667\ \text{s} \pm 1.667\ s$, so $v_{Ax} = v_{Bx}$ for $t = 1.00\ \text{s}$ and $t = 4.33\ \text{s}$.

EVALUATE: At $t = 1.00\ \text{s}$, $v_{Ax} = v_{Bx} = 5.00\ \text{m/s}$. At $t = 4.33\ \text{s}$, $v_{Ax} = v_{Bx} = 13.0\ \text{m/s}$. Now car B is slowing down while A continues to speed up, so their velocities aren't ever equal again.

(d) IDENTIFY and **SET UP:** $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

EXECUTE: $t = \dfrac{\gamma - \beta}{3\delta} = \dfrac{2.80\ \text{m/s}^2 - 1.20\ \text{m/s}^2}{3(0.20\ \text{m/s}^3)} = 2.67\ \text{s}.$

EVALUATE: At $t = 0$, $a_{Bx} > a_{Ax}$, but a_{Bx} is decreasing while a_{Ax} is constant. They are equal at $t = 2.67\ \text{s}$ but for all times after that $a_{Bx} < a_{Ax}$.

2.84. **IDENTIFY:** Interpret the data on a graph to draw conclusions about the motion of a glider having constant acceleration down a frictionless air track, starting from rest at the top.

SET UP: The constant-acceleration kinematics formulas apply. Take the +x-axis along the surface of the track pointing downward.

EXECUTE: **(a)** For constant acceleration starting from rest, we have $x = \frac{1}{2}a_x t^2$. Therefore a plot of x versus t^2 should be a straight line, and the slope of that line should be $a_x/2$.

(b) To construct the graph of x versus t^2, we can use readings from the graph given in the text to construct a table of values for x and t^2, or we could use graphing software if available. The result is a graph similar to the one shown in Figure 2.84, which was obtained using software. A graph done by hand could vary slightly from this one, depending on how one reads the values on the graph in the text. The graph shown is clearly a straight line having slope 3.77 m/s^2 and x-intercept 0.0092 m. Using the slope y-intercept form of the equation of a straight line, the equation of this line is $x = 3.77t^2 + 0.0092$, where x is in meters and t in seconds.

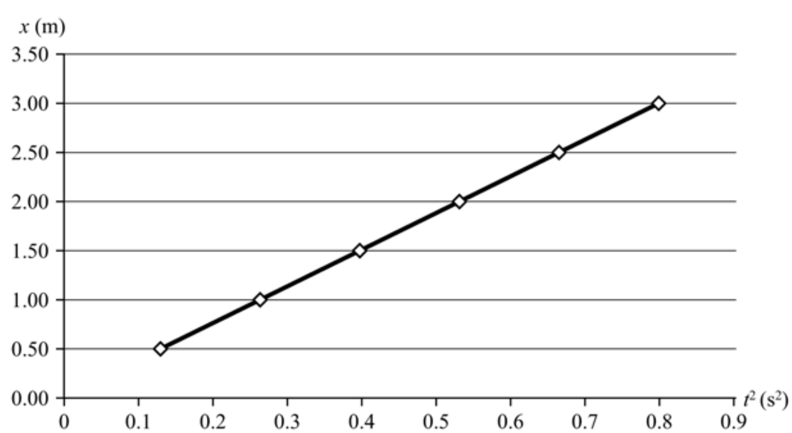


Figure 2.84

(c) The slope of the straight line in the graph is $a_x/2$, so $a_x = 2(3.77\ \text{m/s}^2) = 7.55\ \text{m/s}^2$.
(d) We know the distance traveled is 1.35 m, the acceleration is 7.55 m/s^2, and the initial velocity is zero, so we use the equation $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ and solve for v_x, giving $v_x = 4.51$ m/s.

EVALUATE: For constant acceleration in part (d), the average velocity is (4.51 m/s)/2 = 2.25 m/s. With this average velocity, the time for the glider to travel 1.35 m is x/v_{av} = (1.35 m)/(2.25 m) = 0.6 s, which is approximately the value of t read from the graph in the text for x = 1.35 m.

2.85. **IDENTIFY:** A ball is dropped from rest and falls from various heights with constant acceleration. Interpret a graph of the square of its velocity just as it reaches the floor as a function of its release height.
SET UP: Let $+y$ be downward since all motion is downward. The constant-acceleration kinematics formulas apply for the ball.

EXECUTE: **(a)** The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies to the falling ball. Solving for $y - y_0$ and using $v_{0y} = 0$ and $a_y = g$, we get $y - y_0 = \dfrac{v_y^2}{2g}$. A graph of $y - y_0$ versus v_y^2 will be a straight line with slope $1/2g$ = $1/(19.6\ \text{m/s}^2) = 0.0510\ \text{s}^2/\text{m}$.
(b) With air resistance the acceleration is less than 9.80 m/s^2, so the final speed will be smaller.
(c) The graph will not be a straight line because the acceleration will vary with the speed of the ball. For a given release height, v_y with air resistance is less than without it. Alternatively, with air resistance the ball will have to fall a greater distance to achieve a given velocity than without air resistance. The graph is sketched in Figure 2.85.

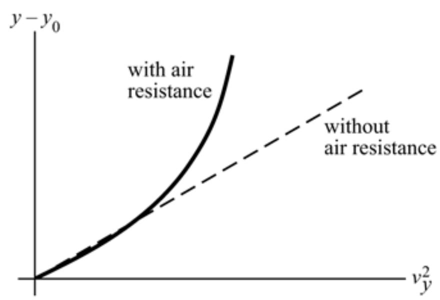


Figure 2.85

EVALUATE: Graphing $y - y_0$ versus v_y^2 for a set of data will tell us if the acceleration is constant. If the graph is a straight line, the acceleration is constant; if not, the acceleration is not constant.

2.86. **IDENTIFY:** Use data of acceleration and time for a model car to find information about its velocity and position.
SET UP: From the table of data in the text, we can see that the acceleration is not constant, so the constant-acceleration kinematics formlas do not apply. Therefore we must use calculus. The equations $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x dt$ apply.

EXECUTE: **(a)** Figure 2.86a shows the graph of a_x versus t. From the graph, we find that the slope of the line is –0.5131 m/s^3 and the a-intercept is 6.026 m/s^2. Using the slope y-intercept equation of a straight line, the equation is $a(t) = -0.513\ \text{m/s}^3\ t + 6.026\ \text{m/s}^2$, where t is in seconds and a is in m/s^2.

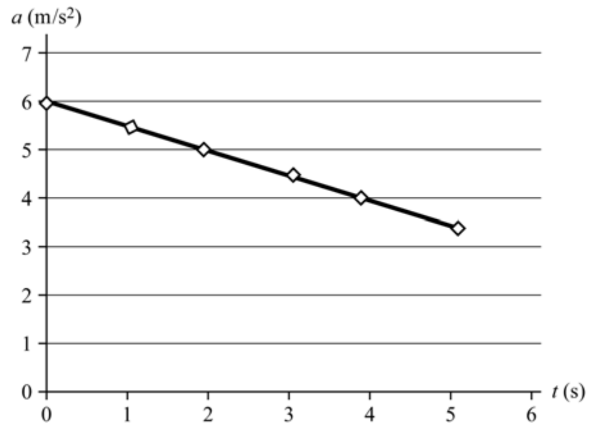


Figure 2.86a

(b) Integrate the acceleration to find the velocity, with the initial velocity equal to zero.
$v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t (6.026 \text{ m/s}^2 - 0.513 \text{ m/s}^3\ t)dt$ $= 6.026 \text{ m/s}^2\ t - 0.2565 \text{ m/s}^3\ t^2$.
Figure 2.86b shows a sketch of the graph of v_x versus t.

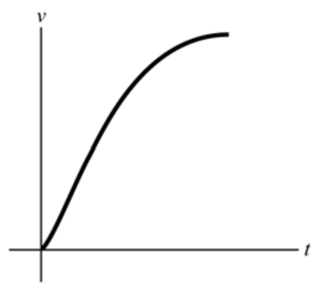


Figure 2.86b

(c) Putting $t = 5.00$ s into the equation we found in (b) gives $v_x = 23.7$ m/s.
(d) Integrate the velocity to find the change in position of the car.
$x - x_0 = \int_0^t v_x dt = \int_0^t [(6.026 \text{ m/s}^2)t - (0.2565 \text{ m/s}^3)t^2]dt$ $= 3.013 \text{ m/s}^2\ t^2 - 0.0855 \text{ m/s}^3\ t^3$
At $t = 5.00$ s, this gives $x - x_0 = 64.6$ m.
EVALUATE: Since the acceleration is not constant, the standard kinematics formulas do not apply, so we must go back to basic definitions involving calculus.

2.87. **IDENTIFY:** Apply $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion from the maximum height, where $v_{0y} = 0$. The time spent above $y_{\text{max}}/2$ on the way down equals the time spent above $y_{\text{max}}/2$ on the way up.

SET UP: Let $+y$ be downward. $a_y = g$. $y - y_0 = y_{\text{max}}/2$ when he is a distance $y_{\text{max}}/2$ above the floor.

EXECUTE: The time from the maximum height to $y_{\text{max}}/2$ above the floor is given by $y_{\text{max}}/2 = \frac{1}{2}gt_1^2$. The time from the maximum height to the floor is given by $y_{\text{max}} = \frac{1}{2}gt_{\text{tot}}^2$ and the time from a height of $y_{\text{max}}/2$ to the floor is $t_2 = t_{\text{tot}} - t_1$.

$$\frac{2t_1}{t_2} = \frac{2\sqrt{y_{\text{max}}/2}}{\sqrt{y_{\text{max}}} - \sqrt{y_{\text{max}}/2}} = \frac{2}{\sqrt{2} - 1} = 4.8.$$

EVALUATE: The person spends over twice as long above $y_{\text{max}}/2$ as below $y_{\text{max}}/2$. His average speed is less above $y_{\text{max}}/2$ than it is when he is below this height.

2.88. **IDENTIFY:** Apply constant acceleration equations to the motion of the two objects, the student and the bus.
SET UP: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then $x_1 = v_0 t$ and $x_2 = x_0 + (1/2)at^2$.

EXECUTE: **(a)** Setting $x_1 = x_2$ and solving for the times t gives $t = \frac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right)$.

$$t = \frac{1}{0.170\ \text{m/s}^2}\left(5.0\ \text{m/s} \pm \sqrt{(5.0\ \text{m/s})^2 - 2(0.170\ \text{m/s}^2)(40.0\ \text{m})}\right) = 9.55\ \text{s and } 49.3\ \text{s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\ \text{m/s})(9.55\ \text{s}) = 47.8\ \text{m}$.

(b) The speed of the bus is $(0.170\ \text{m/s}^2)(9.55\ \text{s}) = 1.62\ \text{m/s}$.

(c) The results can be verified by noting that the x lines for the student and the bus intersect at two points, as shown in Figure 2.88a.

(d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\ \text{m/s}^2)(49.3\ \text{s}) = 8.38\ \text{m/s}$.

(e) No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at $3.5\ \text{m/s}$, Figure 2.88b shows that the two lines do *not* intersect.

(f) For the student to catch the bus, $v_0^2 > 2ax_0$. And so the minimum speed is $\sqrt{2(0.170\ \text{m/s}^2)(40\ \text{m/s})} =$ $3.688\ \text{m/s}$. She would be running for a time $\frac{3.69\ \text{m/s}}{0.170\ \text{m/s}^2} = 21.7\ \text{s}$, and covers a distance $(3.688\ \text{m/s})(21.7\ \text{s}) =$ $80.0\ \text{m}$. However, when the student runs at $3.688\ \text{m/s}$, the lines intersect at *one* point, at $x = 80\ \text{m}$, as shown in Figure 2.88c.

EVALUATE: The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$t_2 = t_{\text{tot}} - t_1$

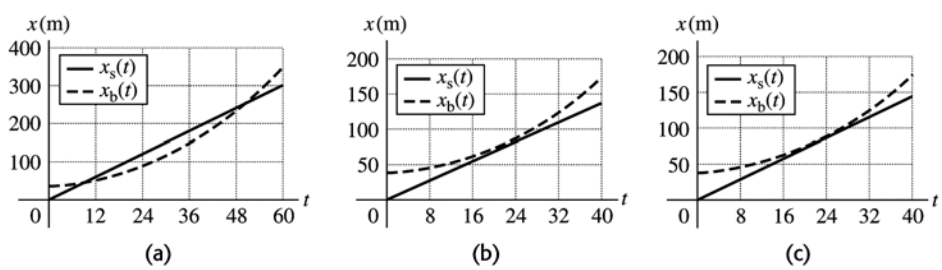


Figure 2.88

2.89. **IDENTIFY:** Apply constant acceleration equations to both objects.
SET UP: Let $+y$ be upward, so each ball has $a_y = -g$. For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is, let $y_1 = h + v_0 t - \frac{1}{2}gt^2$, $y_2 = h - \frac{1}{2}g(t - t_0)^2$. In this case, $t_0 = 1.00\ \text{s}$.

EXECUTE: **(a)** Setting $y_1 = y_2 = 0,$ expanding the binomial $(t-t_0)^2$ and eliminating the common term $\frac{1}{2}gt^2$ yields $v_0 t = gt_0 t - \frac{1}{2}gt_0^2.$ Solving for t: $t = \dfrac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \dfrac{t_0}{2}\left(\dfrac{1}{1 - v_0/(gt_0)}\right).$

Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for h as a function of v_0 yields, after some algebra, $h = \frac{1}{2}gt_0^2 \dfrac{\left(\frac{1}{2}gt_0 - v_0\right)^2}{(gt_0 - v_0)^2}.$ Using the given value $t_0 = 1.00\text{ s}$ and $g = 9.80\text{ m/s}^2,$

$$h = 20.0\text{ m} = (4.9\text{ m})\left(\frac{4.9\text{ m/s} - v_0}{9.8\text{ m/s} - v_0}\right)^2.$$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0, and yields $8.2\text{ m/s}.$ The graph of y versus t for each ball is given in Figure 2.89.

(b) The above expression gives for (i) 0.411 m and for (ii) 1.15 km.

(c) As v_0 approaches $9.8\text{ m/s},$ the height h becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8\text{ m/s},$ the first ball can never catch the second ball.

(d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 < 4.9\text{ m/s},$ the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

EVALUATE: Note that the values of v_0 in parts (a) and (b) are all greater than v_{min} and less than v_{max}.

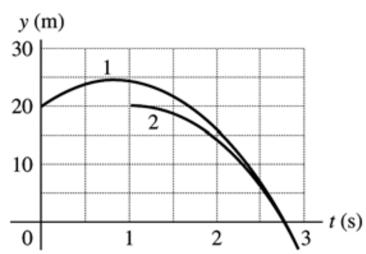


Figure 2.89

2.90. **IDENTIFY:** We know the change in velocity and the time for that change. We can use these quantities to find the average acceleration.

SET UP: The average acceleration is the change in velocity divided by the time for that change.

EXECUTE: $a_{\text{av}} = (v - v_0)/t = (0.80\text{ m/s} - 0)/(250 \times 10^{-3}\text{ s}) = 32\text{ m/s}^2,$ which is choice (c).

EVALUATE: This is about 1/3 the acceleration due to gravity, which is a reasonable acceleration for an organ.

2.91. **IDENTIFY:** The original area is divided into two equal areas. We want the diameter of these two areas, assuming the original and final areas are circular.

SET UP: The area A of a circle or radius r is $A = \pi r^2$ and the diameter d is $d = 2r$. $A_\text{i} = 2A_\text{f}$, and $r = d/2$, where A_f is the area of each of the two arteries.

EXECUTE: Call d the diameter of each artery. $A_\text{i} = \pi(d_\text{a}/2)^2 = 2[\pi(d/2)^2]$, which gives $d = d_\text{a}/\sqrt{2},$ which is choice (b).

EVALUATE: The area of each artery is half the area of the aorta, but the diameters of the arteries are not half the diameter of the aorta.

2.92. **IDENTIFY:** We must interpret a graph of blood velocity during a heartbeat as a function of time.

SET UP: The instantaneous acceleration of a blood molecule is the slope of the velocity-versus-time graph.

EXECUTE: The magnitude of the acceleration is greatest when the slope of the *v*-*t* graph is steepest. That occurs at the upward sloping part of the graph, around $t = 0.10$ s, which makes choice (d) the correct one.
EVALUATE: The slope of the given graph is positive during the first 0.25 s and negative after that. Yet the velocity is positive throughout. Therefore the blood is always flowing forward, but it is increasing in speed during the first 0.25 s and slowing down after that.