

# Chapter 2

## Right Triangle Trigonometry

### 2.1 Definition II: Right Triangle Trigonometry

#### EVEN SOLUTIONS

2. Using Definition II and Figure 8, we would refer to  $a$  as the side opposite  $A$ ,  $b$  as the side adjacent to  $A$ , and  $c$  as the hypotenuse.

4. a. cosine (ii)      b. cosecant (iii)      c. cotangent (i)

6. Using the Pythagorean Theorem, first find  $a$ :

$$a^2 + 8^2 = 17^2$$

$$a^2 + 64 = 289$$

$$a^2 = 225$$

$$a = 15$$

Using  $a = 15$ ,  $b = 8$ , and  $c = 17$ , write the six trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{15}{17}$$

$$\cos A = \frac{b}{c} = \frac{8}{17}$$

$$\tan A = \frac{a}{b} = \frac{15}{8}$$

$$\csc A = \frac{c}{a} = \frac{17}{15}$$

$$\sec A = \frac{c}{b} = \frac{17}{8}$$

$$\cot A = \frac{b}{a} = \frac{8}{15}$$

8. Using the Pythagorean Theorem, first find  $c$ :

$$5^2 + 2^2 = c^2$$

$$25 + 4 = c^2$$

$$c^2 = 29$$

$$c = \sqrt{29}$$

Using  $a = 5$ ,  $b = 2$ , and  $c = \sqrt{29}$ , write the six trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$$

$$\cos A = \frac{b}{c} = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$

$$\tan A = \frac{a}{b} = \frac{5}{2}$$

$$\csc A = \frac{c}{a} = \frac{\sqrt{29}}{5}$$

$$\sec A = \frac{c}{b} = \frac{\sqrt{29}}{2}$$

$$\cot A = \frac{b}{a} = \frac{2}{5}$$

10. Using the Pythagorean Theorem, first find  $c$ :

$$5^2 + (\sqrt{11})^2 = c^2$$

$$25 + 11 = c^2$$

$$c^2 = 36$$

$$c = 6$$

Using  $a = 5$ ,  $b = \sqrt{11}$ , and  $c = 6$ , write the six trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{5}{6}$$

$$\cos A = \frac{b}{c} = \frac{\sqrt{11}}{6}$$

$$\tan A = \frac{a}{b} = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\csc A = \frac{c}{a} = \frac{6}{5}$$

$$\sec A = \frac{c}{b} = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$$

$$\cot A = \frac{b}{a} = \frac{\sqrt{11}}{5}$$

12. Using the Pythagorean Theorem, first find  $a$ :

$$a^2 + 3^2 = 4^2$$

$$a^2 + 9 = 16$$

$$a^2 = 7$$

$$a = \sqrt{7}$$

Using  $a = \sqrt{7}$ ,  $b = 3$ , and  $c = 4$ , find the three trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{\sqrt{7}}{4}$$

$$\cos A = \frac{b}{c} = \frac{3}{4}$$

$$\tan A = \frac{a}{b} = \frac{\sqrt{7}}{3}$$

Now use the Cofunction Theorem to find the three trigonometric functions of  $B$ :

$$\sin B = \cos A = \frac{3}{4}$$

$$\cos B = \sin A = \frac{\sqrt{7}}{4}$$

$$\tan B = \cot A = \frac{b}{a} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

14. Using the Pythagorean Theorem, first find  $c$ :

$$3^2 + 1^2 = c^2$$

$$9 + 1 = c^2$$

$$c^2 = 10$$

$$c = \sqrt{10}$$

Using  $a = 3$ ,  $b = 1$ , and  $c = \sqrt{10}$ , find the three trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos A = \frac{b}{c} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\tan A = \frac{a}{b} = \frac{3}{1} = 3$$

Now use the Cofunction Theorem to find the three trigonometric functions of  $B$ :

$$\sin B = \cos A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos B = \sin A = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan B = \cot A = \frac{b}{a} = \frac{1}{3}$$

16. Using the Pythagorean Theorem, first find  $c$ :

$$1^2 + (\sqrt{5})^2 = c^2$$

$$1 + 5 = c^2$$

$$c^2 = 6$$

$$c = \sqrt{6}$$

Using  $a = 1$ ,  $b = \sqrt{5}$ , and  $c = \sqrt{6}$ , find the three trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

$$\cos A = \frac{b}{c} = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$$

$$\tan A = \frac{a}{b} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Now use the Cofunction Theorem to find the three trigonometric functions of  $B$ :

$$\sin B = \cos A = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{30}}{6}$$

$$\cos B = \sin A = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

$$\tan B = \cot A = \frac{b}{a} = \sqrt{5}$$

18. Using the Pythagorean Theorem, first find  $c$ :

$$x^2 + x^2 = c^2$$

$$c^2 = 2x^2$$

$$c = \sqrt{2}x$$

Using  $a = x$ ,  $b = x$ , and  $c = \sqrt{2}x$ , find the three trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \cos A = \frac{b}{c} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \tan A = \frac{a}{b} = \frac{x}{x} = 1$$

Now use the Cofunction Theorem to find the three trigonometric functions of  $B$ :

$$\sin B = \cos A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \cos B = \sin A = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \tan B = \cot A = \frac{b}{a} = \frac{x}{x} = 1$$

20. The coordinates of point  $B$  are  $B(8, 6)$ . Using the Pythagorean Theorem, first find  $c$ :

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$c^2 = 100$$

$$c = 10$$

Using  $a = 6$ ,  $b = 8$ , and  $c = 10$ , find the three trigonometric functions of  $A$ :

$$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5} \quad \cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5} \quad \tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$$

22. Since  $b \leq c$ ,  $\frac{c}{b} \geq 1$ . Since  $\sec \theta = \frac{c}{b} \geq 1$ , it is impossible for  $\sec \theta = \frac{1}{2}$ .
24. Since  $b \leq c$ ,  $\frac{c}{b} \geq 1$  and can be as large as possible. Since  $\sec \theta = \frac{c}{b}$ ,  $\sec \theta$  can be as large as possible.
26. Using the Cofunction Theorem,  $\cos 70^\circ = \sin 20^\circ$ .
28. Using the Cofunction Theorem,  $\cot 22^\circ = \tan 68^\circ$ .
30. Using the Cofunction Theorem,  $\csc y = \sec(90^\circ - y)$ .
32. Using the Cofunction Theorem,  $\sin(90^\circ - y) = \cos y$ .

34. Complete the table, using the ratio identity  $\sec x = \frac{1}{\cos x}$ :

$x$	$\cos x$	$\sec x$
$0^\circ$	1	1
$30^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{2}{\sqrt{2}} = \sqrt{2}$
$60^\circ$	$\frac{1}{2}$	2
$90^\circ$	0	undefined

36. Simplifying the expression:  $5 \sin^2 60^\circ = 5 \left( \frac{\sqrt{3}}{2} \right)^2 = 5 \cdot \frac{3}{4} = \frac{15}{4}$
38. Simplifying the expression:  $\cos^3 60^\circ = \left( \frac{1}{2} \right)^3 = \frac{1}{8}$
40. Simplifying the expression:  $(\sin 60^\circ + \cos 60^\circ)^2 = \left( \frac{\sqrt{3}}{2} + \frac{1}{2} \right)^2 = \left( \frac{\sqrt{3}+1}{2} \right)^2 = \frac{4+2\sqrt{3}}{4} = \frac{2+\sqrt{3}}{2}$

42. Simplifying the expression:  $(\sin 45^\circ - \cos 45^\circ)^2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)^2 = 0^2 = 0$
44. Simplifying the expression:  $\tan^2 45^\circ + \tan^2 60^\circ = 1^2 + (\sqrt{3})^2 = 1 + 3 = 4$
46. Simplifying the expression:  $6 \cos x = 6 \cos 30^\circ = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$
48. Simplifying the expression:  $-2 \sin(90^\circ - y) = -2 \sin(90^\circ - 45^\circ) = -2 \sin 45^\circ = -2 \cdot \frac{\sqrt{2}}{2} = -\sqrt{2}$
50. Simplifying the expression:  $5 \sin 2y = 5 \sin(2 \cdot 45^\circ) = 5 \sin 90^\circ = 5 \cdot 1 = 5$
52. Simplifying the expression:  $2 \cos(90^\circ - z) = 2 \cos(90^\circ - 60^\circ) = 2 \cos 30^\circ = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$
54. Finding the exact value:  $\csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$
56. Finding the exact value:  $\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2$
58. Finding the exact value:  $\cot 30^\circ = \frac{\cos 30^\circ}{\sin 30^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$
60. Finding the exact value:  $\csc 45^\circ = \frac{1}{\sin 45^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$
62. Finding the exact value:  $\sec 90^\circ = \frac{1}{\cos 90^\circ} = \frac{1}{0}$ , which is undefined
64. Finding the exact value:  $\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0}$ , which is undefined
66. First find  $a$  using the Pythagorean Theorem:
- $$3.68^2 + b^2 = 5.93^2$$
- $$b^2 = 5.93^2 - 3.68^2$$
- $$b^2 = 21.6225$$
- $$b = 4.65$$
- Now find  $\sin A$  and  $\cos A$ :
- $$\sin A = \frac{a}{c} = \frac{3.68}{5.93} \approx 0.62 \qquad \cos A = \frac{b}{c} = \frac{4.65}{5.93} \approx 0.78$$
- Using the Cofunction Theorem:
- $$\sin B = \cos A \approx 0.78 \qquad \cos B = \sin A \approx 0.62$$
68. First find  $c$  using the Pythagorean Theorem:
- $$13.64^2 + 4.77^2 = c^2$$
- $$c^2 = 208.8025$$
- $$c = 14.45$$
- Now find  $\sin A$  and  $\cos A$ :
- $$\sin A = \frac{a}{c} = \frac{13.64}{14.45} \approx 0.94 \qquad \cos A = \frac{b}{c} = \frac{4.77}{14.45} \approx 0.33$$
- Using the Cofunction Theorem:
- $$\sin B = \cos A \approx 0.33 \qquad \cos B = \sin A \approx 0.94$$

70. Since  $CG = CD = 3$ , using the Pythagorean Theorem:

$$(CG)^2 + (CD)^2 = (DG)^2$$

$$3^2 + 3^2 = (DG)^2$$

$$9 + 9 = (DG)^2$$

$$(DG)^2 = 18$$

$$DG = \sqrt{18} = 3\sqrt{2}$$

Now use the Pythagorean Theorem with  $\triangle DGE$ :

$$(DG)^2 + (GE)^2 = (DE)^2$$

$$(3\sqrt{2})^2 + 3^2 = (DE)^2$$

$$18 + 9 = (DE)^2$$

$$(DE)^2 = 27$$

$$DE = \sqrt{27} = 3\sqrt{3}$$

Now, let  $\theta$  represent the angle formed by diagonals  $DE$  and  $DG$ . Therefore:

$$\sin \theta = \frac{GE}{DE} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \cos \theta = \frac{DG}{DE} = \frac{3\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

72. Let  $CG = CD = x$ , using the Pythagorean Theorem:

$$(CG)^2 + (CD)^2 = (DG)^2$$

$$x^2 + x^2 = (DG)^2$$

$$(DG)^2 = 2x^2$$

$$DG = \sqrt{2x^2} = \sqrt{2}x$$

Now use the Pythagorean Theorem with  $\triangle DGE$ :

$$(DG)^2 + (GE)^2 = (DE)^2$$

$$(\sqrt{2}x)^2 + x^2 = (DE)^2$$

$$2x^2 + x^2 = (DE)^2$$

$$(DE)^2 = 3x^2$$

$$DE = \sqrt{3x^2} = \sqrt{3}x$$

Now, let  $\theta$  represent the angle formed by diagonals  $DE$  and  $DG$ . Therefore:

$$\sin \theta = \frac{GE}{DE} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \cos \theta = \frac{DG}{DE} = \frac{\sqrt{2}x}{\sqrt{3}x} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

74. Using the distance formula:

$$\sqrt{(x-1)^2 + (2-5)^2} = (\sqrt{13})^2$$

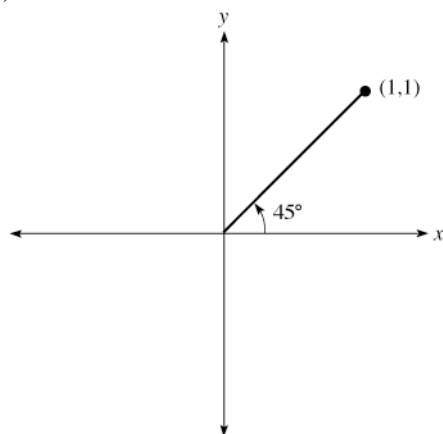
$$(x-1)^2 + 9 = 13$$

$$(x-1)^2 = 4$$

$$x-1 = -2, 2$$

$$x = -1, 3$$

76. A point on the terminal side is  $(1, 1)$ . Drawing the angle in standard position:



78. A coterminal angle to  $-210^\circ$  is  $150^\circ$ .  
 80. Since  $\sin 35^\circ = \cos (90^\circ - 35^\circ) = \cos 55^\circ$ , the correct answer is d.  
 82. Simplifying the expression:  $4 \cos^2 30^\circ + 2 \sin 30^\circ = 4 \left( \frac{\sqrt{3}}{2} \right)^2 + 2 \left( \frac{1}{2} \right) = 4 \cdot \frac{3}{4} + 1 = 3 + 1 = 4$ . The correct answer is c.

### ODD SOLUTIONS

1. triangle measure

$$\begin{aligned} 5. \quad a &= \sqrt{c^2 - b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(5)^2 - (3)^2} && \text{Substitute known values} \\ &= \sqrt{25 - 9} && \text{Simplify} \\ &= \sqrt{16} = 4 \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{4}{5}$$

$$\cos A = \frac{b}{c} = \frac{3}{5}$$

$$\tan A = \frac{a}{b} = \frac{4}{3}$$

$$\cot A = \frac{b}{a} = \frac{3}{4}$$

$$\sec A = \frac{c}{b} = \frac{5}{3}$$

$$\csc A = \frac{c}{a} = \frac{5}{4}$$

$$\begin{aligned} 9. \quad c &= \sqrt{a^2 + b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(2)^2 + (\sqrt{5})^2} && \text{Substitute known values} \\ &= \sqrt{4 + 5} && \text{Simplify} \\ &= \sqrt{9} = 3 \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{2}{3}$$

$$\cos A = \frac{b}{c} = \frac{\sqrt{5}}{3}$$

$$\tan A = \frac{a}{b} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cot A = \frac{b}{a} = \frac{\sqrt{5}}{2}$$

$$\sec A = \frac{c}{b} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\csc A = \frac{c}{a} = \frac{3}{2}$$

3. complement

$$\begin{aligned} 7. \quad c &= \sqrt{a^2 + b^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(2)^2 + (1)^2} && \text{Substitute known values} \\ &= \sqrt{4 + 1} && \text{Simplify} \\ &= \sqrt{5} \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos A = \frac{b}{c} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan A = \frac{a}{b} = \frac{2}{1} = 2$$

$$\cot A = \frac{b}{a} = \frac{1}{2}$$

$$\sec A = \frac{c}{b} = \frac{\sqrt{5}}{1}$$

$$\csc A = \frac{c}{a} = \frac{\sqrt{5}}{2}$$

$$\begin{aligned} 11. \quad b &= \sqrt{c^2 - a^2} && \text{Pythagorean Theorem} \\ &= \sqrt{(6)^2 - (5)^2} && \text{Substitute known values} \\ &= \sqrt{36 - 25} && \text{Simplify} \\ &= \sqrt{11} \end{aligned}$$

$$\sin A = \frac{a}{c} = \frac{5}{6}$$

$$\cos A = \frac{b}{c} = \frac{\sqrt{11}}{6}$$

$$\tan A = \frac{a}{b} = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\sin B = \frac{b}{c} = \frac{\sqrt{11}}{6}$$

$$\cos B = \frac{a}{c} = \frac{5}{6}$$

$$\tan B = \frac{b}{a} = \frac{\sqrt{11}}{5}$$

13.  $c = \sqrt{a^2 + b^2}$  Pythagorean Theorem  $\sin A = \frac{a}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$   $\sin B = \frac{b}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$   
 $= \sqrt{(1)^2 + (1)^2}$  Substitute known values  $\cos A = \frac{b}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$   $\cos B = \frac{a}{c} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$   
 $= \sqrt{1+1}$  Simplify  $\tan A = \frac{a}{b} = \frac{1}{1} = 1$   $\tan B = \frac{b}{a} = \frac{1}{1} = 1$   
 $= \sqrt{2}$
15.  $b = \sqrt{c^2 - a^2}$  Pythagorean Theorem  $\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$   $\sin B = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$   
 $= \sqrt{10^2 - 6^2}$  Substitute known values  $\cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5}$   $\cos B = \frac{a}{c} = \frac{6}{10} = \frac{3}{5}$   
 $= \sqrt{100 - 36}$  Simplify  $\tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4}$   $\tan B = \frac{b}{a} = \frac{8}{6} = \frac{4}{3}$   
 $= \sqrt{64} = 8$
17.  $a = \sqrt{c^2 - b^2}$  Pythagorean Theorem **19.** The coordinates of B are (4, 3).  
 $= \sqrt{(2x)^2 - (x)^2}$  Substitute known values  $a = 3, b = 4, c = 5$   
 $= \sqrt{4x^2 - x^2}$  Simplify  $\sin A = \frac{a}{c} = \frac{3}{5}$   
 $= \sqrt{3x^2}$   $\cos A = \frac{b}{c} = \frac{4}{5}$   
 $= x\sqrt{3}$   $\tan A = \frac{a}{b} = \frac{3}{4}$   
 $\sin A = \frac{a}{c} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$   $\sin B = \frac{b}{c} = \frac{x}{2x} = \frac{1}{2}$   
 $\cos A = \frac{b}{c} = \frac{x}{2x} = \frac{1}{2}$   $\cos B = \frac{a}{c} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$   
 $\tan A = \frac{a}{b} = \frac{x\sqrt{3}}{x} = \sqrt{3}$   $\tan B = \frac{b}{a} = \frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
21.  $\cos \theta = \frac{\text{adj side}}{\text{hyp}} = \frac{3}{1}$  For this to be true, the adjacent side would have to be three times larger than the hypotenuse.  
This is impossible since the hypotenuse is the longest side of a right triangle.
23.  $\tan \theta = \frac{\text{opp side}}{\text{adj side}}$  The opposite and adjacent sides can be any number greater than 0. If we choose a very large number for the opposite side and a very small number for the adjacent side, the ratio will approach infinity.
25.  $\sin 10^\circ = \cos(90^\circ - 10^\circ) = \cos 80^\circ$  **27.**  $\tan 8^\circ = \cot(90^\circ - 8^\circ) = \cot 82^\circ$
29.  $\sin x^\circ = \cos(90^\circ - x^\circ)$  **31.**  $\tan(90^\circ - x^\circ) = \cot x^\circ$
33.  $\csc x = \frac{1}{\sin x}$   $\csc 45^\circ = \frac{1}{1/\sqrt{2}} = \sqrt{2}$   
 $\csc 0^\circ = \frac{1}{0}$  undefined  $\csc 60^\circ = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$   
 $\csc 30^\circ = \frac{1}{1/2} = 2$   $\csc 90^\circ = \frac{1}{1} = 1$
35.  $4 \sin 30^\circ = 4\left(\frac{1}{2}\right) = 2$

$$37. \quad (2 \cos 30^\circ)^2 = \left[ 2 \left( \frac{\sqrt{3}}{2} \right) \right]^2 = (\sqrt{3})^2 = 3$$

$$39. \quad \sin^2 60^\circ + \cos^2 60^\circ = \left( \frac{\sqrt{3}}{2} \right)^2 + \left( \frac{1}{2} \right)^2 \\ = \frac{3}{4} + \frac{1}{4} \\ = \frac{4}{4} \\ = 1$$

$$41. \quad \sin^2 45^\circ - 2 \sin 45^\circ \cos 45^\circ + \cos^2 45^\circ = \left( \frac{\sqrt{2}}{2} \right)^2 - 2 \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{2}}{2} \right) + \left( \frac{\sqrt{2}}{2} \right)^2 \\ = \frac{2}{4} - 2 \left( \frac{2}{4} \right) + \frac{2}{4} = 0$$

$$43. \quad (\tan 45^\circ + \tan 60^\circ)^2 = (1 + \sqrt{3})^2 \\ = (1 + \sqrt{3})(1 + \sqrt{3}) \\ = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$$

$$45. \quad 2 \sin 30^\circ = 2 \left( \frac{1}{2} \right) \\ = 1$$

$$47. \quad 4 \cos(z - 30^\circ) = 4 \cos(60^\circ - 30^\circ) \\ = 4 \cos 30^\circ \\ = 4 \left( \frac{\sqrt{3}}{2} \right) = 2\sqrt{3}$$

$$49. \quad -3 \sin 2(30^\circ) = -3 \sin 60^\circ \\ = -3 \left( \frac{\sqrt{3}}{2} \right) \\ = -\frac{3\sqrt{3}}{2}$$

$$51. \quad 2 \cos(3x - 45^\circ) = 2 \cos(3 \cdot 30^\circ - 45^\circ) \\ = 2 \cos(90^\circ - 45^\circ) \\ = 2 \cos 45^\circ \\ = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$53. \quad \sec 30^\circ = \frac{1}{\cos 30^\circ} \\ = \frac{1}{\sqrt{3}/2} \\ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Reciprocal identity

Substitute exact value from Table 1

Division of fractions

$$55. \quad \csc 60^\circ = \frac{1}{\sin 60^\circ} \\ = \frac{1}{\sqrt{3}/2} \\ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$57. \quad \cot 45^\circ = \frac{\cos 45^\circ}{\sin 45^\circ} \quad \text{Ratio identity}$$

$$= \frac{\sqrt{2}/2}{\sqrt{2}/2} \quad \text{Substitute values from Table 1}$$

$$= 1 \quad \text{Simplify}$$



$$\begin{aligned}
 59. \quad \sec 45^\circ &= \frac{1}{\cos 45^\circ} \\
 &= \frac{1}{1/\sqrt{2}} \\
 &= \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 63. \quad \csc 90^\circ &= \frac{1}{\sin 90^\circ} \\
 &= \frac{1}{1} = 1
 \end{aligned}$$

$$\begin{aligned}
 65. \quad a &= \sqrt{c^2 - b^2} \\
 &= \sqrt{(9.62)^2 - (8.88)^2} \\
 &= \sqrt{13.69} \\
 &= 3.70
 \end{aligned}$$

$$\begin{aligned}
 67. \quad c &= \sqrt{a^2 + b^2} \\
 &= \sqrt{(19.44)^2 + (5.67)^2} \\
 &= \sqrt{410.0625} \\
 &= 20.25
 \end{aligned}$$

$$\begin{aligned}
 69. \quad CH &= \sqrt{(CD^2) + (DH)^2} \\
 &= \sqrt{5^2 + 5^2} \\
 &= \sqrt{25 + 25} \\
 &= \sqrt{50} = 5\sqrt{2} \\
 \sin \theta &= \frac{FH}{CF} \\
 &= \frac{5}{5\sqrt{3}} \\
 &= \frac{\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 71. \quad CH &= \sqrt{(CD^2) + (DH)^2} \\
 &= \sqrt{x^2 + x^2} \\
 &= \sqrt{2x^2} \\
 &= x\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 61. \quad \cot 60^\circ &= \frac{\cos 60^\circ}{\sin 60^\circ} \\
 &= \frac{1/2}{\sqrt{3}/2} \\
 &= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
 \end{aligned}$$

Reciprocal identity

Substitute values and simplify

$$\begin{aligned}
 \sin A &= \frac{a}{c} = \frac{3.70}{9.62} = 0.38 \\
 \cos A &= \frac{b}{c} = \frac{8.88}{9.62} = 0.92 \\
 \sin B &= \frac{b}{c} = \frac{8.88}{9.62} = 0.92 \\
 \cos B &= \frac{a}{c} = \frac{3.70}{9.62} = 0.38
 \end{aligned}$$

$$\begin{aligned}
 \sin A &= \frac{a}{c} = \frac{19.44}{20.25} = 0.96 \\
 \cos A &= \frac{b}{c} = \frac{5.67}{20.25} = 0.28 \\
 \sin B &= \frac{b}{c} = \frac{5.67}{20.25} = 0.28 \\
 \cos B &= \frac{a}{c} = \frac{19.44}{20.25} = 0.96
 \end{aligned}$$

$$\begin{aligned}
 CF &= \sqrt{(CH)^2 + (FH)^2} \\
 &= \sqrt{(5\sqrt{2})^2 + (5)^2} \\
 &= \sqrt{50 + 25} \\
 &= \sqrt{75} = 5\sqrt{3} \\
 \cos \theta &= \frac{CH}{CF} \\
 &= \frac{5\sqrt{2}}{5\sqrt{3}} \\
 &= \frac{\sqrt{2}}{\sqrt{3}} \text{ or } \frac{\sqrt{6}}{3}
 \end{aligned}$$

$$\begin{aligned}
 CF &= \sqrt{(CH)^2 + (FH)^2} \\
 &= \sqrt{(x\sqrt{2})^2 + x^2} \\
 &= \sqrt{2x^2 + x^2} \\
 &= \sqrt{3x^2} = x\sqrt{3}
 \end{aligned}$$

Ratio identity

Substitute values from Table 1

Simplify

$$\begin{aligned}\sin \theta &= \frac{FH}{CF} \\ &= \frac{x}{x\sqrt{3}} \\ &= \frac{\sqrt{3}}{3}\end{aligned}$$

$$\begin{aligned}\cos \theta &= \frac{CH}{CF} \\ &= \frac{x\sqrt{2}}{x\sqrt{3}} \\ &= \frac{\sqrt{2}}{\sqrt{3}} \text{ or } \frac{\sqrt{6}}{3}\end{aligned}$$

$$\begin{aligned}73. \quad r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance formula} \\ &= \sqrt{[3 - (-1)]^2 + [-2 - (-4)]^2} && \text{Substitute known values} \\ &= \sqrt{4^2 + 2^2} && \text{Simplify} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20} = 2\sqrt{5}\end{aligned}$$

75. The terminal side is the line  $y = -x$ . Some points in quadrant II on the line  $y = -x$  are  $(-1, 1)$ ,  $(-2, 2)$ , and  $(-3, 3)$ .

77.  $-135^\circ + 360^\circ = 225^\circ$

79.  $\sin A = \frac{a}{c} = \frac{16}{20} = \frac{4}{5}$  The answer is c.

81. Statement a is false because  $\sin 30^\circ = \frac{1}{2}$ .

## 2.2 Calculators and Trigonometric Functions of an Acute Angle

### EVEN SOLUTIONS

2. If  $\theta = 7.25^\circ$  in decimal degrees, then the 7 represents the number of degrees, the 2 represents the number of tenths of a degree, and the 5 represents the number of hundredths of a degree.
4. On a calculator, the  $\text{SIN}^{-1}$ ,  $\text{COS}^{-1}$ , and  $\text{TAN}^{-1}$  keys allow us to find an angle given the value of a trigonometric function.
6. Adding the angles:  $11^\circ 41' + 32^\circ 16' = 43^\circ 57'$
8. Adding the angles:  $63^\circ 38' + 24^\circ 52' = 87^\circ 90' = 88^\circ 30'$
10. Adding the angles:  $77^\circ 21' + 26^\circ 44' = 104^\circ 5'$
12. Subtracting the angles:  $90^\circ - 62^\circ 25' = 89^\circ 60' - 62^\circ 25' = 27^\circ 35'$
14. Subtracting the angles:  $180^\circ - 132^\circ 39' = 179^\circ 60' - 132^\circ 39' = 47^\circ 21'$
16. Subtracting the angles:  $89^\circ 38' - 28^\circ 58' = 88^\circ 98' - 28^\circ 58' = 60^\circ 40'$
18. Converting to degrees and minutes:  $83.6^\circ = 83^\circ + 0.6^\circ = 83^\circ + 0.6(60') = 83^\circ 36'$
20. Converting to degrees and minutes:  $78.5^\circ = 78^\circ + 0.5^\circ = 78^\circ + 0.5(60') = 78^\circ 30'$
22. Converting to degrees and minutes:  $43.85^\circ = 43^\circ + 0.85^\circ = 43^\circ + 0.85(60') = 43^\circ 51'$
24. Converting to degrees and minutes:  $8.3^\circ = 8^\circ + 0.3^\circ = 8^\circ + 0.3(60') = 8^\circ 18'$
26. Converting to decimal degrees:  $74^\circ 54' = 74^\circ + 54' = 74^\circ + \left(\frac{54}{60}\right)^\circ = 74.9^\circ$
28. Converting to decimal degrees:  $21^\circ 15' = 21^\circ + 15' = 21^\circ + \left(\frac{15}{60}\right)^\circ = 21.25^\circ$
30. Converting to decimal degrees:  $39^\circ 10' = 39^\circ + 10' = 39^\circ + \left(\frac{10}{60}\right)^\circ \approx 39.17^\circ$
32. Converting to decimal degrees:  $78^\circ 37' = 78^\circ + 37' = 78^\circ + \left(\frac{37}{60}\right)^\circ = 78.62^\circ$
34. Calculating the value:  $\cos 79.2^\circ \approx 0.1874$
36. Calculating the value:  $\sin 4^\circ \approx 0.0698$

38. Calculating the value:  $\tan 41.88^\circ \approx 0.8966$
40. Calculating the value:  $\cot 29^\circ = \frac{1}{\tan 29^\circ} \approx 1.8040$
42. Calculating the value:  $\sec 18.7^\circ = \frac{1}{\cos 18.7^\circ} \approx 1.0557$
44. Calculating the value:  $\csc 77.77^\circ = \frac{1}{\sin 77.77^\circ} \approx 1.0232$
46. Calculating the value:  $\sin 75^\circ 50' = \sin \left( 75 \frac{5}{6} \right)^\circ \approx 0.9696$
48. Calculating the value:  $\tan 45^\circ 19' = \tan \left( 45 \frac{19}{60} \right)^\circ \approx 1.0111$
50. Calculating the value:  $\cos 6^\circ 4' = \cos \left( 6 \frac{1}{15} \right)^\circ \approx 0.9944$
52. Calculating the value:  $\csc 48^\circ 48' = \csc \left( 48 \frac{48}{60} \right)^\circ = \csc 48.8^\circ = \frac{1}{\sin 48.8^\circ} \approx 1.3291$

54. Completing the table:

$x$	$\csc x$	$\sec x$	$\cot x$
$0^\circ$	error (undefined)	1	error (undefined)
$15^\circ$	3.8637	1.0353	3.7321
$30^\circ$	2	1.1547	1.7321
$45^\circ$	1.4142	1.4142	1
$60^\circ$	1.1547	2	0.5774
$75^\circ$	1.0353	3.8637	0.2679
$90^\circ$	1	error (undefined)	error (undefined)

56. Finding the angle  $\theta$ :  $\theta = \sin^{-1}(0.7139) \approx 45.6^\circ$
58. Finding the angle  $\theta$ :  $\theta = \cos^{-1}(0.0945) \approx 84.6^\circ$
60. Finding the angle  $\theta$ :  $\theta = \tan^{-1}(6.2703) \approx 80.9^\circ$
62. Since  $\sec \theta = 8.0101$ ,  $\cos \theta = \frac{1}{8.0101}$ , so  $\theta = \cos^{-1} \left( \frac{1}{8.0101} \right) \approx 82.8^\circ$ .
64. Since  $\csc \theta = 4.2319$ ,  $\sin \theta = \frac{1}{4.2319}$ , so  $\theta = \sin^{-1} \left( \frac{1}{4.2319} \right) \approx 13.7^\circ$ .
66. Since  $\cot \theta = 7.0234$ ,  $\tan \theta = \frac{1}{7.0234}$ , so  $\theta = \tan^{-1} \left( \frac{1}{7.0234} \right) \approx 8.1^\circ$ .
68. Finding the angle  $\theta$ :  $\theta = \sin^{-1}(0.9459) \approx 71.0672^\circ = 71^\circ + 0.0672(60') = 71^\circ 4'$
70. Finding the angle  $\theta$ :  $\theta = \tan^{-1}(2.4652) \approx 67.9202^\circ = 67^\circ + 0.9202(60') = 67^\circ 55'$
72. Since  $\sec \theta = 1.9102$ ,  $\cos \theta = \frac{1}{1.9102}$ .  
Finding the angle  $\theta$ :  $\theta = \cos^{-1} \left( \frac{1}{1.9102} \right) \approx 58.4323^\circ = 58^\circ + 0.4323(60') = 58^\circ 26'$
74. Calculating the values:  $\sin 13^\circ \approx 0.2250$  and  $\cos 77^\circ \approx 0.2250$
76. Calculating the values:  $\sec 6.7^\circ \approx 1.0069$  and  $\csc 83.3^\circ \approx 1.0069$
78. Calculating the values:  $\tan 35^\circ 15' = \tan 35.25^\circ \approx 0.7067$  and  $\cot 54^\circ 45' = \cot 54.75^\circ \approx 0.7067$
80. Calculating the value:  $\cos^2 58^\circ + \sin^2 58^\circ = 1$
82. To calculate  $B$ ,  $B = \sin^{-1}(4.321)$ , which results in an error message. Since, for any angle  $B$ ,  $\sin B \leq 1$ , it is impossible to find an angle  $B$  such that  $\sin B = 4.321$ .

84. To calculate  $\cot 0^\circ$ , we would find  $\tan 0^\circ = 0$  then find the reciprocal. This results in an error message. Since  $\frac{1}{0}$  is an undefined value,  $\cot 0^\circ$  is undefined.

86. a. Completing the table:

$x$	$3^\circ$	$2.5^\circ$	$2^\circ$	$1.5^\circ$	$1^\circ$	$0.5^\circ$	$0^\circ$
$\cot x$	19.1	22.9	28.6	38.2	57.3	114.6	undefined

b. Completing the table:

$x$	$0.6^\circ$	$0.5^\circ$	$0.4^\circ$	$0.3^\circ$	$0.2^\circ$	$0.1^\circ$	$0^\circ$
$\cot x$	95.5	114.6	143.2	191.0	286.5	573.0	undefined

88. Using  $\alpha = 36.597^\circ$  and  $h = 5$  in the shadow angle formula:

$$\tan \theta = (\sin 36.597^\circ)(\tan(5 \cdot 15^\circ)) \approx 2.2250$$

$$\theta = \tan^{-1}(2.2250) \approx 65.8^\circ$$

90. First find the value of  $r$ :  $r = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$

Finding the three trigonometric functions using  $x = -\sqrt{3}$ ,  $y = 1$ , and  $r = 2$ :

$$\sin \theta = \frac{y}{r} = \frac{1}{2}$$

$$\cos \theta = \frac{x}{r} = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

92. Let  $(-1, -1)$  be a point on the terminal side of  $-135^\circ$ . First find the value of  $r$ :  $r = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

Finding the three trigonometric functions using  $x = -1$ ,  $y = -1$ , and  $r = \sqrt{2}$ :

$$\sin \theta = \frac{y}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{x}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = \frac{y}{x} = \frac{-1}{-1} = 1$$

94. Since  $\tan \theta = -\frac{3}{4}$  and  $\theta$  terminates in quadrant II (where  $x < 0$  and  $y > 0$ ), choose  $x = -4$  and  $y = 3$ . Finding  $r$ :

$$r = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Finding the remaining trigonometric functions using  $x = -4$ ,  $y = 3$ , and  $r = 5$ :

$$\sin \theta = \frac{y}{r} = \frac{3}{5}$$

$$\cos \theta = \frac{x}{r} = -\frac{4}{5}$$

$$\cot \theta = \frac{x}{y} = -\frac{4}{3}$$

$$\csc \theta = \frac{r}{y} = \frac{5}{3}$$

$$\sec \theta = \frac{r}{x} = -\frac{5}{4}$$

96. Since  $\sec \theta > 0$ ,  $x > 0$ . Thus for  $\tan \theta < 0$ , we must have  $y < 0$ . Thus the terminal side of  $\theta$  lies in quadrant IV.

98. Converting to decimal degrees:  $76^\circ 36' = 76^\circ + 36' = 76^\circ + \left(\frac{36}{60}\right)^\circ = 76.6^\circ$ . The correct answer is b.

100. Since  $\cot \theta = x$ ,  $\tan \theta = \frac{1}{x}$ . Then  $\theta = \tan^{-1}\left(\frac{1}{x}\right)$ . The correct answer is a.

## ODD SOLUTIONS

1. minutes, seconds

5.  $37^\circ 45'$

$+26^\circ 24'$

$63^\circ 69' = 64^\circ 9'$  since  $60' = 1^\circ$

9.  $61^\circ 33'$

$+45^\circ 16'$

$106^\circ 49'$

3. value, angle

7.  $51^\circ 55'$

$+37^\circ 45'$

$88^\circ 100' = 89^\circ 40'$

11.  $90^\circ = 89^\circ 60'$

$-34^\circ 12' \quad -34^\circ 12'$

$55^\circ 48'$

$$13. \quad \begin{array}{rcl} 180^\circ & = & 179^\circ 60' \quad \text{Change } 1^\circ \text{ to } 60' \\ -120^\circ 17' & & -120^\circ 17' \\ \hline & & 59^\circ 43' \end{array}$$

$$17. \quad \begin{aligned} 35.4^\circ &= 35^\circ + 0.4(60)' \\ &= 35^\circ + 24' \\ &= 35^\circ 24' \end{aligned}$$

$$21. \quad \begin{aligned} 92.55^\circ &= 92^\circ + 0.55(60)' \\ &= 92^\circ + 33' \\ &= 92^\circ 33' \end{aligned}$$

$$25. \quad \begin{aligned} 45^\circ 12' &= 45 + \frac{12}{60} \\ &= 45.2^\circ \end{aligned}$$

$$29. \quad \begin{aligned} 17^\circ 20' &= 17 + \frac{20}{60} \\ &= 17.33^\circ \end{aligned}$$

33. Scientific Calculator: 27.2  $\boxed{\sin}$   
 Graphing Calculator:  $\boxed{\sin} \boxed{(} \boxed{27.2} \boxed{)} \boxed{ENTER}$   
 Answer to 4 places: 0.4571

35. Scientific Calculator: 18  $\boxed{\cos}$   
 Graphing Calculator:  $\boxed{\cos} \boxed{(} \boxed{18} \boxed{)} \boxed{ENTER}$   
 Answer to 4 places: 0.9511

37. Scientific Calculator: 87.32  $\boxed{\tan}$   
 Graphing Calculator:  $\boxed{\tan} \boxed{(} \boxed{87.32} \boxed{)} \boxed{ENTER}$   
 Answer to 4 places: 21.3634

39.  $\cot 31^\circ = \frac{1}{\tan 31^\circ}$   
 Scientific Calculator: 31  $\boxed{\tan} \boxed{1/x}$   
 Graphing Calculator:  $\boxed{\tan} \boxed{(} \boxed{31} \boxed{)} \boxed{x^{-1}} \boxed{ENTER}$   
 Answer: 1.6643

41.  $\sec 48.2^\circ = \frac{1}{\cos 48.2^\circ}$   
 Scientific Calculator: 48.2  $\boxed{\cos} \boxed{1/x}$   
 Graphing Calculator:  $\boxed{\cos} \boxed{(} \boxed{48.2} \boxed{)} \boxed{x^{-1}}$   
 Answer: 1.5003

43.  $\csc 14.15^\circ = \frac{1}{\sin 14.15^\circ}$   
 Scientific Calculator: 14.15  $\boxed{\sin} \boxed{1/x}$   
 Graphing Calculator:  $\boxed{\sin} \boxed{(} \boxed{14.15} \boxed{)} \boxed{x^{-1}} \boxed{ENTER}$   
 Answer: 4.0906

$$15. \quad \begin{array}{rcl} 76^\circ 24' & = & 75^\circ 84' \\ -22^\circ 34' & & -22^\circ 34' \\ \hline & & 53^\circ 50' \end{array}$$

$$19. \quad \begin{aligned} 16.25^\circ &= 16^\circ + 0.25(60)' \\ &= 16^\circ + 15' \\ &= 16^\circ 15' \end{aligned}$$

$$23. \quad \begin{aligned} 19.9^\circ &= 19^\circ + 0.9(60)' \\ &= 19^\circ + 54' \\ &= 19^\circ 54' \end{aligned}$$

$$27. \quad \begin{aligned} 62^\circ 36' &= 62 + \frac{36}{60} \\ &= 62.6^\circ \end{aligned}$$

$$31. \quad \begin{aligned} 48^\circ 27' &= 48 + \frac{27}{60} \\ &= 48.45^\circ \end{aligned}$$

45.  $24^\circ 30' = 24 + \frac{30}{60} = 24.5^\circ$

Scientific Calculator: 24.5  $\boxed{\cos}$

Graphing Calculator:  $\boxed{\cos} \boxed{(} 24.5 \boxed{)} \boxed{ENTER}$

Answer: 0.9100

47.  $42^\circ 15' = 42 + \frac{15}{60}$   
 $= 42.25^\circ$

Scientific Calculator: 42.25  $\boxed{\tan}$

Graphing Calculator:  $\boxed{\tan} \boxed{(} 42.25 \boxed{)} \boxed{ENTER}$

Answer: 0.9083

49.  $56^\circ 40' = 56 + \frac{40}{60} = 56.67^\circ$

Scientific Calculator: 56.67  $\boxed{\sin}$

Graphing Calculator:  $\boxed{\sin} \boxed{(} 56.67 \boxed{)} \boxed{ENTER}$

Answer: 0.8355

51.  $45^\circ 54' = 45 + \frac{54}{60}$   
 $= 45.9^\circ$

$\sec 45.9^\circ = \frac{1}{\cos 45.9^\circ}$

Scientific Calculator: 45.9  $\boxed{\cos} \boxed{1/x}$

Graphing Calculator:  $\boxed{\cos} \boxed{(} 45.9 \boxed{)} \boxed{x^{-1}} \boxed{ENTER}$

Answer: 1.4370

53. Use your calculator to find the values of the sine, cosine, and tangent of each angle:

$x$	$\sin x$	$\cos x$	$\tan x$
$0^\circ$	0	1	0
$15^\circ$	0.2588	0.9659	0.2679
$30^\circ$	0.5	0.8660	0.5774
$45^\circ$	0.7071	0.7071	1
$60^\circ$	0.8660	0.5	1.7321
$75^\circ$	0.9659	0.2588	3.7321
$90^\circ$	1	0	Error (undefined)

55. Scientific Calculator: 0.9770  $\boxed{inv} \boxed{\cos}$

Graphing Calculator:  $\boxed{2nd} \boxed{\cos} \boxed{(} 0.9770 \boxed{)} \boxed{ENTER}$

Answer:  $12.3^\circ$

57. Scientific Calculator: 0.6873  $\boxed{inv} \boxed{\tan}$

Graphing Calculator:  $\boxed{2nd} \boxed{\tan} \boxed{(} 0.6873 \boxed{)} \boxed{ENTER}$

Answer:  $34.5^\circ$

59. Scientific Calculator: 0.9813  $\boxed{inv} \boxed{\sin}$

Graphing Calculator:  $\boxed{2nd} \boxed{\sin} \boxed{(} 0.9813 \boxed{)} \boxed{ENTER}$

Answer:  $78.9^\circ$

61.  $\sec \theta = 1.0191$   
 $\frac{1}{\cos \theta} = 1.0191$   
 $\cos \theta = \frac{1}{1.0191}$   
Scientific Calculator:  $1 \div 1.0191 = \boxed{\text{inv}} \boxed{\cos}$   
Graphing Calculator:  $\boxed{2nd} \boxed{\cos} \boxed{(} 1 \div 1.0191 \boxed{)} \boxed{ENTER}$   
Answer:  $11.1^\circ$
63.  $\csc \theta = 1.8214$   
 $\frac{1}{\sin \theta} = 1.8214$   
 $\sin \theta = \frac{1}{1.8214}$   
Scientific Calculator:  $1 \div 1.8214 = \boxed{\text{inv}} \boxed{\sin}$   
Graphing Calculator:  $\boxed{2nd} \boxed{\sin} \boxed{(} 1 \div 1.8214 \boxed{)} \boxed{ENTER}$   
Answer:  $33.3^\circ$
65.  $\cot \theta = 0.6873$   
 $\frac{1}{\tan \theta} = 0.6873$   
 $\tan \theta = \frac{1}{0.6873}$   
Scientific Calculator:  $1 \div 0.6873 = \boxed{\text{inv}} \boxed{\tan}$   
Graphing Calculator:  $\boxed{2nd} \boxed{\tan} \boxed{(} 1 \div 0.6873 \boxed{)} \boxed{ENTER}$   
Answer:  $55.5^\circ$
67. Scientific Calculator:  $0.4112 \boxed{\text{inv}} \boxed{\cos}$   
Answer in decimal degrees is  $65.719^\circ$   
Convert the decimal part to minutes:  $0.719 \times 60 =$   
Graphing Calculator:  $\boxed{2nd} \boxed{\cos} \boxed{(} 0.4112 \boxed{)} \boxed{2nd} \boxed{APPS} \boxed{\text{DMS}} \boxed{ENTER}$   
Answer:  $\theta = 65^\circ 43'$
69.  $\cot \theta = 5.5764$   
 $\frac{1}{\tan \theta} = 5.5764$   
 $\tan \theta = \frac{1}{5.5764}$   
Scientific Calculator:  $1 \div 5.5764 = \boxed{\text{inv}} \boxed{\tan}$   
Answer in decimal degrees is  $10.1666^\circ$   
Convert the decimal part to minutes:  $0.1666 \times 60 =$   
Graphing Calculator:  $\boxed{2nd} \boxed{\tan} \boxed{(} 5.5764 \boxed{)} \boxed{2nd} \boxed{APPS} \boxed{\text{DMS}} \boxed{ENTER}$   
Answer:  $\theta = 10^\circ 10'$
71.  $\csc \theta = 7.0683$   
 $\frac{1}{\sin \theta} = 7.0683$   
 $\sin \theta = \frac{1}{7.0683}$   
Scientific Calculator:  $1 \div 7.0683 = \boxed{\text{inv}} \boxed{\sin}$   
Answer in decimal degrees is  $8.1333^\circ$   
Convert the decimal part to minutes:  $0.133 \times 60 =$   
Graphing Calculator:  $\boxed{2nd} \boxed{\sin} \boxed{(} 1 \div 7.0683 \boxed{)} \boxed{2nd} \boxed{APPS} \boxed{\text{DMS}} \boxed{ENTER}$   
Answer:  $\theta = 8^\circ 8'$
73. Scientific Calculator:  $23 \boxed{\sin}$  and  $67 \boxed{\cos}$   
Graphing Calculator:  $\boxed{\sin} \boxed{(} 23 \boxed{)} \boxed{ENTER}$   $\cos \boxed{(} 67 \boxed{)} \boxed{ENTER}$   
Both answers should be 0.3907.

75. To calculate  $\sec 34.5^\circ = \frac{1}{\cos 34.5^\circ}$ :

Scientific Calculator: 34.5  $\boxed{\cos}$   $\boxed{1/x}$

Graphing Calculator:  $\boxed{\cos}$   $\boxed{(}$  34.5  $\boxed{)}$   $\boxed{x^{-1}}$   $\boxed{ENTER}$

To calculate  $\csc 55.5^\circ = \frac{1}{\sin 55.5^\circ}$ :

Scientific Calculator: 55.5  $\boxed{\sin}$   $\boxed{1/x}$

Graphing Calculator:  $\boxed{\sin}$   $\boxed{(}$  55.5  $\boxed{)}$   $\boxed{x^{-1}}$   $\boxed{ENTER}$

Both answers should be 1.2134.

77. Scientific Calculator: 4.5  $\boxed{\tan}$

Graphing Calculator:  $\boxed{\tan}$   $\boxed{(}$  4.5  $\boxed{)}$   $\boxed{ENTER}$

To calculate  $\cot 85.5^\circ = \frac{1}{\tan 85.5^\circ}$ :

Scientific Calculator: 85.5  $\boxed{\tan}$   $\boxed{1/x}$

Graphing Calculator:  $\boxed{\tan}$   $\boxed{(}$  85.5  $\boxed{)}$   $\boxed{x^{-1}}$   $\boxed{ENTER}$

Both answers should be 0.0787.

79. Scientific Calculator: 37  $\boxed{\cos}$   $\boxed{x^2}$   $\boxed{+}$  37  $\boxed{\sin}$   $\boxed{x^2}$   $\boxed{=}$

Graphing Calculator:  $\boxed{\cos}$   $\boxed{(}$  37  $\boxed{)}$   $\boxed{x^2}$   $\boxed{+}$   $\boxed{\sin}$   $\boxed{(}$  37  $\boxed{)}$   $\boxed{x^2}$   $\boxed{ENTER}$

Answer should be 1.

81. Scientific Calculator: 1.234  $\boxed{inv}$   $\boxed{\sin}$

Graphing Calculator:  $\boxed{2nd}$   $\boxed{\sin}$  1.234  $\boxed{ENTER}$

You should get an error message. The sine of an angle can never be greater than 1.

83. Scientific Calculator: 90  $\boxed{\tan}$

Graphing Calculator:  $\boxed{\tan}$   $\boxed{(}$  90  $\boxed{)}$   $\boxed{ENTER}$

You should get an error message. The tangent of  $90^\circ$  is undefined.

87.  $\tan \theta = \sin \alpha \tan(h \cdot 15^\circ)$  where  $\alpha = 35.282^\circ$  and  $h = 2$

$$\tan \theta = \sin(35.282^\circ) \tan(2 \cdot 15^\circ)$$

$$= .333478$$

$$\theta = \tan^{-1}(.333478)$$

$$\theta = 18.4^\circ$$

89.  $(x, y) = (3, -2)$

$$x = 3 \text{ and } y = -2$$

$$r = \sqrt{3^2 + (-2)^2}$$

$$= \sqrt{9 + 4} = \sqrt{13}$$

$$\sin \theta = \frac{y}{r} = \frac{-2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{-2}{3} = -\frac{2}{3}$$



91. A point on the terminal side of an angle of  $90^\circ$  in standard position is  $(0, 1)$ , where  $x = 0$ ,  $y = 1$ , and  $r = 1$ .

$$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$$

$$\cos 90^\circ = \frac{x}{r} = \frac{0}{1} = 0$$

$$\tan 90^\circ = \frac{y}{x} = \frac{1}{0} \text{ is undefined}$$

93.  $\cos \theta = -\frac{5}{13}$  and  $\theta$  is in QIII. In QIII, both  $x$  and  $y$  are negative.

$$\cos \theta = \frac{x}{r} = \frac{-5}{13}$$

$$x = -5 \text{ and } r = 13$$

$$x^2 + y^2 = r^2$$

$$(-5)^2 + y^2 = 13^2$$

$$25 + y^2 = 169$$

$$y^2 = 144$$

$$y = \pm 12$$

$$y = -12 \text{ because } \theta \text{ is in QIII}$$

95. The  $\sin \theta$  is positive in QI and QII.  
The  $\cos \theta$  is negative in QII and QIII.  
Therefore,  $\theta$  must lie in QII.

97.  $67^\circ 22' = 66^\circ 82'$  Change  $1^\circ$  to  $60'$

$$\begin{array}{rcl} -34^\circ 30' & & -34^\circ 30' \\ & & 32^\circ 52' \end{array}$$

The answer is d.

$$\sin \theta = \frac{y}{r} = \frac{-12}{13} = -\frac{12}{13}$$

$$\tan \theta = \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5}$$

$$\cot \theta = \frac{x}{y} = \frac{-5}{-12} = \frac{5}{12}$$

$$\sec \theta = \frac{r}{x} = \frac{13}{-5} = -\frac{13}{5}$$

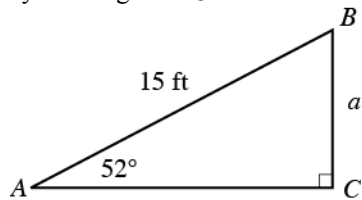
$$\csc \theta = \frac{r}{y} = \frac{13}{-12} = -\frac{13}{12}$$

## 2.3 Solving Right Triangles

### EVEN SOLUTIONS

2. If the sides of a triangle are accurate to three significant digits, then angles should be measured to the nearest tenth of a degree, or the nearest ten minutes.
4. In general, round answers so that the number of significant digits in your answer matches the number of significant digits in the least significant number given in the problem.
6.
  - a. three
  - b. three
  - c. five
  - d. three
8.
  - a. five
  - b. five
  - c. five
  - d. seven

10. Begin by drawing  $\triangle ABC$ :

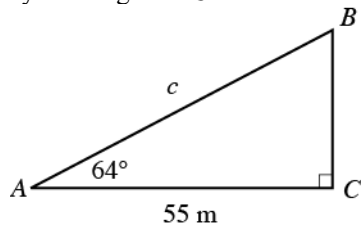


Therefore:

$$\sin 52^\circ = \frac{a}{15}$$

$$a = 15 \sin 52^\circ \approx 12 \text{ ft}$$

12. Begin by drawing  $\triangle ABC$ :



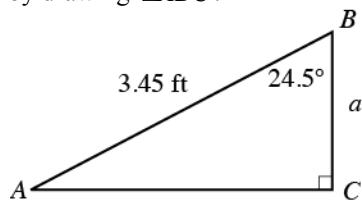
Therefore:

$$\cos 64^\circ = \frac{55}{c}$$

$$c \cos 64^\circ = 55$$

$$c = \frac{55}{\cos 64^\circ} \approx 125 \text{ m} \approx 130 \text{ m}$$

14. Begin by drawing  $\triangle ABC$ :

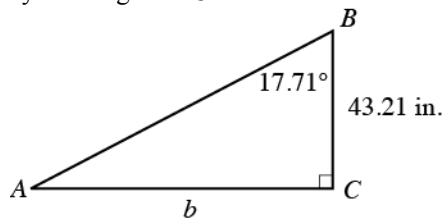


Therefore:

$$\cos 24.5^\circ = \frac{a}{3.45}$$

$$a = 3.45 \cos 24.5^\circ \approx 3.14 \text{ ft}$$

16. Begin by drawing  $\triangle ABC$ :

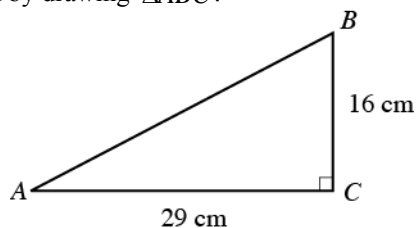


Therefore:

$$\tan 17.71^\circ = \frac{b}{43.21}$$

$$b = 43.21 \tan 17.71^\circ \approx 13.80 \text{ in.}$$

18. Begin by drawing  $\triangle ABC$ :

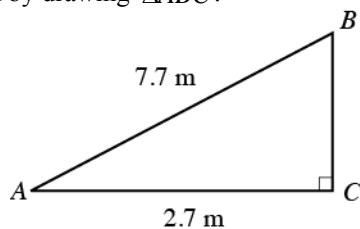


Therefore:

$$\tan A = \frac{16}{29}$$

$$A = \tan^{-1}\left(\frac{16}{29}\right) \approx 29^\circ$$

20. Begin by drawing  $\triangle ABC$ :

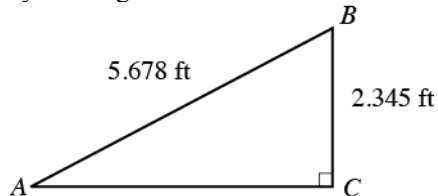


Therefore:

$$\cos A = \frac{2.7}{7.7}$$

$$A = \cos^{-1}\left(\frac{2.7}{7.7}\right) \approx 69^\circ$$

22. Begin by drawing  $\triangle ABC$ :

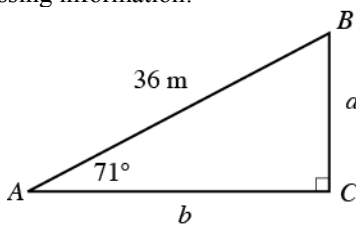


Therefore:

$$\sin A = \frac{2.345}{5.678}$$

$$A = \sin^{-1}\left(\frac{2.345}{5.678}\right) \approx 24.39^\circ$$

24. Begin by drawing  $\triangle ABC$  and label missing information:



Note that  $B = 90^\circ - 71^\circ = 19^\circ$ . Therefore:

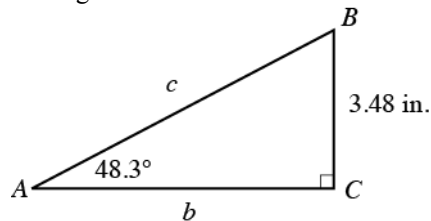
$$\sin 71^\circ = \frac{a}{36}$$

$$a = 36 \sin 71^\circ \approx 34 \text{ m}$$

$$\cos 71^\circ = \frac{b}{36}$$

$$b = 36 \cos 71^\circ \approx 12 \text{ m}$$

26. Begin by drawing  $\triangle ABC$  and label missing information:



Note that  $B = 90^\circ - 48.3^\circ = 41.7^\circ$ . Therefore:

$$\sin 48.3^\circ = \frac{3.48}{c}$$

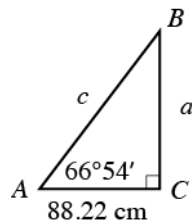
$$c \sin 48.3^\circ = 3.48$$

$$c = \frac{3.48}{\sin 48.3^\circ} \approx 4.66 \text{ in.}$$

$$\cos 48.3^\circ = \frac{b}{4.66}$$

$$b = 4.66 \cos 48.3^\circ \approx 3.10 \text{ in.}$$

28. Begin by drawing  $\triangle ABC$  and label missing information:



Note that  $B = 90^\circ - 66^\circ 54' = 89^\circ 60' - 66^\circ 54' = 23^\circ 6'$ . So:

$$\cos 66^\circ 54' = \frac{88.22}{c}$$

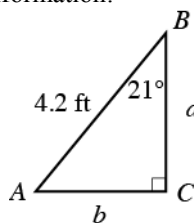
$$c \cos 66^\circ 54' = 88.22$$

$$c = \frac{88.22}{\cos 66^\circ 54'} \approx 224.9 \text{ cm}$$

$$\tan 66^\circ 54' = \frac{a}{88.22}$$

$$a = 88.22 \tan 66^\circ 54' \approx 206.8 \text{ cm}$$

30. Begin by drawing  $\triangle ABC$  and label missing information:



Note that  $A = 90^\circ - 21^\circ = 69^\circ$ . Therefore:

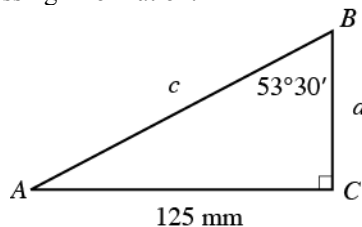
$$\cos 69^\circ = \frac{b}{4.2}$$

$$b = 4.2 \cos 69^\circ \approx 1.5 \text{ ft}$$

$$\sin 69^\circ = \frac{a}{4.2}$$

$$a = 4.2 \sin 69^\circ \approx 3.9 \text{ ft}$$

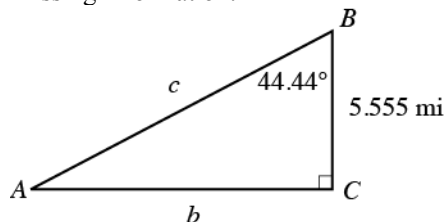
32. Begin by drawing  $\triangle ABC$  and label missing information:



Note that  $A = 90^\circ - 53^\circ 30' = 89^\circ 60' - 53^\circ 30' = 36^\circ 30'$ . So:

$$\begin{aligned}\cos 36^\circ 30' &= \frac{125}{c} \\ c \cos 36^\circ 30' &= 125 \\ c &= \frac{125}{\cos 36^\circ 30'} \approx 156 \text{ mm}\end{aligned}\qquad \begin{aligned}\tan 36^\circ 30' &= \frac{a}{125} \\ a &= 125 \tan 36^\circ 30' \approx 92.5 \text{ mm}\end{aligned}$$

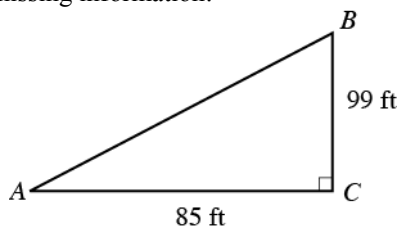
34. Begin by drawing  $\triangle ABC$  and label missing information:



Note that  $A = 90^\circ - 44.44^\circ = 45.56^\circ$ . Therefore:

$$\begin{aligned}\sin 45.56^\circ &= \frac{5.555}{c} \\ c \sin 45.56^\circ &= 5.555 \\ c &= \frac{5.555}{\sin 45.56^\circ} \approx 7.780 \text{ mi}\end{aligned}\qquad \begin{aligned}\tan 45.56^\circ &= \frac{5.555}{b} \\ b \tan 45.56^\circ &= 5.555 \\ b &= \frac{5.555}{\tan 45.56^\circ} \approx 5.447 \text{ mi}\end{aligned}$$

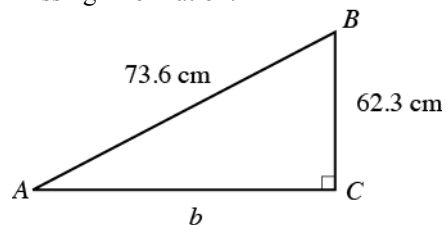
36. Begin by drawing  $\triangle ABC$  and label missing information:



Therefore:  $c = \sqrt{85^2 + 99^2} \approx 130 \text{ ft}$ . Finding the angles:

$$\begin{aligned}\tan A &= \frac{99}{85} \\ A &= \tan^{-1}\left(\frac{99}{85}\right) \approx 49^\circ \\ B &= 90^\circ - 49^\circ = 41^\circ\end{aligned}$$

38. Begin by drawing  $\triangle ABC$  and label missing information:



Therefore:  $b = \sqrt{73.6^2 - 62.3^2} \approx 39.2 \text{ cm}$ . Finding the angles:

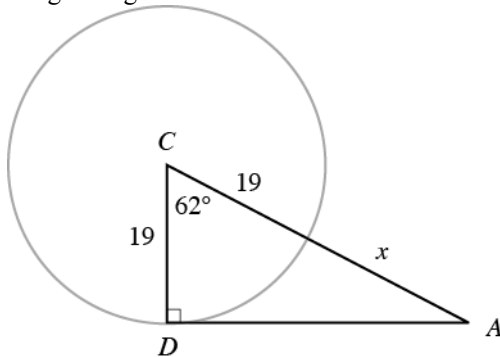
$$\begin{aligned}\sin A &= \frac{62.3}{73.6} \\ A &= \sin^{-1}\left(\frac{62.3}{73.6}\right) \approx 57.8^\circ \\ B &= 90^\circ - 57.8^\circ = 32.2^\circ\end{aligned}$$

40. Since the right triangle is a  $45^\circ-45^\circ-90^\circ$  triangle, its height is 2.0. Therefore:

$$\tan A = \frac{2.0}{3.0}$$

$$A = \tan^{-1}\left(\frac{2.0}{3.0}\right) \approx 34^\circ$$

42. Re-drawing the figure:



Using the right triangle:

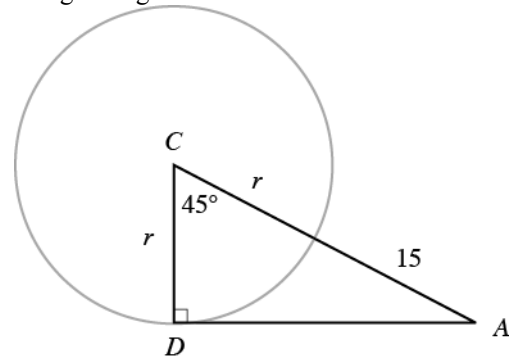
$$\cos 62^\circ = \frac{19}{19+x}$$

$$(19+x) \cos 62^\circ = 19$$

$$19+x = \frac{19}{\cos 62^\circ}$$

$$x = \frac{19}{\cos 62^\circ} - 19 \approx 21$$

44. Re-drawing the figure:



Using the right triangle:

$$\cos 45^\circ = \frac{r}{r+15}$$

$$(r+15) \cos 45^\circ = r$$

$$r \cos 45^\circ + 15 \cos 45^\circ = r$$

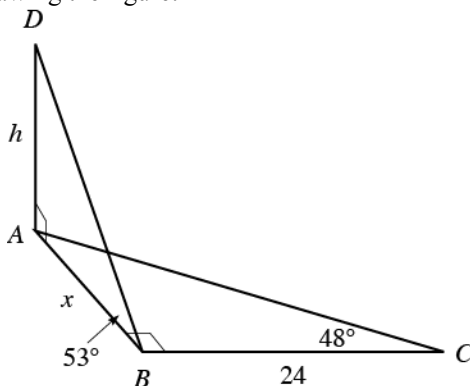
$$15 \cos 45^\circ = r - r \cos 45^\circ$$

$$15 \cos 45^\circ = r(1 - \cos 45^\circ)$$

$$r = \frac{15 \cos 45^\circ}{1 - \cos 45^\circ}$$

$$r = \frac{15 \left( \frac{\sqrt{2}}{2} \right)}{1 - \frac{\sqrt{2}}{2}} = \frac{15\sqrt{2}}{2 - \sqrt{2}} \approx 36$$

46. Re-drawing the figure:

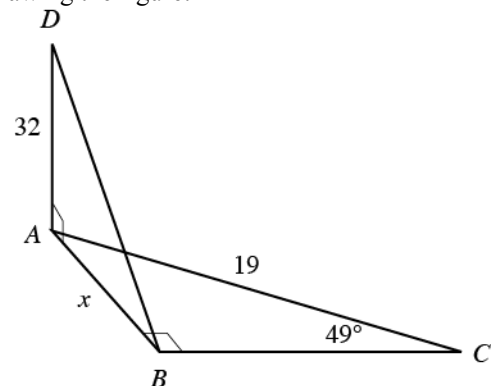


First find  $x$ :

$$\tan 48^\circ = \frac{x}{24}$$

$$x = 24 \tan 48^\circ \approx 27$$

48. Re-drawing the figure:



First find  $x$ :

$$\sin 49^\circ = \frac{x}{19}$$

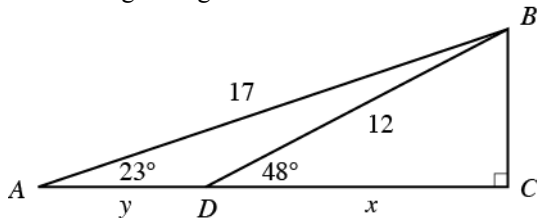
$$x = 19 \sin 49^\circ \approx 14$$

Now find  $h$ :

$$\tan 53^\circ = \frac{h}{47}$$

$$h = 27 \tan 53^\circ \approx 35$$

50. Re-drawing the figure:



First find  $x$ :

$$\cos 48^\circ = \frac{x}{12}$$

$$x = 12 \cos 48^\circ \approx 8.0$$

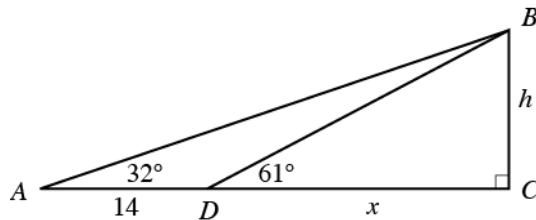
Now find  $y$ :

$$\cos 23^\circ = \frac{8+y}{17}$$

$$8+y = 17 \cos 23^\circ$$

$$y = 17 \cos 23^\circ - 8 \approx 7.6$$

54. Re-drawing the figure:



Note that  $\tan 61^\circ = \frac{h}{x}$ , so  $h = x \tan 61^\circ$ . Also note that  $\tan 32^\circ = \frac{h}{14+x}$ , so  $h = (14+x) \tan 32^\circ$ . Setting these two expressions equal:

$$x \tan 61^\circ = (14+x) \tan 32^\circ$$

$$x \tan 61^\circ = 14 \tan 32^\circ + x \tan 32^\circ$$

$$x \tan 61^\circ - x \tan 32^\circ = 14 \tan 32^\circ$$

$$x(\tan 61^\circ - \tan 32^\circ) = 14 \tan 32^\circ$$

$$x = \frac{14 \tan 32^\circ}{\tan 61^\circ - \tan 32^\circ} \approx 7.4$$

56. Since  $GC = CD = 3.00$ , using the Pythagorean Theorem:  $GD = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$ . Therefore:

$$\tan \angle GDE = \frac{GE}{GD} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

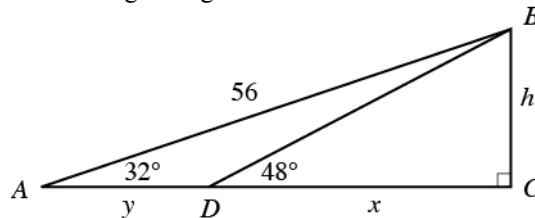
$$\angle GDE = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 35.3^\circ$$

Now find  $\angle ABD$ :

$$\tan \angle ABD = \frac{32}{14}$$

$$\angle ABD = \tan^{-1}\left(\frac{32}{14}\right) \approx 66^\circ$$

52. Re-drawing the figure:



First find  $h$ :

$$\sin 32^\circ = \frac{h}{56}$$

$$h = 56 \sin 32^\circ \approx 30$$

Now find  $x$ :

$$\tan 48^\circ = \frac{30}{x}$$

$$x \tan 48^\circ = 30$$

$$x = \frac{30}{\tan 48^\circ} \approx 27$$

58. First find  $\angle CAB$ :

$$\tan(\angle CAB) = \frac{66}{54}$$

$$\angle CAB = \tan^{-1}\left(\frac{66}{54}\right) \approx 50.71^\circ$$

- Now find  $\angle EAB$ :

$$\tan(\angle EAB) = \frac{78}{54}$$

$$\angle EAB = \tan^{-1}\left(\frac{78}{54}\right) \approx 55.30^\circ$$

Therefore:  $\angle CAE = \angle EAB - \angle CAB = 55.30^\circ - 50.71^\circ \approx 4.6^\circ$

60. Let  $O$  represent the center of the goal.

First find  $\angle OAD$ :

$$\tan(\angle OAD) = \frac{6}{54}$$

$$\angle OAD = \tan^{-1}\left(\frac{6}{54}\right) \approx 6.34^\circ$$

- Now find  $\angle OAF$ :

$$\tan(\angle OAF) = \frac{12}{54}$$

$$\angle OAF = \tan^{-1}\left(\frac{12}{54}\right) \approx 12.53^\circ$$

Therefore:  $\angle DAF = \angle OAF - \angle OAD = 12.53^\circ - 6.34^\circ \approx 6.2^\circ$

Since  $\angle CAE$  is also  $6.2^\circ$ , the sum of the angles is  $12.4^\circ$ .

62. Since  $12.4^\circ$  is much greater than  $4.6^\circ$ , the chance of scoring is much better from the center than from the corner of the penalty area.

64. From Example 5, we have:

$$\cos 135^\circ = \frac{139 - h}{125}$$

$$-\frac{1}{\sqrt{2}} = \frac{139 - h}{125}$$

$$-\frac{125}{\sqrt{2}} = 139 - h$$

$$h = 139 + \frac{125}{\sqrt{2}} \approx 227.4$$

The rider is approximately 230 ft above the ground.

66. First note that the distance from the ground to the low point of Colossus is  $174 - 165 = 9$  ft. The radius is 82.5 ft. Since  $x + h = 82.5 + 9 = 91.5$ ,  $x = 91.5 - h$ . Therefore:

$$\cos \theta = \frac{x}{82.5} = \frac{91.5 - h}{82.5}$$

$$91.5 - h = 82.5 \cos \theta$$

$$h = 91.5 - 82.5 \cos \theta$$

- a. Substituting  $\theta = 150^\circ$ :  $h = 91.5 - 82.5 \cos 150^\circ \approx 163$  ft  
b. Substituting  $\theta = 240^\circ$ :  $h = 91.5 - 82.5 \cos 240^\circ \approx 133$  ft  
c. Substituting  $\theta = 315^\circ$ :  $h = 91.5 - 82.5 \cos 315^\circ \approx 33.2$  ft

68. First note that the distance from the ground to the low point of the High Roller is  $550 - 520 = 30$  ft. The radius is 260 ft. Since  $x + h = 260 + 30 = 290$ ,  $x = 290 - h$ . Therefore:

$$\cos \theta = \frac{x}{260} = \frac{290 - h}{260}$$

$$290 - h = 260 \cos \theta$$

$$h = 290 - 260 \cos \theta$$

Substituting  $\theta = 110^\circ$ :  $h = 290 - 260 \cos 110^\circ \approx 379$  ft  $\approx 380$  ft

70. Entering the functions  $Y_1 = -\frac{7}{640}(X - 80)^2 + 70$  and  $Y_2 = \tan^{-1}\left(\frac{Y_1}{X}\right)$ , complete the table:

$X$	10	5	1	0.5	0.1	0.01
$Y_1$	16.4063	8.4766	1.7391	0.8723	0.1749	0.0175
$Y_2$	58.6°	59.5°	60.1°	60.2°	60.2°	60.3°

Based on these results, it appears the angle between the cannon and the horizontal is approximately  $60.3^\circ$ .



72. Since  $\csc B = 5$ ,  $\sin B = \frac{1}{5}$ , so  $\sin^2 B = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$ .

74. Finding  $\cos^2 A$ :  $\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$

So  $\cos A = \pm \frac{\sqrt{7}}{4}$ . Since  $A$  terminates in quadrant II, where  $x < 0$ ,  $\cos A < 0$ . Thus  $\cos A = -\frac{\sqrt{7}}{4}$ .

76. First find  $\sin \theta$  (note  $\sin \theta < 0$  since  $\theta$  terminates in quadrant IV):

$$\sin \theta = -\sqrt{1 - \cos^2 \theta} = -\sqrt{1 - \frac{1}{5}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Now find the other four trigonometric ratios using  $x = 1$ ,  $y = -2$ , and  $r = \sqrt{5}$ :

$$\sec \theta = \frac{r}{x} = \sqrt{5} \quad \csc \theta = \frac{r}{y} = -\frac{\sqrt{5}}{2} \quad \tan \theta = \frac{y}{x} = -2 \quad \cot \theta = \frac{x}{y} = -\frac{1}{2}$$

78. Since  $\csc \theta = -2$ ,  $\sin \theta = -\frac{1}{2}$ . Now find  $\cos \theta$  (note that  $\cos \theta < 0$  since  $\theta$  terminates in quadrant III):

$$\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{1 - \left(-\frac{1}{2}\right)^2} = -\sqrt{1 - \frac{1}{4}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

Now find the other three trigonometric ratios using  $x = -\sqrt{3}$ ,  $y = -1$ , and  $r = 2$ :

$$\sec \theta = \frac{r}{x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3} \quad \tan \theta = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \cot \theta = \frac{x}{y} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

80. Finding side  $b$ :

$$\cos A = \frac{b}{c}$$

$$\cos 58^\circ = \frac{b}{15}$$

$$b = 15 \cos 58^\circ \approx 7.9 \text{ ft}$$

The correct answer is c.

82. Let  $x$  represent the height of the rider above the center of the wheel (which is 51.5 feet above the ground). Since the point of the rider is  $50^\circ$  above the horizontal, we have:

$$\sin 50^\circ = \frac{h - 51.5}{45}$$

$$h - 51.5 = 45 \sin 50^\circ$$

$$h = 51.5 + 45 \sin 50^\circ \approx 86 \text{ ft}$$

The correct answer is c.

## ODD SOLUTIONS

1. left, right, first nonzero, not

5. a. 2 b. 3 c. 2 d. 2

9.  $\cos 42^\circ = \frac{b}{89}$

$$b = 89 \cos 42^\circ$$

$$= 89(0.7431)$$

$$= 66 \text{ cm}$$

3. sides, angles

7. a. 4 b. 6 c. 4 d. 4

Cosine relationship

Multiply both sides by 89

Substitute value for  $\cos 42^\circ$

Answer rounded to 2 significant digits

- 11.**  $\sin 34^\circ = \frac{22}{c}$   
 $c \sin 34^\circ = 22$   
 $c = \frac{22}{\sin 34^\circ}$   
 $c = \frac{22}{0.5592}$   
 $c = 39 \text{ m}$   
Sine relationship  
Multiply both sides by  $c$   
Divide both sides by  $\sin 34^\circ$   
Substitute value for  $\sin 34^\circ$   
Answer rounded to 2 significant digits
- 13.**  $\sin 16.9^\circ = \frac{b}{7.55}$   
 $b = 7.55 \sin 16.9^\circ$   
 $= 7.55(0.2907)$   
 $= 2.19 \text{ cm}$   
Sine relationship  
Multiply both sides by 7.55  
Substitute value for  $\sin 16.9^\circ$   
Answer rounded to 3 significant digits
- 15.**  $\tan 55.33^\circ = \frac{12.34}{a}$   
 $a \tan 55.33^\circ = 12.34$   
 $a = \frac{12.34}{\tan 55.33^\circ}$   
 $a = \frac{12.34}{1.4458}$   
 $a = 8.535 \text{ yd}$   
Tangent relationship  
Multiply both sides by  $a$   
Divide both sides by  $\tan 55.33^\circ$   
Substitute value for  $\tan 55.33^\circ$   
Answer rounded to 4 significant digits
- 17.**  $\tan B = \frac{32.4}{42.3}$   
 $= 0.7659$   
 $B = \tan^{-1}(0.7659)$   
 $B = 37.5^\circ$   
Tangent relationship  
Divide  
Use calculator to find angle  
Answer rounded to the nearest tenth of a degree
- 19.**  $\sin B = \frac{9.8}{12}$   
 $= 0.8166$   
 $B = \sin^{-1}(0.8166)$   
 $= 55^\circ$   
Sine relationship  
Divide  
Use calculator to find angle  
Answer rounded to the nearest degree
- 21.**  $\cos B = \frac{23.32}{45.54}$   
 $= 0.5120$   
 $B = \cos^{-1}(0.5120)$   
 $= 59.20^\circ$   
Cosine relationship  
Divide 23.32 by 45.54  
Use calculator to find angle  
Answer rounded to the nearest hundredth of a degree
- 23.** First, we find  $\angle B$ :  
Next, we find side  $a$ :  
 $\sin 25^\circ = \frac{a}{24}$   
 $a = 24 \sin 25^\circ$   
 $a = 10 \text{ m}$   
Last, we find side  $b$ :  
 $\cos 25^\circ = \frac{b}{24}$   
 $b = 24 \cos 25^\circ$   
 $b = 22 \text{ m}$   
 $\angle B = 90^\circ - \angle A = 90^\circ - 25^\circ = 65^\circ$   
Sine relationship  
Multiply both sides by 24  
Answer rounded to 2 significant digits  
Cosine relationship  
Multiply both sides by 24  
Answer rounded to 2 significant digits

25. First, we find  $\angle B$  :
- $$\angle B = 90^\circ - \angle A$$
- $$= 90^\circ - 32.6^\circ = 57.4^\circ$$
- Next, we find side  $c$ :
- $$\sin 32.6^\circ = \frac{43.4}{c}$$
- Sine relationship
- $$c = \frac{43.4}{\sin 32.6^\circ}$$
- Multiply both sides by  $c$  then divide by  $\sin 32.6^\circ$
- $$= 80.6 \text{ in}$$
- Answer rounded to 3 significant digits
- Last, we find side  $b$ :
- $$\tan 57.4^\circ = \frac{b}{43.4}$$
- Tangent relationship
- $$b = 43.4 \tan 57.4^\circ$$
- Multiply both sides by 43.4
- $$= 67.9 \text{ in}$$
- Answer rounded to 2 significant digits
27. First, we find  $\angle B$  :
- $$\angle B = 90^\circ - \angle A$$
- $$= 90^\circ - 10^\circ 42'$$
- $$= 79^\circ 18'$$
- Next, we find side  $a$ :
- $$\tan 10^\circ 42' = \frac{a}{5.932}$$
- Tangent relationship
- $$\tan 10.7^\circ = \frac{a}{5.932}$$
- Change angle to decimal degrees
- $$a = 5.932 \tan 10.7^\circ$$
- Multiply both sides by 5.932
- $$a = 1.121 \text{ cm}$$
- Answer rounded to 4 significant digits
- Last, we find side  $c$ :
- $$\cos 10.7^\circ = \frac{5.932}{c}$$
- Cosine relationship
- $$c = \frac{5.932}{\cos 10.7^\circ}$$
- Multiply both sides by  $c$  then divide by  $\cos 10.7^\circ$
- $$c = 6.037 \text{ cm}$$
- Answer rounded to 4 significant digits
29. First, we find  $\angle A$  :
- $$\angle A = 90^\circ - 76^\circ$$
- $$= 14^\circ$$
- Next, we find side  $a$ :
- $$\cos 76^\circ = \frac{a}{5.8}$$
- Cosine relationship
- $$a = 5.8 \cos 76^\circ$$
- Multiply both sides by 5.8
- $$= 1.4 \text{ ft}$$
- Answer rounded to 2 significant digits
- Last, we find side  $b$ :
- $$\sin 76^\circ = \frac{b}{5.8}$$
- Sine relationship
- $$b = 5.8 \sin 76^\circ = 5.6 \text{ ft}$$
- Multiply both sides by 5.8 and round to 2 significant digits

31. First, we find  $\angle A$  :

$$\begin{aligned}\angle A &= 90^\circ - \angle B \\ &= 90^\circ - 26^\circ 30' \\ &= 63^\circ 30'\end{aligned}$$

Next, we find side  $a$ :

$$\tan 26^\circ 30' = \frac{324}{a}$$

Tangent relationship

$$\tan 26.5^\circ = \frac{324}{a}$$

Change angle to decimal degrees

$$a = \frac{324}{\tan 26.5^\circ}$$

Multiply both sides by  $a$  then divide by  $\tan 26.5^\circ$

$$a = 650 \text{ mm}$$

Answer rounded to 3 significant digits

Last, we find side  $c$ :

$$\sin 26.5^\circ = \frac{324}{c}$$

Sine relationship

$$c = \frac{324}{\sin 26.5^\circ}$$

Multiply both sides by  $c$  then divide by  $\sin 26.5^\circ$

$$= 726 \text{ mm}$$

Answer rounded to 3 significant digits

33. First, we find  $\angle A$  :

$$\begin{aligned}\angle A &= 90^\circ - 23.45^\circ \\ &= 66.55^\circ\end{aligned}$$

Next, we find side  $b$ :

$$\tan 23.45^\circ = \frac{b}{5.432}$$

Tangent relationship

$$b = 5.432 \tan 23.45^\circ$$

Multiply both sides by 5.432

$$= 2.356 \text{ mi}$$

Answer rounded to 4 significant digits

Last, we find side  $c$ :

$$\cos 23.45^\circ = \frac{5.432}{c}$$

Cosine relationship

$$c = \frac{5.432}{\cos 23.45^\circ}$$

Multiply both sides by  $c$  and then divide by  $\cos 23.45^\circ$

$$= 5.921 \text{ mi}$$

Answer rounded to 4 significant digits

35. First, we find  $\angle A$  :

$$\tan A = \frac{37}{87}$$

Tangent relationship

$$= 0.4253$$

Divide 37 by 87

$$A = \tan^{-1}(0.4253)$$

Use calculator to find angle

$$= 23^\circ$$

Answer rounded to nearest degree

Next, we find  $\angle B$  :

$$\angle B = 90^\circ - \angle A = 90^\circ - 23^\circ = 67^\circ$$

Last, we find  $c$ :

$$\begin{aligned}c^2 &= 37^2 + 87^2 \\ &= 1369 + 7569\end{aligned}$$

Pythagorean Theorem

$$= 8938$$

Simplify

$$c = \pm 95$$

Simplify

$$= 95 \text{ ft}$$

Take square root of both sides

$c$  must be positive

37. First, we find  $\angle A$  :
- $$\cos A = \frac{377.3}{588.5}$$
- $$= 0.6411$$
- $$A = \cos^{-1}(0.6411)$$
- $$= 50.12^\circ$$
- Next, we find  $\angle B$  :
- $$\angle B = 90^\circ - \angle A$$
- $$= 90^\circ - 50.12^\circ$$
- $$= 39.88^\circ$$
- Last, we find side  $a$ :
- $$a^2 + 377.3^2 = 588.5^2$$
- $$a^2 = 203,976.96$$
- $$a = \pm 451.6$$
- $$= 451.6 \text{ in}$$
- Pythagorean Theorem  
Subtract and simplify  
Take square root of both sides  
 $a$  must be positive
39. Using  $\triangle BCD$ , we find  $BD$ :
- $$\sin 30^\circ = \frac{BD}{6.0}$$
- $$BD = 6.0 \sin 30^\circ$$
- $$= 3$$
- Sine relationship  
Multiply both sides by 6  
Exact answer
- Next, we find  $\angle A$
- $$\sin A = \frac{3}{4.0}$$
- $$= 0.75$$
- $$A = \sin^{-1}(0.75)$$
- $$A = 49^\circ$$
- Sine relationship  
Divide  
Use calculator to find angle  
Answer rounded to the nearest degree
41.  $\sin 31^\circ = \frac{12}{x+12}$
- Sine relationship
- $$(x+12)\sin 31^\circ = 12$$
- Multiply both sides by  $x+12$
- $$x+12 = \frac{12}{\sin 31^\circ}$$
- Divide both sides by  $\sin 31^\circ$
- $$x = \frac{12}{\sin 31^\circ} - 12 = 11$$
- Subtract 12 from both sides and round to 2 significant digits
43.  $\cos 65^\circ = \frac{r}{r+22}$
- Sine relationship
- $$r = (r+22)\cos 65^\circ$$
- Multiply both side by  $r+22$
- $$r = r \cos 65^\circ + 22 \cos 65^\circ$$
- Use distributive property
- $$r - r \cos 65^\circ = 22 \cos 65^\circ$$
- Subtract  $r \cos 65^\circ$  from both sides
- $$r(1 - \cos 65^\circ) = 22 \cos 65^\circ$$
- Factor left side
- $$r = \frac{22 \cos 65^\circ}{1 - \cos 65^\circ}$$
- Divide both sides by  $1 - \cos 65^\circ$
- $$= 16$$
- Answer rounded to 2 significant digits

45. Using  $\triangle ABC$ , we find side  $x$ :

$$\tan 62^\circ = \frac{x}{42}$$

Tangent relationship

$$x = 42 \tan 62^\circ$$

Multiply both sides by 42

$$= 79$$

Answer rounded to 2 significant digits

Next, using  $\triangle ABD$ , we find side  $h$ :

$$\tan 27^\circ = \frac{h}{x}$$

Tangent relationship

$$= \frac{h}{79}$$

Substitute value for  $x$

$$h = 79 \tan 27^\circ$$

Multiply both sides by 79

$$h = 40$$

Answer rounded to 2 significant digits

47. Using  $\triangle ABC$ , we find side  $x$ :

$$\sin 41^\circ = \frac{x}{32}$$

Sine relationship

$$x = 32 \sin 41^\circ$$

Multiply both sides by 32

$$= 21$$

Answer rounded to 2 significant digits

Next, using  $\triangle ABD$ , we find  $\angle ABD$ :

$$\tan \angle ABD = \frac{h}{x}$$

Tangent relationship

$$= \frac{19}{21}$$

Substitute known values

$$= 0.9047$$

Divide 19 by 21

$$\angle ABD = \tan^{-1}(0.9047)$$

Use calculator to find angle

$$\angle ABD = 42^\circ$$

Answer rounded to the nearest degree

49. Using  $\triangle BCD$ , we find side  $x$ :

$$\cos 58^\circ = \frac{x}{14}$$

Cosine relationship

$$x = 14 \cos 58^\circ$$

Multiply both sides by 14

$$x = 7.4$$

Answer rounded to 2 significant digits

Next, using  $\triangle ABC$ , we find  $y$ :

$$\cos 41^\circ = \frac{x+y}{18}$$

Cosine relationship

$$x+y = 18 \cos 41^\circ$$

Multiply both sides by 18

$$x+y = 13.58$$

Evaluate right side

$$7.4+y = 13.58$$

Substitute value for  $x$

$$y = 6.18 \approx 6.2$$

Subtract 7.4 from both sides and round to 2 significant digits

51. Using  $\triangle ABC$ , we find side  $h$ :

$$\sin 41^\circ = \frac{h}{28}$$

Sine relationship

$$h = 28 \sin 41^\circ$$

Multiply both sides by 28

$$= 18$$

Answer rounded to 2 significant digits

Next, using  $\triangle BCD$ , we find side  $x$ :

$$\tan 58^\circ = \frac{h}{x}$$

Tangent relationship

$$\tan 58^\circ = \frac{18}{x}$$

Substitute value found for  $h$

$$x = \frac{18}{\tan 58^\circ} = 11$$

Solve for  $x$  and round to 2 significant digits

53. Since  $h$  is in both  $\triangle ABC$  and  $\triangle BCD$ , we will solve for  $h$  in the two triangles:

$$\text{In } \triangle BCD, \tan 57^\circ = \frac{h}{x}$$

Tangent relationship

$$h = x \tan 57^\circ$$

Multiply both sides by  $x$

$$\text{In } \triangle ABC, \tan 43^\circ = \frac{h}{x+y}$$

Tangent relationship

$$h = (x+y) \tan 43^\circ$$

Multiply both sides by  $x+y$

$$h = (x+11) \tan 43^\circ$$

Substitute value for  $y$

$$\text{Therefore, } x \tan 57^\circ = (x+11) \tan 43^\circ$$

Property of equality

$$x \tan 57^\circ = x \tan 43^\circ + 11 \tan 43^\circ$$

Distribution Property

$$x \tan 57^\circ - x \tan 43^\circ = 11 \tan 43^\circ$$

Subtract  $x \tan 43^\circ$  from both sides

$$x(\tan 57^\circ - \tan 43^\circ) = 11 \tan 43^\circ$$

Factor left side

$$x = \frac{11 \tan 43^\circ}{\tan 57^\circ - \tan 43^\circ} = 17$$

Divide both sides by  $\tan 57^\circ - \tan 43^\circ$

55. From Problem 69 in Problem Set 2.1, we found that

$$\sin \theta = \frac{1}{\sqrt{3}}$$

$$= 0.5774$$

$$\theta = \sin^{-1}(0.5774) = 35.3^\circ$$

57. From Problem 69 in Problem Set 2.1, we found that

$$\cos \theta = \frac{\sqrt{2}}{\sqrt{3}}$$

$$= 0.8165$$

$$\theta = \cos^{-1}(0.8165) = 35.3^\circ$$

59. We know that  $EC = DF = 6$  ft,  $EB = 78$  ft,  $CB = 72$  ft,  $DB = 60$  ft, and  $\angle FAB = 45^\circ$ .

$$\tan \angle EAB = \frac{78}{54}$$

$$\tan \angle CAB = \frac{72}{54}$$

$$\tan \angle DAB = \frac{60}{54}$$

$$\angle EAB = \tan^{-1} \frac{78}{54}$$

$$\angle CAB = \tan^{-1} \frac{72}{54}$$

$$\angle DAB = \tan^{-1} \frac{60}{54}$$

$$\angle EAB = 55.3^\circ$$

$$\angle CAB = 53.1^\circ$$

$$\angle DAB = 48.0^\circ$$

$$\angle EAC = \angle EAB - \angle CAB$$

and

$$\angle DAF = \angle DAB - \angle FAB$$

$$= 55.3^\circ - 53.1^\circ = 2.2^\circ$$

$$= 48.0^\circ - 45^\circ = 3.0^\circ$$

Therefore, the sum of the angles is  $5.2^\circ$ .

$$\begin{aligned}
 63. \quad \cos 120^\circ &= \frac{x}{125} = \frac{139-h}{125} \\
 125 \cos 120^\circ &= 139-h \\
 h &= 139-125 \cos 120^\circ \\
 &= 201.5 \\
 &= 200 \text{ ft}
 \end{aligned}$$

Solve for  $h$

Round to 2 significant digits

$$\begin{aligned}
 65. \quad r &= 98.5 \\
 \text{a.} \quad h &= 12 + 98.5 + x \\
 \cos 60.0^\circ &= \frac{x}{98.5} \\
 x &= 98.5 \cos 60.0^\circ \\
 &= 49.25 \\
 h &= 12 + 98.5 + 49.25 \\
 &= 159.8 \approx 160 \text{ ft} \\
 \text{b.} \quad h &= 12 + 98.5 + x \\
 \cos 30.0^\circ &= \frac{x}{98.5} \\
 x &= 98.5 \cos 30.0^\circ \\
 &= 85.3 \\
 h &= 12 + 98.5 + 85.3 \\
 &= 195.8 \approx 196 \text{ ft}
 \end{aligned}$$

$$\begin{aligned}
 \text{c.} \quad r + 12 &= 98.5 + 12 = 110.5 \\
 h &= 110.5 - x \\
 \cos 45.0^\circ &= \frac{x}{98.5} \\
 x &= 98.5 \cos 45.0^\circ \\
 &= 69.7 \\
 h &= 110.5 - 69.7 \\
 &= 40.8 \text{ ft}
 \end{aligned}$$

$$67. \quad \text{The radius of the London Eye is } \frac{135}{2} = 67.5.$$

$$\begin{aligned}
 \cos \theta &= \frac{67.5 - 44.5}{67.5} \\
 &= \frac{23}{67.5} \\
 \theta &= \cos^{-1}(0.6592)
 \end{aligned}$$

$$\begin{aligned}
 71. \quad \theta &= 70.1^\circ \\
 \sec \theta &= 2
 \end{aligned}$$

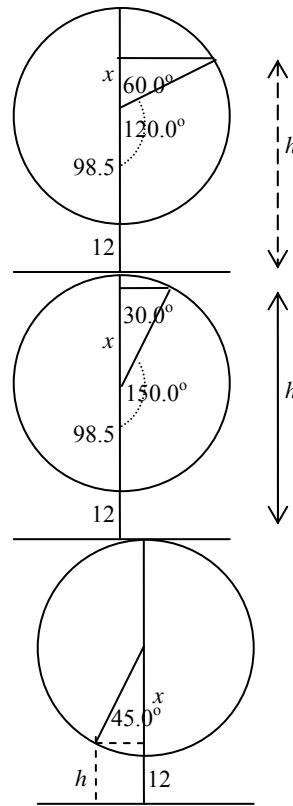
$$\begin{aligned}
 \cos \theta &= \frac{1}{\sec \theta} \\
 &= \frac{1}{2}
 \end{aligned}$$

$$\cos^2 \theta = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Reciprocal identity

Substitute known value

Square both sides





73.  $\cos \theta = -\sqrt{1 - \sin^2 \theta}$  Pythagorean identity,  $\theta$  in QIII  
 $= -\sqrt{1 - \left(-\frac{2}{3}\right)^2} = -\sqrt{1 - \frac{4}{9}}$   
 $= -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$

75.  $\cos \theta = -\sqrt{1 - \sin^2 \theta}$  Pythagorean identity,  $\theta$  in QII  
 $= -\sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$  Substitute known value  
 $= -\sqrt{1 - \frac{3}{4}}$  Simplify  
 $= -\sqrt{\frac{1}{4}} = -\frac{1}{2}$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$  Ratio identity  
 $= \frac{\sqrt{3}/2}{-1/2}$  Substitute known values  
 $= -\sqrt{3}$  Simplify

$\csc \theta = \frac{1}{\sin \theta}$  Reciprocal identity  
 $= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec \theta = \frac{1}{\cos \theta}$  Reciprocal identity  
 $= \frac{1}{-1/2} = -2$

$\cot \theta = \frac{1}{\tan \theta}$  Reciprocal identity  
 $= \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

77.  $\cos \theta = \frac{1}{\sec \theta}$  Reciprocal identity  
 $= -\frac{1}{2}$  Substitute known values

$\tan \theta = \frac{\sin \theta}{\cos \theta}$  Ratio identity  
 $= \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$  Substitute values

$\sin \theta = -\sqrt{1 - \cos^2 \theta}$  Pythagorean identity,  $\theta$  in QIII  
 $= -\sqrt{1 - \left(-\frac{1}{2}\right)^2}$  Substitute value for  $\cos \theta$   
 $= -\sqrt{1 - \frac{1}{4}} = -\sqrt{\frac{3}{4}}$  Simplify  
 $= -\frac{\sqrt{3}}{2}$

$\cot \theta = \frac{1}{\tan \theta}$  Reciprocal identity  
 $= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

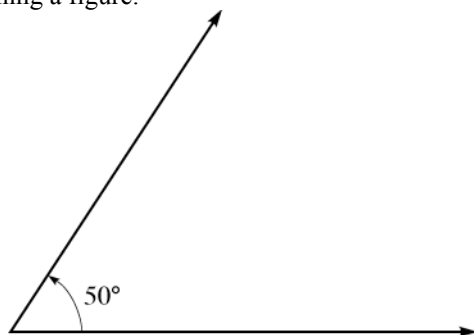
$\csc \theta = \frac{1}{\sin \theta}$  Reciprocal identity  
 $= \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

81.  $\tan B = \frac{35}{58}$   
 $B = 31^\circ$   
The answer is b.

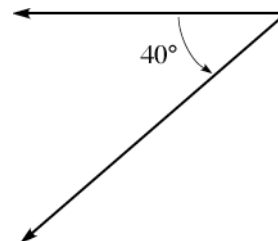
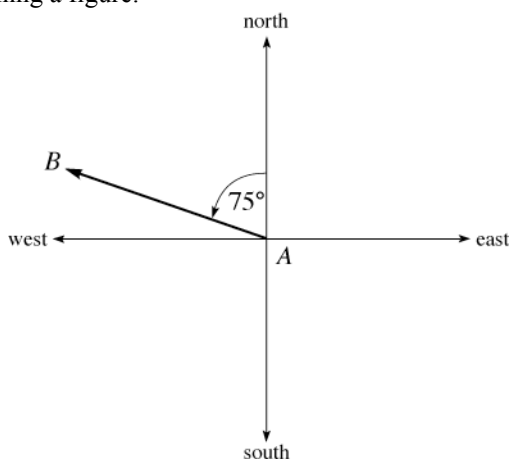
## 2.4 Applications

### EVEN SOLUTIONS

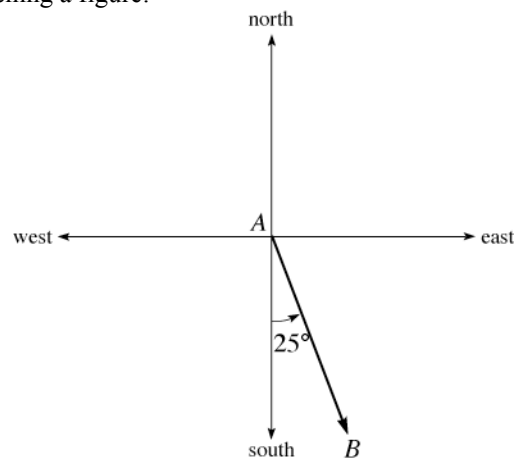
2. If an observer positioned at the vertex of an angle views an object in the direction of the non-horizontal side of the angle, then this side is called the line of sight of the observer.
4. The bearing of a line is always measured as an angle from the north or south rotating toward the east or west.
6. Sketching a figure:
8. Sketching a figure:



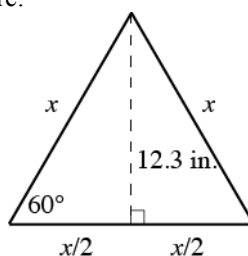
10. Sketching a figure:



12. Sketching a figure:



14. Call  $x$  the length of each side. Note the figure:



Therefore:

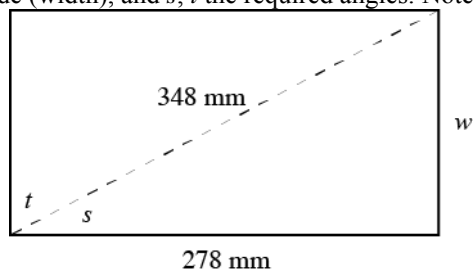
$$\sin 60^\circ = \frac{12.3}{x}$$

$$x \sin 60^\circ = 12.3$$

$$x = \frac{12.3}{\sin 60^\circ} \approx 14.2$$

The length of each side is 14.2 in.

16. Call  $w$  the length of the shorter side (width), and  $s, t$  the required angles. Note the figure:



Find  $w$  using the Pythagorean Theorem:

$$278^2 + w^2 = 348^2$$

$$w^2 = 348^2 - 278^2 = 43820$$

$$w \approx 209$$

Now find angles  $s$  and  $t$ :

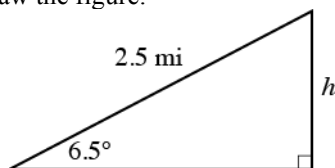
$$\cos s = \frac{278}{348}$$

$$s = \cos^{-1}\left(\frac{278}{348}\right) \approx 37.0^\circ$$

$$t = 90^\circ - 37.0^\circ = 53.0^\circ$$

The shorter side is 209 mm, and the two angles are  $37.0^\circ$  and  $53.0^\circ$ .

18. Let  $h$  represent the height of the hill. Draw the figure:



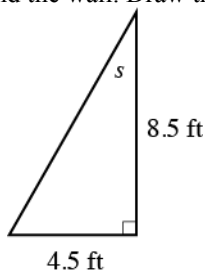
Therefore:

$$\sin 6.5^\circ = \frac{h}{2.5}$$

$$h = 2.5 \sin 6.5^\circ \approx 0.28$$

The hill is approximately 0.28 mi high, which is approximately 1,480 feet.

20. Let  $s$  represent the angle between the ladder and the wall. Draw the figure:



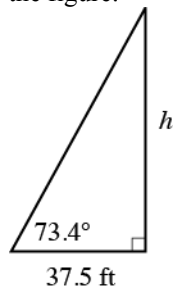
Therefore:

$$\tan s = \frac{4.5}{8.5}$$

$$s = \tan^{-1}\left(\frac{4.5}{8.5}\right) \approx 28^\circ$$

The angle between the ladder and the wall is approximately  $28^\circ$ .

22. Let  $h$  represent the height of the building. Draw the figure:



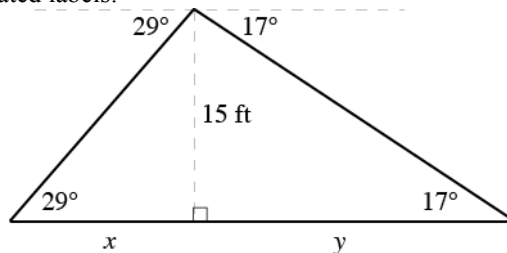
Therefore:

$$\tan 73.4^\circ = \frac{h}{37.5}$$

$$h = 37.5 \tan 73.4^\circ \approx 126$$

The height of the building is approximately 126 feet.

24. Draw the figure with the associated labels:



The sum  $x + y$  represents the width of the sand pile. Using the two triangles:

$$\tan 29^\circ = \frac{15}{x}$$

$$x \tan 29^\circ = 15$$

$$x = \frac{15}{\tan 29^\circ} \approx 27.1$$

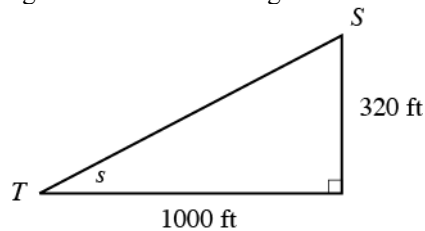
$$\tan 17^\circ = \frac{15}{y}$$

$$y \tan 17^\circ = 15$$

$$y = \frac{15}{\tan 17^\circ} \approx 49.1$$

The width of the sand pile is therefore  $27.1 + 49.1 \approx 76$  feet.

26. a. First note that  $\frac{5}{8}$  in.  $= \frac{5}{8} \cdot 1600 = 1000$  ft, which is the horizontal distance between Stacey and Travis.  
 b. There are 8 contour intervals between Stacey and Travis, which corresponds to a vertical distance of  $8 \cdot 40 = 320$  ft.  
 c. Let  $s$  represent the elevation angle. Construct the triangle:



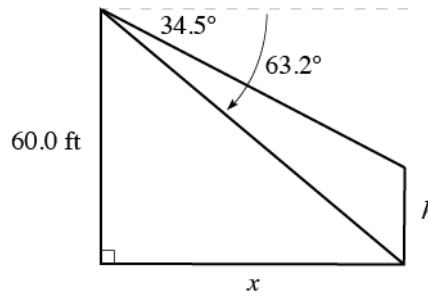
Therefore:

$$\tan s = \frac{320}{1000} = 0.32$$

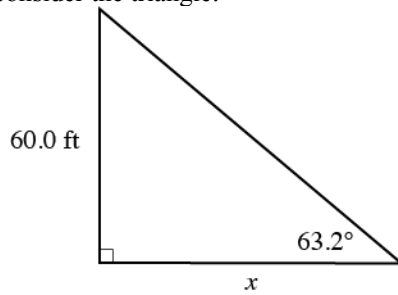
$$s = \tan^{-1}(0.32) \approx 17.7^\circ$$

The elevation angle from Travis to Stacey is approximately  $17.7^\circ$ .

28. Construct the figure:



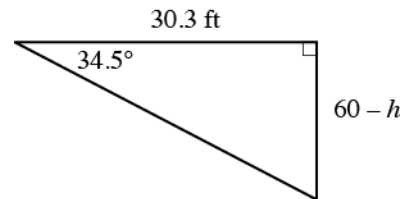
First consider the triangle:



Therefore:

$$\begin{aligned}\tan 63.2^\circ &= \frac{60.0}{x} \\ x \tan 63.2^\circ &= 60.0 \\ x &= \frac{60.0}{\tan 63.2^\circ} \approx 30.3 \text{ ft}\end{aligned}$$

Now consider the triangle:

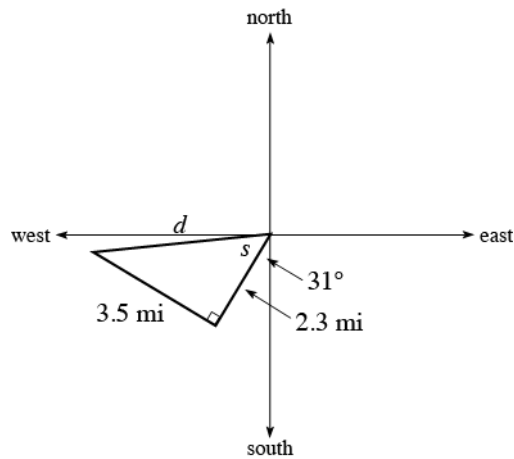


Therefore:

$$\begin{aligned}\tan 34.5^\circ &= \frac{60 - h}{30.3} \\ 60 - h &= 30.3 \tan 34.5^\circ \\ h &= 60 - 30.3 \tan 34.5^\circ \approx 39.2 \text{ ft}\end{aligned}$$

The building next door is approximately 39.2 feet tall.

30. Construct the figure:



To find his distance from the starting point, use the Pythagorean Theorem:

$$d^2 = 3.5^2 + 2.3^2 = 17.54$$

$$d = \sqrt{17.54} \approx 4.2 \text{ mi}$$

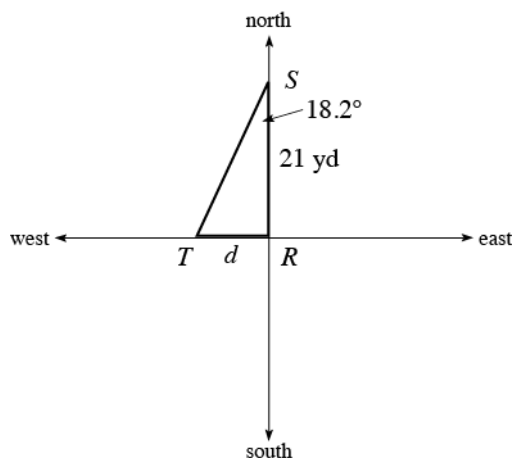
Now find angle  $s$ :

$$\tan s = \frac{3.5}{2.3}$$

$$s = \tan^{-1}\left(\frac{3.5}{2.3}\right) \approx 57^\circ$$

His bearing is S  $88^\circ$  W.

32. Construct the figure:



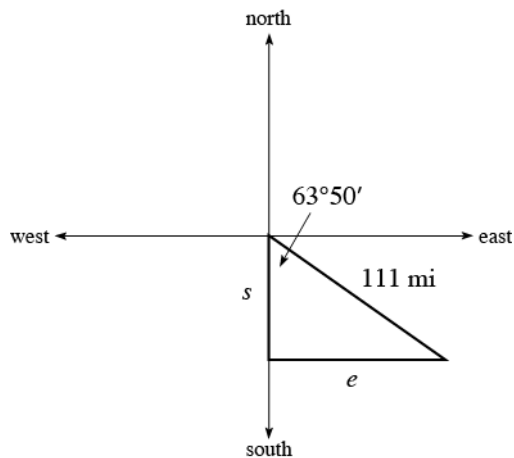
Therefore:

$$\tan 18.2^\circ = \frac{d}{21.0}$$

$$d = 21.0 \tan 18.2^\circ \approx 6.90$$

The distance from the tree to the rock is 6.90 yards.

34. Construct the figure:



Therefore:

$$\sin 63^\circ 50' = \frac{e}{111}$$

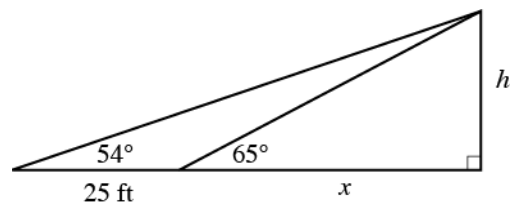
$$e = 111 \sin 63^\circ 50' \approx 99.6 \text{ mi}$$

The boat travels 48.9 mi south and 99.6 mi east.

$$\cos 63^\circ 50' = \frac{s}{111}$$

$$s = 111 \cos 63^\circ 50' \approx 48.9 \text{ mi}$$

36. Draw the figure:



From the smaller triangle:

$$\begin{aligned}\tan 65^\circ &= \frac{h}{x} \\ x \tan 65^\circ &= h \\ x &= \frac{h}{\tan 65^\circ}\end{aligned}$$

Setting these two expressions equal:

$$\begin{aligned}\frac{h}{\tan 65^\circ} &= \frac{h}{\tan 54^\circ} - 25 \\ h \cot 65^\circ &= h \cot 54^\circ - 25 \\ h \cot 65^\circ - h \cot 54^\circ &= -25 \\ h(\cot 65^\circ - \cot 54^\circ) &= -25 \\ h &= \frac{25}{\cot 54^\circ - \cot 65^\circ} \approx 96\end{aligned}$$

The height of the obelisk is approximately 96 feet.

38. First find the length  $CB$ :

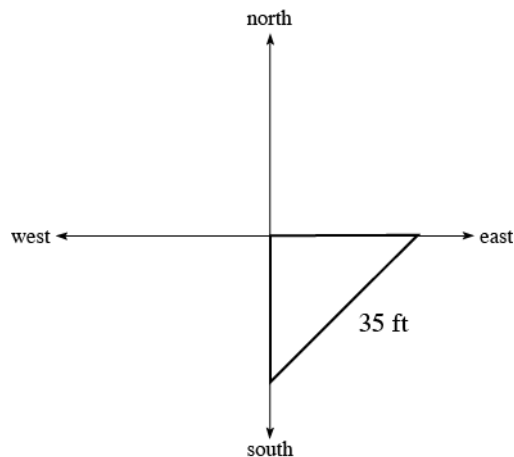
$$\begin{aligned}\tan 12.3^\circ &= \frac{426}{CB} \\ CB \tan 12.3^\circ &= 426 \\ CB &= \frac{426}{\tan 12.3^\circ} \approx 1,954 \text{ ft}\end{aligned}$$

Therefore:

$$\begin{aligned}\sin \angle BCA &= \frac{AB}{CB} \\ \sin 57.5^\circ &= \frac{AB}{1954} \\ AB &= 1954 \sin 57.5^\circ \approx 1,650 \text{ ft}\end{aligned}$$

A rescue boat at  $A$  will have to travel approximately 1,650 feet to reach any survivors at point  $B$ .

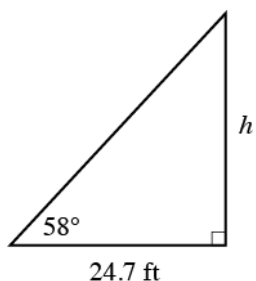
40. Construct a figure:



From the larger triangle:

$$\begin{aligned}\tan 54^\circ &= \frac{h}{25+x} \\ (25+x) \tan 54^\circ &= h \\ 25+x &= \frac{h}{\tan 54^\circ} \\ x &= \frac{h}{\tan 54^\circ} - 25\end{aligned}$$

First note that each person is  $\frac{35}{\sqrt{2}}$  ft  $\approx 24.7$  ft from the base of the tree. Let  $h$  represent the height of the tree. Now construct the triangle:



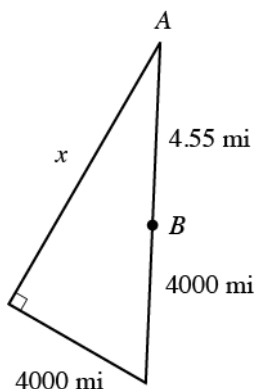
Therefore:

$$\tan 58^\circ = \frac{h}{24.7}$$

$$h = 24.7 \tan 58^\circ \approx 40 \text{ ft}$$

The tree is approximately 27 feet tall.

42. Construct the figure (not drawn to scale):



Using the Pythagorean Theorem:

$$x^2 + 3960^2 = 3964.55^2$$

$$x^2 + 15,681,600 = 15,717,656.7$$

$$x^2 = 36,056.7$$

$$x \approx 190$$

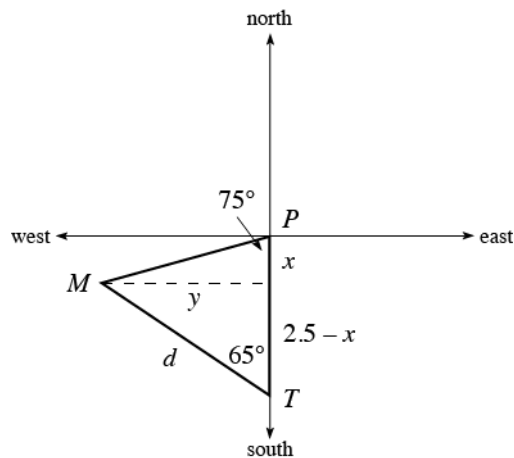
The plane is 190 miles from the horizon. Now find angle  $A$ :

$$\sin A = \frac{3960}{3964.55}$$

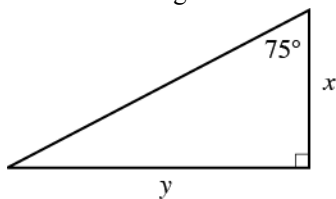
$$A = \sin^{-1}\left(\frac{3960}{3964.55}\right) \approx 87.3^\circ$$



44. Construct the figure:



Consider the two triangles:



Therefore:

$$\tan 75^\circ = \frac{y}{x}$$

$$y = x \tan 75^\circ$$

Setting these two expressions equal:

$$(2.5 - x) \tan 65^\circ = x \tan 75^\circ$$

$$2.5 \tan 65^\circ - x \tan 65^\circ = x \tan 75^\circ$$

$$2.5 \tan 65^\circ = x \tan 75^\circ + x \tan 65^\circ$$

$$2.5 \tan 65^\circ = x (\tan 75^\circ + \tan 65^\circ)$$

$$x = \frac{2.5 \tan 65^\circ}{\tan 75^\circ + \tan 65^\circ} \approx 0.91$$

Therefore:

$$\cos 65^\circ = \frac{2.5 - x}{d} = \frac{2.5 - 0.91}{d} = \frac{1.59}{d}$$

$$d \cos 65^\circ = 1.59$$

$$d = \frac{1.59}{\cos 65^\circ} \approx 3.8 \text{ mi}$$

Tim is approximately 3.8 miles from the missile when it is launched.

46. Note that  $\sin \theta_1 = \frac{1}{\sqrt{2}}$ ,  $\sin \theta_2 = \frac{1}{\sqrt{3}}$ ,  $\sin \theta_3 = \frac{1}{\sqrt{4}}$ , ..., thus  $\sin \theta_n = \frac{1}{\sqrt{n+1}}$ .

48. a. Let  $x$  represent the height that is illuminated on the floor. Then:

$$\tan 84^\circ = \frac{4}{x}$$

$$x = \frac{4}{\tan 84^\circ} \approx 0.42$$

The illuminated area is then:  $(0.42)(6.5) \approx 2.7 \text{ ft}^2$ .

b. Following the procedure from part a:

$$\tan 37^\circ = \frac{4}{x}$$

$$x = \frac{4}{\tan 37^\circ} \approx 5.31$$

The illuminated area is then:  $(5.31)(6.5) \approx 34.5 \text{ ft}^2$ . The area is much larger on the winter day.

50. Simplifying:  $\frac{1}{\sin \theta} - \sin \theta = \frac{1}{\sin \theta} - \sin \theta \cdot \frac{\sin \theta}{\sin \theta} = \frac{1 - \sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$

52. Working from the left side:  $\cos \theta \csc \theta \tan \theta = \cos \theta \cdot \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} = 1$

54. Working from the left side:  $(1 - \cos \theta)(1 + \cos \theta) = 1 - \cos \theta + \cos \theta - \cos^2 \theta = 1 - \cos^2 \theta = \sin^2 \theta$

56. Working from the left side:  $1 - \frac{\cos \theta}{\sec \theta} = 1 - \frac{\cos \theta}{1/\cos \theta} = 1 - \cos^2 \theta = \sin^2 \theta$

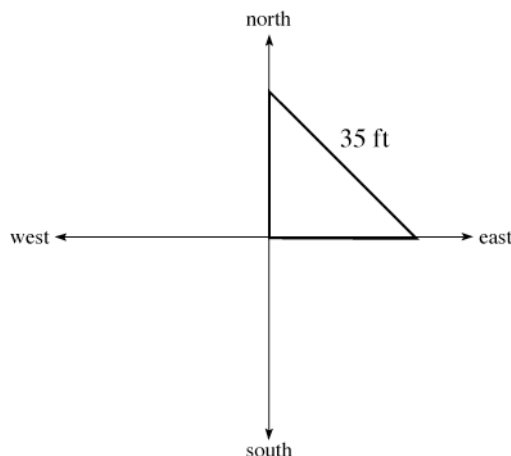
58. Let  $h$  represent the height of the flagpole. Then:

$$\tan 74.3^\circ = \frac{h}{22.5}$$

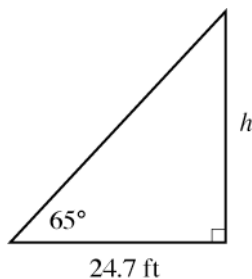
$$h = 22.5 \tan 74.3^\circ \approx 80.0$$

The flagpole is 80.0 feet tall. The correct answer is d.

60. Construct a figure:



First note that each person is  $\frac{35}{\sqrt{2}} \text{ ft} \approx 24.7 \text{ ft}$  from the base of the tree. Let  $h$  represent the height of the tree. Now construct the triangle:



Therefore:

$$\tan 65^\circ = \frac{h}{24.7}$$

$$h = 24.7 \tan 65^\circ \approx 53 \text{ ft}$$

The tree is approximately 53 feet tall. The correct answer is a.

## ODD SOLUTIONS

1. elevation, depression

3. north-south

For problems 5 through 11, see diagrams in textbook answer section.

13. To find the height,  $h$ , we can use the Pythagorean Theorem:

$$h^2 + (16)^2 = (42)^2$$

$$h^2 + 256 = 1,764$$

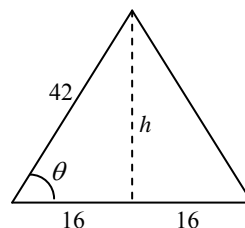
$$h^2 = 1,508$$

$$h = \pm\sqrt{1,508} = 39\text{ cm}$$

To find angle  $\theta$ , we can use the cosine ratio:

$$\cos \theta = \frac{16}{42}$$

$$\theta = \cos^{-1}\left(\frac{16}{42}\right) = 68^\circ$$



The height is 39 cm and the two equal angles are  $68^\circ$ .

15. Consider the right triangle with sides of 25.3 cm and 5.2 cm (one-half of the diameter):

$$\tan \theta = \frac{25.3}{5.2}$$

$$= 4.8654$$

$$\theta = \tan^{-1}(4.8654)$$

$$= 78.4^\circ$$

The angle the side makes with the base is  $78.4^\circ$ .

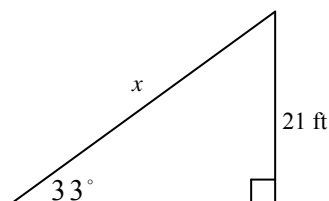
17. To find the length of the escalator,  $x$ , we use the sine ratio:

$$\sin 33^\circ = \frac{21}{x}$$

$$x = \frac{21}{\sin 33^\circ}$$

$$= \frac{21}{0.5446} = 39 \text{ ft}$$

The length of the escalator is 39 feet.



19. 
$$\sin \theta = \frac{43.2}{72.5}$$

$$= 0.5959$$

$$\theta = \sin^{-1}(0.5959)$$

$$= 36.6^\circ$$

The angle the rope makes with the pole is  $36.6^\circ$

21. We use the tangent ratio to find the angle of elevation to the sun,  $\theta$ :

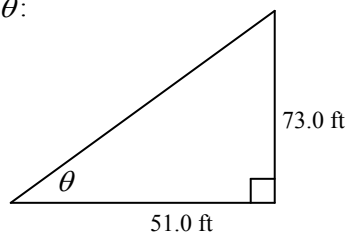
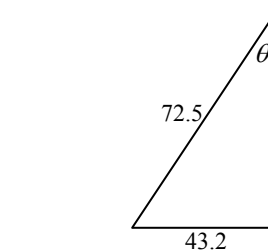
$$\tan \theta = \frac{73.0}{51.0}$$

$$= 1.4313$$

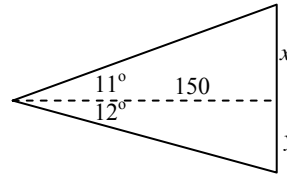
$$\theta = \tan^{-1}(1.4313)$$

$$= 55.1^\circ$$

The angle of elevation to the sun is  $55.1^\circ$ .



23.  $\tan 11^\circ = \frac{x}{150}$   
 $x = 150 \tan 11^\circ = 29 \text{ cm}$   
 $\tan 12^\circ = \frac{y}{150}$   
 $y = 150 \tan 12^\circ$   
 $= 32 \text{ cm}$



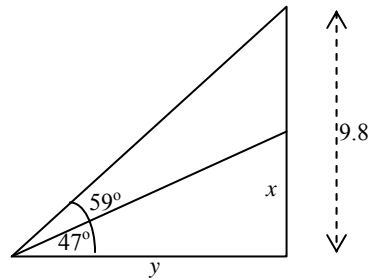
The vertical dimension of the mirror is  $x + y$  or 61 cm.

25. a. horizontal distance =  $0.50(1,600) = 800 \text{ ft}$   
b. vertical distance = (number of contour intervals)(40)  
 $= 5(40)$   
 $= 200 \text{ ft}$

c.  $\tan \theta = \frac{\text{vertical distance}}{\text{horizontal distance}}$   
 $= \frac{200}{800}$   
 $= 0.25$   
 $\theta = \tan^{-1}(0.25)$   
 $= 14^\circ$

27.  $\tan 59^\circ = \frac{9.8}{y}$   
 $y = \frac{9.8}{\tan 59^\circ} = \frac{9.8}{1.6643} = 5.9$

$\tan 47^\circ = \frac{9.8}{x}$   
 $x = y \tan 47^\circ$   
 $= 5.9(1.0724) = 6.3 \text{ ft}$



The vertical dimension of the door is 6.3 feet.

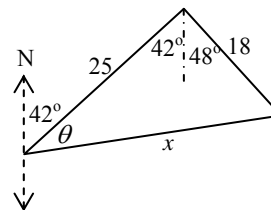
29. We use the Pythagorean Theorem to find the distance  $x$ :

$x^2 = 25^2 + 18^2$   
 $= 625 + 324$   
 $= 949$

$x = 31 \text{ mi}$

We use the tangent relationship to find angle  $\theta$ :

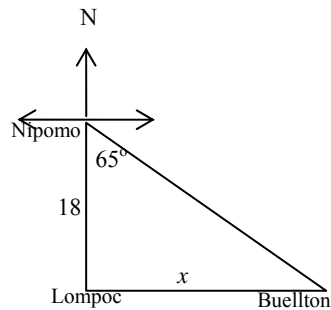
$\tan \theta = \frac{18}{25}$   
 $= 0.72$   
 $\theta = \tan^{-1}(0.72)$   
 $= 36^\circ$



To find the bearing we add  $42^\circ + 36^\circ = 78^\circ$ . The boat is 31 miles from the harbor entrance and its bearing is N  $78^\circ$  E.

$$\begin{aligned}
 31. \quad \tan 65^\circ &= \frac{x}{18} \\
 x &= 18 \tan 65^\circ \\
 &= 18(2.1445) \\
 &= 39 \text{ mi}
 \end{aligned}$$

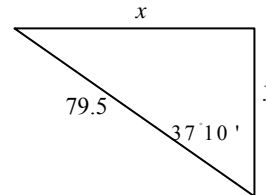
The distance from Lompoc to Buellton is 39 miles.



33. We will call the west distance,  $x$  and the north distance,  $y$ :

$$\begin{aligned}
 \sin 37^\circ 10' &= \frac{x}{79.5} & \cos 37^\circ 10' &= \frac{y}{79.5} \\
 x &= 79.5 \sin 37^\circ 10' & y &= 79.5 \cos 37^\circ 10' \\
 &= 48.0 \text{ mi} & &= 63.4 \text{ mi}
 \end{aligned}$$

The boat has traveled 48.0 miles west and 63.4 miles north.



$$\begin{aligned}
 35. \quad \text{In } \triangle ABC, \tan 42.17^\circ &= \frac{h}{x+33} \\
 h &= (x+33) \tan 42.17^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{In } \triangle BCD, \tan 47.5^\circ &= \frac{h}{x} \\
 h &= x \tan 47.5^\circ
 \end{aligned}$$

$$\text{Therefore, } x \tan 47.5^\circ = (x+33) \tan 42.17^\circ$$

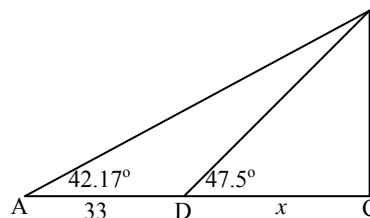
$$x \tan 47.5^\circ = x \tan 42.17^\circ + 33 \tan 42.17^\circ$$

$$x \tan 47.5^\circ - x \tan 42.17^\circ = 33 \tan 42.17^\circ$$

$$x (\tan 47.5^\circ - \tan 42.17^\circ) = 33 \tan 42.17^\circ$$

$$x = \frac{33 \tan 42.17^\circ}{\tan 47.5^\circ - \tan 42.17^\circ} = 161 \text{ ft}$$

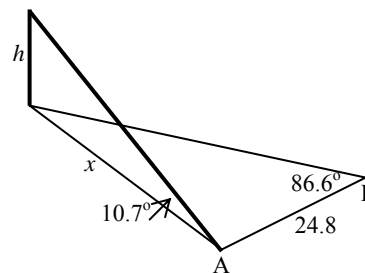
The person at point A is 161 feet from the base of the antenna.



$$\begin{aligned}
 37. \quad \tan 86.6^\circ &= \frac{x}{24.8} \\
 x &= 24.8 \tan 86.6^\circ \\
 &= 24.8(16.8319) \\
 &= 417.431
 \end{aligned}$$

$$\begin{aligned}
 \tan 10.7^\circ &= \frac{h}{x} \\
 h &= x \tan 10.7^\circ \\
 &= (417.431)(0.18895) \\
 &= 78.9 \text{ ft}
 \end{aligned}$$

The tree is 78.9 feet high.



39. First, we will find each person's distance from the pole,  $x$ , using the Pythagorean Theorem:

$$x^2 + x^2 = 25^2$$

$$2x^2 = 625$$

$$x^2 = 312.5$$

$$x = 17.678 \text{ ft}$$

Next, we will find the height of the pole,  $h$ , using the tangent relationship:

$$\tan 56^\circ = \frac{h}{17.678}$$

$$h = 17.678 \tan 56^\circ$$

$$= 26 \text{ ft}$$

The height of the pole is 26 feet.

41.

$$\sin 76.6^\circ = \frac{r}{r + 112}$$

$$r = (r + 112) \sin 76.6^\circ$$

$$r = r \sin 76.6^\circ + 112 \sin 76.6^\circ$$

$$r - r \sin 76.6^\circ = 112 \sin 76.6^\circ$$

$$r(1 - \sin 76.6^\circ) = 112 \sin 76.6^\circ$$

$$r = \frac{112 \sin 76.6^\circ}{1 - \sin 76.6^\circ}$$

$$= \frac{112(0.9728)}{1 - 0.9728}$$

$$= \frac{108.9509}{0.02722}$$

$$= 4,000 \text{ mi}$$

The radius of the earth is 4,000 miles.

43. We want to find  $x$  and  $y$  in terms of  $h$

$$\tan 53^\circ = \frac{h}{x}$$

$$\tan 31^\circ = \frac{h}{y}$$

$$x \tan 53^\circ = h$$

$$y \tan 31^\circ = h$$

$$x = \frac{h}{\tan 53^\circ}$$

$$y = \frac{h}{\tan 31^\circ}$$

We know that  $x + y = 15$ . Therefore,

$$\frac{h}{\tan 53^\circ} + \frac{h}{\tan 31^\circ} = 15$$

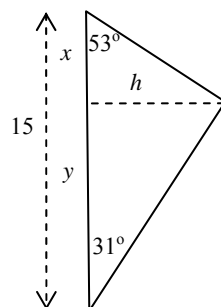
$$h \left( \frac{1}{\tan 53^\circ} + \frac{1}{\tan 31^\circ} \right) = 15$$

$$h(0.7536 + 1.6643) = 15$$

$$2.4179h = 15$$

$$h = \frac{15}{2.4179} = 6.2 \text{ mi}$$

The ship is 6.2 miles from the shore.

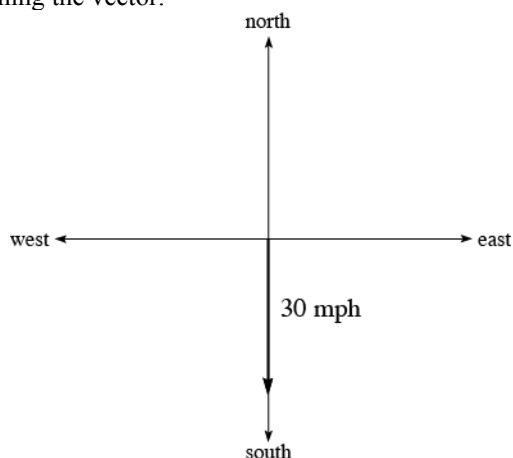


45.  $\tan \theta_1 = \frac{1}{1}$        $\tan \theta_2 = \frac{1}{\sqrt{2}}$        $\tan \theta_3 = \frac{1}{\sqrt{3}}$   
 $= 1$        $= 0.7071$        $= 0.5774$   
 $\theta_1 = \tan^{-1}(1)$        $\theta_2 = \tan^{-1}(0.7071)$        $\theta_3 = \tan^{-1}(0.5774)$   
 $\theta_1 = 45.00^\circ$        $\theta_2 = 35.26^\circ$        $\theta_3 = 30.00^\circ$
49.  $(\sin \theta - \cos \theta)^2 = (\sin \theta - \cos \theta)(\sin \theta - \cos \theta)$   
 $= \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta$   
 $= \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$   
 $= 1 - 2 \sin \theta \cos \theta$
51.  $\sin \theta \cot \theta = \sin \theta \cdot \frac{\cos \theta}{\sin \theta}$       Ratio identity  
 $= \frac{\sin \theta \cos \theta}{\sin \theta}$       Multiplication of fractions  
 $= \cos \theta$       Division of common factor
53.  $\frac{\sec \theta}{\tan \theta} = \frac{\frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}}$       Reciprocal and ratio identity  
 $= \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}$       Division of fractions  
 $= \frac{1}{\sin \theta}$       Multiplication of fractions and divide common factor  
 $= \csc \theta$
55.  $\sec \theta - \cos \theta = \frac{1}{\cos \theta} - \cos \theta$       Reciprocal identity  
 $= \frac{1}{\cos \theta} - \cos \theta \cdot \frac{\cos \theta}{\cos \theta}$       L.C.D. is  $\cos \theta$   
 $= \frac{1 - \cos^2 \theta}{\cos \theta}$       Subtraction of fractions  
 $= \frac{\sin^2 \theta}{\cos \theta}$       Pythagorean identity

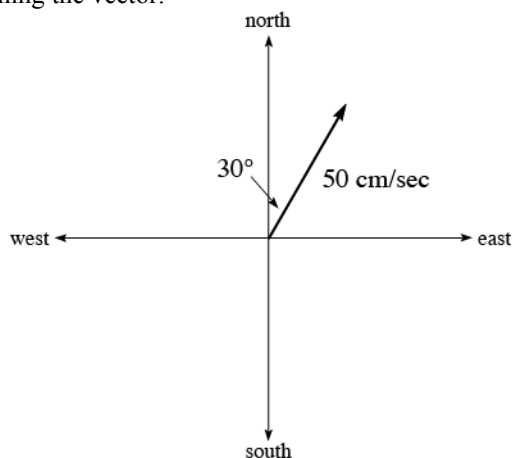
## 2.5 Vectors: A Geometric Approach

### EVEN SOLUTIONS

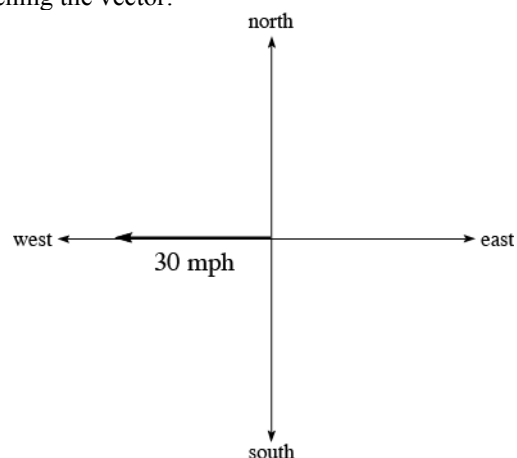
2. Two vectors are equivalent if they have the same magnitude and direction.
4. A vector is in standard position if the tail of the vector is placed at the origin of a rectangular coordinate system.
6. If  $\mathbf{V}$  makes an angle  $\theta$  with the positive  $x$ -axis when in standard position, then  $|\mathbf{V}_x| = |\mathbf{V}| \cos \theta$  and  $|\mathbf{V}_y| = |\mathbf{V}| \sin \theta$ .
8. If a constant force  $\mathbf{F}$  is applied to an object and moves the object in a straight line a distance  $d$  at an angle  $\theta$  with the force, then the work performed by the force is found by multiplying  $|\mathbf{F}| \cos \theta$  and  $d$ .
10. Sketching the vector:



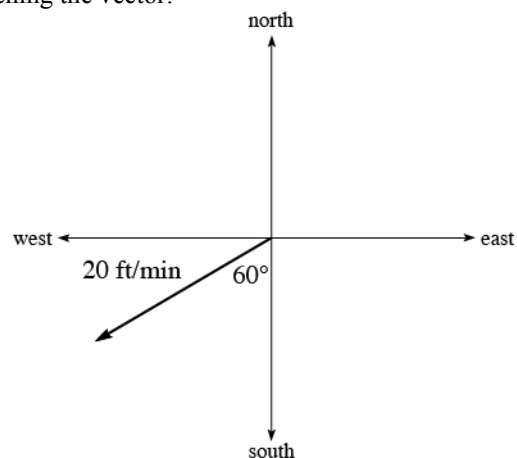
14. Sketching the vector:



12. Sketching the vector:

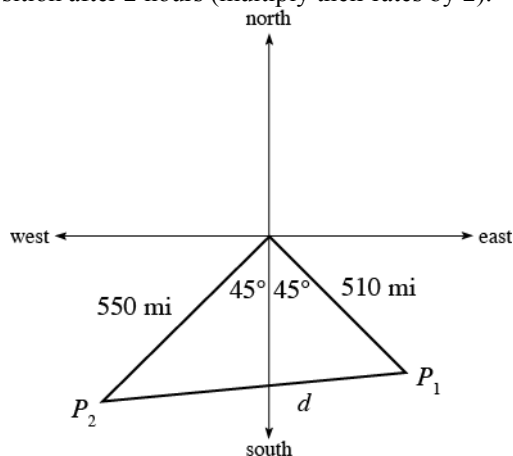


16. Sketching the vector:





18. Construct the figure for their position after 2 hours (multiply their rates by 2):

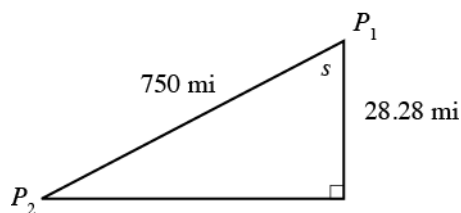


Using the Pythagorean Theorem:  $d = \sqrt{550^2 + 510^2} \approx 750$  miles

To find the bearing from  $P_1$  to  $P_2$ , first find the vertical (north-south) change in their positions. This is given by:

$$550 \sin 45^\circ - 510 \sin 45^\circ \approx 28.28 \text{ miles}$$

Construct the triangle:



Therefore:

$$\cos s = \frac{28.28}{750}$$

$$s = \cos^{-1}\left(\frac{28.28}{750}\right) \approx 87.8^\circ$$

The bearing from  $P_1$  to  $P_2$  is S  $87.8^\circ$  W.

20. Computing the magnitudes of  $\mathbf{V}_x$  and  $\mathbf{V}_y$ :

$$|\mathbf{V}_x| = 17.6 \cos 72.6^\circ \approx 5.26$$

$$|\mathbf{V}_y| = 17.6 \sin 72.6^\circ \approx 16.8$$

22. Computing the magnitudes of  $\mathbf{V}_x$  and  $\mathbf{V}_y$ :

$$|\mathbf{V}_x| = 383 \cos 12^\circ 20' \approx 374$$

$$|\mathbf{V}_y| = 383 \sin 12^\circ 20' \approx 81.8$$

24. Computing the magnitudes of  $\mathbf{V}_x$  and  $\mathbf{V}_y$ :

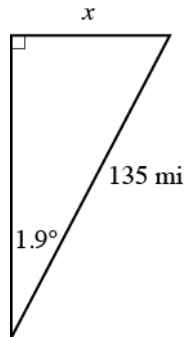
$$|\mathbf{V}_x| = 84 \cos 90^\circ = 0$$

$$|\mathbf{V}_y| = 84 \sin 90^\circ = 84$$

26. Using the Pythagorean Theorem:  $|\mathbf{V}| = \sqrt{54.2^2 + 14.5^2} \approx 56.1$

28. Using the Pythagorean Theorem:  $|\mathbf{V}| = \sqrt{2.2^2 + 8.8^2} \approx 9.1$

30. Construct the triangle:



Therefore:

$$\sin 1.9^\circ = \frac{x}{135}$$

$$x = 135 \sin 1.9^\circ \approx 4.48 \text{ miles}$$

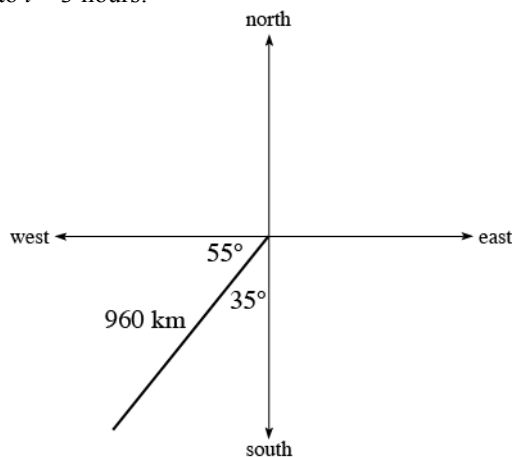
The plane will be approximately 4.48 miles off course.

32. Computing the magnitudes of  $\mathbf{V}_x$  and  $\mathbf{V}_y$ :

$$|\mathbf{V}_x| = 1,800 \cos 55^\circ = 1,032 \frac{\text{ft}}{\text{sec}} \approx 1,000 \frac{\text{ft}}{\text{sec}}$$

$$|\mathbf{V}_y| = 1,800 \sin 55^\circ \approx 1,474 \frac{\text{ft}}{\text{sec}} \approx 1,500 \frac{\text{ft}}{\text{sec}}$$

34. The horizontal distance traveled is  $1.5 \times 1,032 = 1,548 \text{ feet} \approx 1,500 \text{ feet}$ .  
 36. Draw the figure corresponding to  $t = 3 \text{ hours}$ :



The west and south distances are given by:

$$\text{west: } 960 \cos 55^\circ \approx 550 \text{ km}$$

$$\text{south: } 960 \sin 55^\circ \approx 790 \text{ km}$$

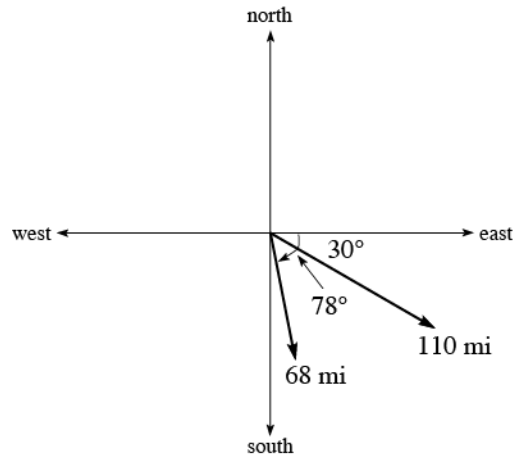
38. Using the Pythagorean Theorem:  $|\mathbf{V}| = \sqrt{16.5^2 + 24.3^2} \approx 29.4 \text{ ft/sec}$

The elevation angle is given by:

$$\tan \theta = \frac{24.3}{16.5}$$

$$\theta = \tan^{-1} \left( \frac{24.3}{16.5} \right) \approx 55.8^\circ$$

40. Construct the figure:

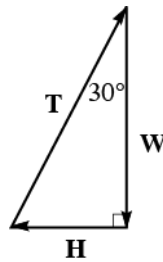


The total distance south and east is given by:

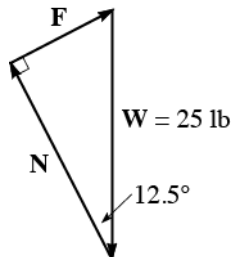
east:  $68 \cos 78^\circ + 110 \cos 30^\circ \approx 110$  miles

south:  $68 \sin 78^\circ + 110 \sin 30^\circ \approx 120$  miles

42. The corresponding force diagram would be:



44. The corresponding force diagram would be:



Therefore:

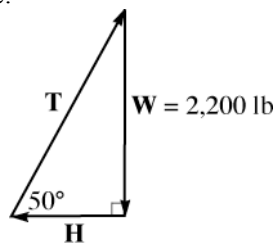
$$\sin 12.5^\circ = \frac{|F|}{25.0}$$

$$|F| = 25.0 \sin 12.5^\circ \approx 5.41 \text{ lb}$$

$$\cos 12.5^\circ = \frac{|N|}{25.0}$$

$$|N| = 25.0 \cos 12.5^\circ \approx 24.4 \text{ lb}$$

46. The corresponding force diagram would be:



Therefore:

$$\sin 50^\circ = \frac{2,200}{|\mathbf{T}|}$$

$$|\mathbf{T}| \sin 50^\circ = 2,200$$

$$|\mathbf{T}| = \frac{2,200}{\sin 50^\circ} \approx 2,900 \text{ lb}$$

$$\tan 50^\circ = \frac{2,200}{|\mathbf{H}|}$$

$$|\mathbf{H}| \tan 50^\circ = 2,200$$

$$|\mathbf{H}| = \frac{2,200}{\tan 50^\circ} \approx 1,800 \text{ lb}$$

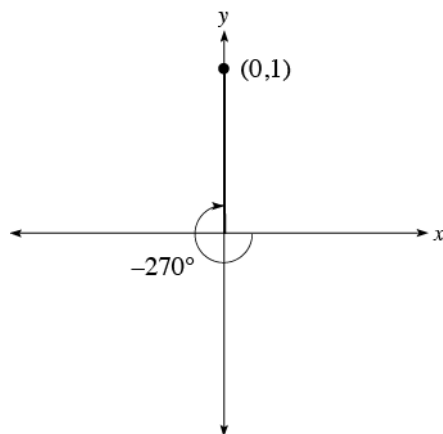
48. The horizontal portion of the force is given by:  $|\mathbf{F}_x| = |\mathbf{F}| \cos 35^\circ = 15 \cos 35^\circ \text{ lb}$

The work is then given by:  $\text{Work} = (15 \cos 35^\circ)(52) \approx 640 \text{ ft-lb}$

50. The horizontal portion of the force is given by:  $|\mathbf{F}_x| = |\mathbf{F}| \cos 15^\circ = 85 \cos 15^\circ \text{ lb}$

The work is then given by:  $\text{Work} = (85 \cos 15^\circ)(110) \approx 9,000 \text{ ft-lb}$

52. Drawing the angle in standard position:



Since  $r = 1$ ,  $\sin(-270^\circ) = 1$ ,  $\cos(-270^\circ) = 0$ , and  $\tan(-270^\circ)$  is undefined.

54. Choose  $(-1, 1)$  as a point on the terminal side of  $\theta$ . Then  $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ . Therefore:

$$\sin \theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{x}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

56. Since  $\cos \theta = \frac{x}{r} = -\frac{3}{5} = -\frac{6}{10}$ , choose  $x = -6$  and  $r = 10$ . Now find  $y$ :

$$(-6)^2 + y^2 = 10^2$$

$$36 + y^2 = 100$$

$$y^2 = 64$$

$$y = \pm 8$$

58. Using the Pythagorean Theorem:  $|\mathbf{V}| = \sqrt{9.6^2 + 2.3^2} \approx 9.9$

Finding the angle:

$$\tan \theta = \frac{2.3}{9.6}$$

$$\theta = \tan^{-1}\left(\frac{2.3}{9.6}\right) \approx 13^\circ$$

The correct answer is c.

60. The horizontal portion of the force is given by:  $|\mathbf{F}_x| = |\mathbf{F}| \cos 35^\circ = 28 \cos 35^\circ \text{ lb}$

The work is then given by:  $\text{Work} = (28 \cos 35^\circ)(150) \approx 3,400 \text{ ft-lb}$

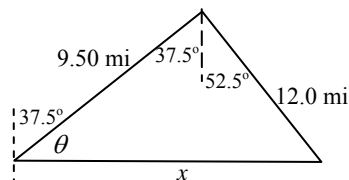
The correct answer is b.

## ODD SOLUTIONS

1. scalar, vector
3. resultant, diagonal
5. horizontal, component, vertical, component
7. zero, static equilibrium

For problems 9 through 15, see textbook answer section for diagrams.

17. The first hour, the distance traveled is  
 $(9.50 \text{ mph})(1 \text{ hr}) = 9.50 \text{ miles}$   
 The next hour and a half, the distance traveled is  
 $(8.00 \text{ mph})(1.5 \text{ hr}) = 12.0 \text{ miles}$   
 We will use the Pythagorean Theorem to find  $x$ :  
 $x^2 = 9.50^2 + 12.0^2$   
 $x^2 = 234.25$   
 $x = 15.3 \text{ mi}$



We will use the tangent ratio to find  $\theta$  and then add  $37.5^\circ$ :

$$\tan \theta = \frac{12.0}{9.50}$$

$$= 1.2632$$

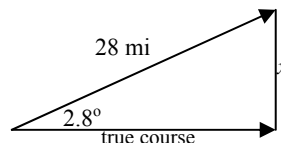
$$\theta = \tan^{-1}(1.2632)$$

$$= 51.6^\circ$$

$$51.6^\circ + 37.5^\circ = 89.1^\circ$$

The balloon is 15.3 miles from its starting point. The bearing is N  $89.1^\circ$  E.

19.  $|V_x| = |V| \cos \theta$   
 $= 13.8 \cos 24.2^\circ$   
 $= 12.6$
21.  $|V_x| = |V| \cos \theta$   
 $= 425 \cos 36^\circ 10'$   
 $= 425 \cos 36.17^\circ$   
 $= 425(0.8073)$   
 $= 343$
23.  $|V_x| = |V| \cos \theta$   
 $= 64 \cos 0^\circ$   
 $= 64(1) = 64$
25.  $|V| = \sqrt{|V_x|^2 + |V_y|^2}$   
 $= \sqrt{(35.0)^2 + (26.0)^2}$   
 $= \sqrt{1,225 + 676}$   
 $= \sqrt{1,901}$   
 $= 43.6$
27.  $|V| = \sqrt{|V_x|^2 + |V_y|^2}$   
 $= \sqrt{(4.5)^2 + (3.8)^2}$   
 $= \sqrt{20.25 + 14.44}$   
 $= \sqrt{34.69}$   
 $= 5.9$
29. To find the distance,  $x$ , the plane has flown off its course, we can use the sine ratio:  
 $\sin 2.8^\circ = \frac{x}{28}$   
 $x = 28 \sin 2.8^\circ$   
 $= 1.37 \text{ miles}$



$$\begin{aligned}
 31. \quad |V_x| &= |V| \cos \theta & |V_y| &= |V| \sin \theta \\
 &= 1,200 \cos 45^\circ & &= 1,200 \sin 45^\circ \\
 &= 1200(0.7071) & &= 1200(0.7071) \\
 &= 850 \text{ feet per second} & &= 850 \text{ feet per second}
 \end{aligned}$$

33. In 3 seconds, the bullet travels  $3(850 \text{ ft/sec}) = 2,550 \text{ ft}$ .

$$\begin{aligned}
 35. \quad |V_x| &= 130 \cos 48^\circ & |V_y| &= 130 \sin 48^\circ \\
 &= 87 & &= 97
 \end{aligned}$$

The ship has traveled 97 km south and 87 km east.

37. We are given that  $|V_x| = 35.0$  and  $|V_y| = 15.0$

$$\begin{aligned}
 |V| &= \sqrt{|V_x|^2 + |V_y|^2} & \tan \theta &= \frac{|V_y|}{|V_x|} \\
 &= \sqrt{(35.0)^2 + (15.0)^2} & &= \frac{15.0}{35.0} \\
 &= \sqrt{1,225 + 225} & &= 0.4285 \\
 &= \sqrt{1,450} & &= 38.1 \text{ feet per second} \\
 & & \theta &= 23.2^\circ
 \end{aligned}$$

Therefore, the velocity of the arrow is 38.1 feet per second at an elevation of  $23.2^\circ$ .

39. To find the total distance traveled north, we must find the sum of  $|V_y|$  and  $|W_y|$  and to find the total distance traveled east, we must find the sum of  $|V_x|$  and  $|W_x|$ .

We are given that  $|V|$  is 170 mi. at an angle of inclination of  $90^\circ - 18^\circ$  or  $72^\circ$  and

also that  $|W|$  is 120 mi. at an angle of inclination of  $90^\circ - 49^\circ$  or  $41^\circ$

$$\begin{aligned}
 |V_x| &= |V| \cos \theta_1 & |V_y| &= |V| \sin \theta_1 \\
 &= 170 \cos 72^\circ & &= 170 \sin 72^\circ \\
 &= 170(0.3090) & &= 170(0.9510) \\
 &= 53 \text{ mi} & &= 162 \text{ mi}
 \end{aligned}$$

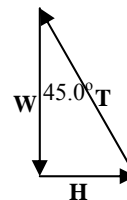
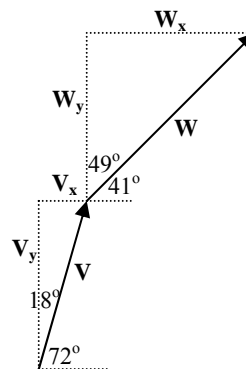
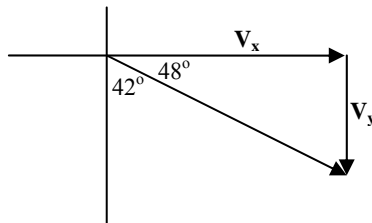
$$\begin{aligned}
 |W_x| &= |W| \cos \theta_2 & |W_y| &= |W| \sin \theta_2 \\
 &= 120 \cos 41^\circ & &= 120 \sin 41^\circ \\
 &= 120(0.7547) & &= 120(0.6560) \\
 &= 90 \text{ mi} & &= 80 \text{ mi}
 \end{aligned}$$

$$|V_y| + |W_y| = 162 + 80 = 242 \text{ and } |V_x| + |W_x| = 53 + 90 = 143$$

The total distance north is 240 miles and the total distance east is 140 miles, rounded to 2 significant digits.

41.  $|W| = 42.0$

$$\begin{aligned}
 \cos 45.0^\circ &= \frac{|W|}{|T|} & \tan 45.0^\circ &= \frac{|H|}{|W|} \\
 |T| &= \frac{|W|}{\cos 45.0^\circ} & |H| &= |W| \tan 45.0^\circ \\
 &= \frac{42.0}{\cos 45.0^\circ} & &= 42.0 \tan 45.0^\circ \\
 &= 59.4 \text{ lb.} & &= 42.0 \text{ lb.}
 \end{aligned}$$



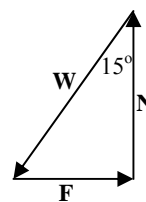
43. We are given that  $|W| = 8.0$

$$\cos 15^\circ = \frac{|N|}{|W|}$$

$$\begin{aligned}|N| &= |W| \cos 15^\circ \\ &= 8.0(0.9659) \\ &= 7.7 \text{ pounds}\end{aligned}$$

$$\sin 15^\circ = \frac{|F|}{|W|}$$

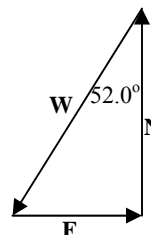
$$\begin{aligned}|F| &= |W| \sin 15^\circ \\ &= 8.0(0.2588) \\ &= 2.1 \text{ pounds}\end{aligned}$$



45.  $|W| = 42.0$

$$\sin 52.0^\circ = \frac{|F|}{|W|}$$

$$\begin{aligned}|F| &= |W| \sin 52.0^\circ \\ &= 42.0 \sin 52.0^\circ \\ &= 33.1 \text{ lb}\end{aligned}$$



47.  $\theta = 20^\circ$ ,  $|F| = 40$  lb, and  $d = 75$  ft

$$\begin{aligned}|F_x| &= |F| \cos \theta \\ &= 41 \cos 20^\circ\end{aligned}$$

$$\begin{aligned}\text{Work} &= |F_x| \cdot d \\ &= (41 \cos 20^\circ)(75) \\ &= 2900 \text{ ft} \cdot \text{lb}.\end{aligned}$$

49.  $\theta = 30^\circ$ ,  $|F| = 25$  lb, and  $d = 350$  ft

$$\begin{aligned}|F_x| &= |F| \cos \theta \\ &= 25 \cos 30^\circ\end{aligned}$$

$$\begin{aligned}\text{Work} &= |F_x| \cdot d \\ &= (25 \cos 30^\circ)(350) \\ &= 7,600 \text{ ft} \cdot \text{lb}.\end{aligned}$$

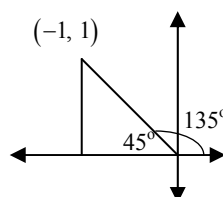
51.  $(x, y) = (-1, 1)$

$$x = -1, y = 1 \text{ and } r = \sqrt{2}$$

$$\sin 135^\circ = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 135^\circ = \frac{x}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\tan 135^\circ = \frac{y}{x} = \frac{1}{-1} = -1$$



53. A point on the line  $y = 2x$  in quadrant I is  $(1, 2)$ .  $x = 1$ ,  $y = 2$ , and  $r = \sqrt{1^2 + 2^2} = \sqrt{5}$

$$\sin \theta = \frac{y}{r} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{x}{r} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

55.  $\sin \theta = \frac{y}{r} = \frac{-4}{5} = \frac{-8}{10}$   
 $y = -8$  and  $r = 10$

$$x^2 + y^2 = r^2$$

$$x^2 + (-8)^2 = 10^2$$

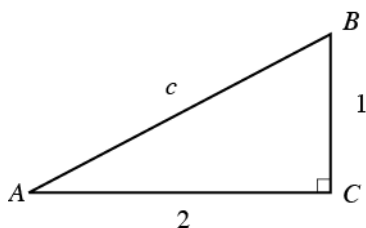
$$x^2 + 64 = 100$$

$$x^2 = 36$$

$$x = \pm 6$$

## Chapter 2 Test

1. First draw the triangle:



Note that  $c = \sqrt{2^2 + 1^2} = \sqrt{5}$ . Therefore:

$$\sin A = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

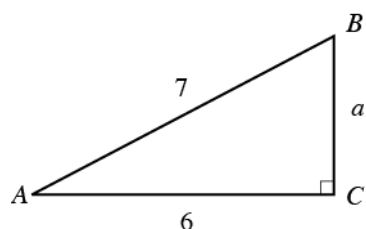
$$\tan A = \frac{1}{2}$$

$$\sin B = \cos A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos B = \sin A = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan B = \cot A = \frac{2}{1} = 2$$

2. First draw the triangle:



Note that  $a = \sqrt{7^2 - 6^2} = \sqrt{13}$ . Therefore:

$$\sin A = \frac{\sqrt{13}}{7}$$

$$\cos A = \frac{6}{7}$$

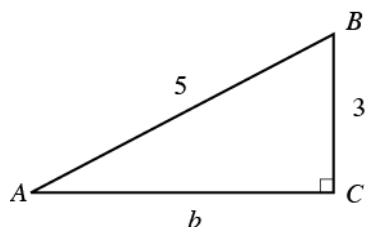
$$\tan A = \frac{\sqrt{13}}{6}$$

$$\sin B = \cos A = \frac{6}{7}$$

$$\cos B = \sin A = \frac{\sqrt{13}}{7}$$

$$\tan B = \cot A = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$$

3. First draw the triangle:



Note that  $b = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$ . Therefore:

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

$$\tan A = \frac{3}{4}$$

$$\sin B = \cos A = \frac{4}{5}$$

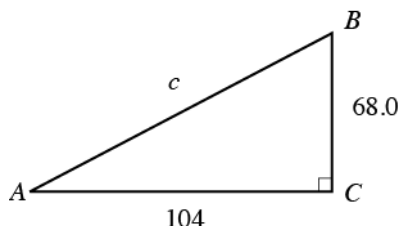
$$\cos B = \sin A = \frac{3}{5}$$

$$\tan B = \cot A = \frac{4}{3}$$

4. Since  $y \leq r$ ,  $\frac{y}{r} \leq 1$ . Therefore  $\sin \theta = \frac{y}{r} \leq 1$ , so it is impossible for  $\sin \theta = 2$ .
5.  $\sin 14^\circ = \cos(90^\circ - 14^\circ) = \cos 76^\circ$
6. Simplifying:  $\sin^2 45^\circ + \cos^2 60^\circ = \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
7. Simplifying:  $\tan 45^\circ + \cot 45^\circ = 1 + 1 = 2$



8. Simplifying:  $\sin^2 60^\circ - \cos^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} - \frac{3}{4} = 0$
9. Simplifying:  $\frac{1}{\csc 30^\circ} = \sin 30^\circ = \frac{1}{2}$
10. Adding:  $48^\circ 31' + 24^\circ 52' = 72^\circ 83' = 73^\circ 23'$
11. Converting to degrees and minutes:  $73.2^\circ = 73^\circ + 0.2^\circ = 73^\circ + 0.2(60') = 73^\circ 12'$
12. Converting to decimal degrees:  $2^\circ 48' = 2^\circ + 48' = 2^\circ + \left(\frac{48}{60}\right)^\circ = 2.8^\circ$
13. Calculating the value:  $\sin 24^\circ 20' = \sin\left(24\frac{1}{3}\right)^\circ \approx 0.4120$
14. Calculating the value:  $\cos 48.3^\circ \approx 0.6652$
15. Calculating the value:  $\cot 71^\circ 20' = \cot\left(71\frac{1}{3}\right)^\circ = \frac{1}{\tan\left(71\frac{1}{3}\right)^\circ} \approx 0.3378$
16. Since  $\sin \theta = 0.6459$ ,  $\theta = \sin^{-1}(0.6459) \approx 40.2^\circ$ .
17. Since  $\sec \theta = 1.923$ ,  $\cos \theta = \frac{1}{1.923}$ , so  $\theta = \cos^{-1}\left(\frac{1}{1.923}\right) \approx 58.7^\circ$ .
18. First sketch the triangle:



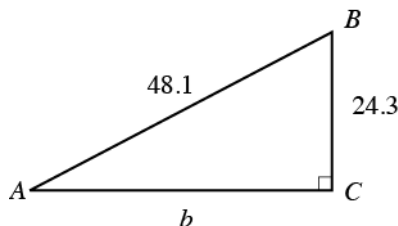
Using the Pythagorean Theorem:  $c = \sqrt{104^2 + 68^2} \approx 124$ . Therefore:

$$\tan A = \frac{68}{104}$$

$$A = \tan^{-1}\left(\frac{68}{104}\right) \approx 33.2^\circ$$

$$B = 90^\circ - 33.2^\circ = 56.8^\circ$$

19. First sketch the triangle:



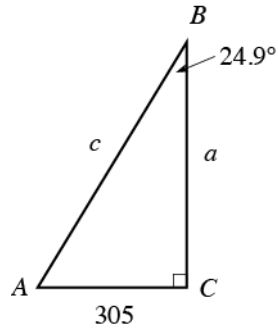
Using the Pythagorean Theorem:  $b = \sqrt{48.1^2 - 24.3^2} \approx 41.5$ . Therefore:

$$\sin A = \frac{24.3}{48.1}$$

$$A = \sin^{-1}\left(\frac{24.3}{48.1}\right) \approx 30.3^\circ$$

$$B = 90^\circ - 30.3^\circ = 59.7^\circ$$

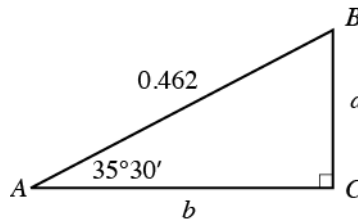
20. First sketch the triangle:



Note that  $A = 90^\circ - 24.9^\circ = 65.1^\circ$ . Therefore:

$$\begin{aligned}\tan 65.1^\circ &= \frac{a}{305} & \cos 65.1^\circ &= \frac{305}{c} \\ a &= 305 \tan 65.1^\circ \approx 657 & c \cos 65.1^\circ &= 305 \\ & & c &= \frac{305}{\cos 65.1^\circ} \approx 724\end{aligned}$$

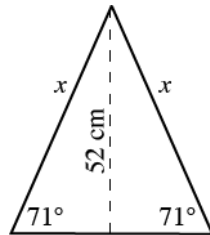
21. First sketch the triangle:



Note that  $B = 90^\circ - 35^\circ 30' = 89^\circ 60' - 35^\circ 30' = 54^\circ 30'$ . Also:

$$\begin{aligned}\sin 35.5^\circ &= \frac{a}{0.462} & \cos 35.5^\circ &= \frac{b}{0.462} \\ a &= 0.462 \sin 35.5^\circ \approx 0.268 & b &= 0.462 \cos 35.5^\circ \approx 0.376\end{aligned}$$

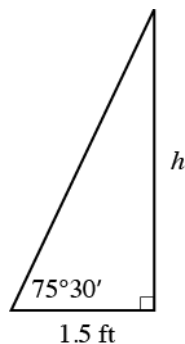
22. First sketch the triangle:



Therefore:

$$\begin{aligned}\sin 71^\circ &= \frac{52}{x} \\ x \sin 71^\circ &= 52 \\ x &= \frac{52}{\sin 71^\circ} \approx 55 \text{ cm}\end{aligned}$$

23. Sketch the figure:



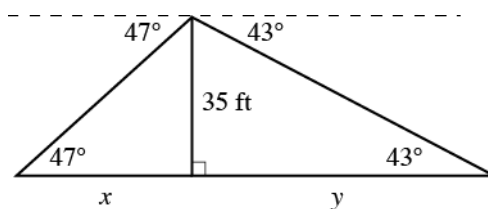
Therefore:

$$\tan 75.5^\circ = \frac{h}{1.5}$$

$$h = 1.5 \tan 75.5^\circ \approx 5.8$$

The post is approximately 5.8 feet tall.

24. Draw the figure:



Therefore:

$$\tan 47^\circ = \frac{35}{x}$$

$$x \tan 47^\circ = 35$$

$$x = \frac{35}{\tan 47^\circ} \approx 32.64 \text{ feet}$$

$$\tan 43^\circ = \frac{35}{y}$$

$$y \tan 43^\circ = 35$$

$$y = \frac{35}{\tan 43^\circ} \approx 37.53 \text{ feet}$$

The stakes are  $32.6 + 37.5 \approx 70$  feet apart.

25. Let  $\theta$  represent the required angle. Then:

$$\tan \theta = \frac{31}{11}$$

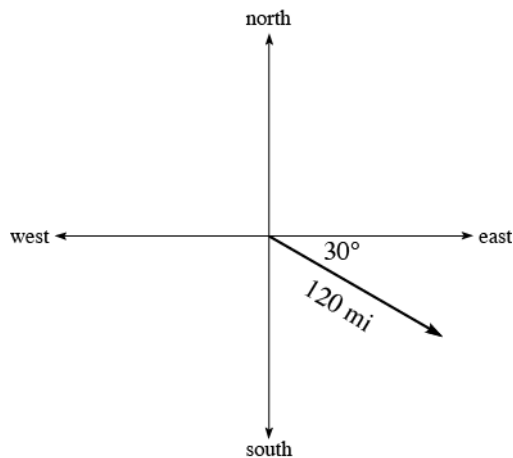
$$\theta = \tan^{-1}\left(\frac{31}{11}\right) \approx 70^\circ$$

26. The magnitudes are given by:

$$|\mathbf{V}_x| = 850 \cos 52^\circ \approx 523 \text{ ft/sec} \approx 520 \text{ ft/sec}$$

$$|\mathbf{V}_y| = 850 \sin 52^\circ \approx 670 \text{ ft/sec}$$

27. Draw the figure:

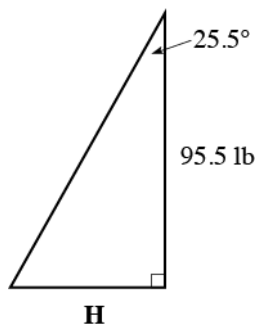


Therefore the distances the ship has traveled are:

east:  $120 \cos 30^\circ \approx 100$  miles

south:  $120 \sin 30^\circ = 60$  miles

28. Drawing the figure:



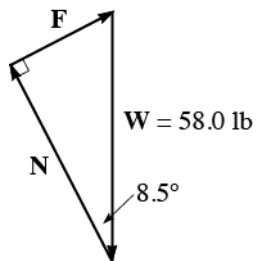
Now find the magnitude of **H**:

$$\tan 25.5^\circ = \frac{|\mathbf{H}|}{95.5}$$

$$|\mathbf{H}| = 95.5 \tan 25.5^\circ \approx 45.6$$

Kelly must push horizontally with a force of 45.6 lb.

29. The corresponding force diagram would be:



Now find the magnitude of **F**:

$$\sin 8.5^\circ = \frac{|\mathbf{F}|}{58.0}$$

$$|\mathbf{F}| = 58.0 \sin 8.5^\circ \approx 8.57 \text{ lb}$$

Tyler must push with a force of 8.57 lb.

30. The horizontal portion of the force is given by:  $|\mathbf{F}_x| = |\mathbf{F}| \cos 40^\circ = 44 \cos 40^\circ \text{ lb}$

The work is then given by:  $\text{Work} = (44 \cos 40^\circ)(85) \approx 2,865 \text{ ft-lb} \approx 2,900 \text{ ft-lb}$