Chapter 2 Solutions

Solutions to Section 2.1

2.1 (a)

Die Result	Coin(s) Result	Number of Outcomes
1	{(H),(T)}	2
2	{(H,H),...,(T,T)}	4
3	{(H,H,H),...,(T,T,T)}	8
4	{ (H,H,H,H),...,(T,T,T,T) }	16
5	{ (H,H,H,H,H),...,(T,T,T,T,T) }	32
6	{ (H,H,H,H,H,H),...,(T,T,T,T,T,T) }	64

There are a total of 126 outcomes.

(b)

Die Result	Number of Heads	Number of Outcomes
1	{(0),(1)}	2
2	{(0),(1),(2)}	3
3	{(0),(1),...,(3)}	4
4	{ (0),(1),...,(4) }	5
5	{ (0),(1),...,(5) }	6
6	{ (0),(1),...,(6) }	7

There are a total of 27 outcomes.

2.2 (a) $T \cap N^c$.

(b) $(T \cup R)^c$.

(c) $(T \cup R \cup N)^c$.

2.3 **Result 1**: Since A and A^c are mutually exclusive,

$$P(A) + P(A^c) = P(A \cup A^c) = P(S) = 1$$

and

$$P(A^c) = 1 - P(A).$$

Result 3:

$$P(A) = P((A \cap B) \cup (A \cap B^c)) = P(A \cap B) + P(A \cap B^c).$$

since $A \cap B$ and $A \cap B^c$ are mutually exclusive.

Result 2: (Uses Result 3)

$$P(A \cup B) = P((A \cap B) \cup (A \cap B^c) \cup (A^c \cap B)).$$

Since all of these are mutually exclusive,

$$\begin{aligned}
P(A \cup B) &= P(A \cap B) + P((A \cap B^c) \cup (A^c \cap B)) \\
&= P(A \cap B) + P(A \cap B^c) + P(A^c \cap B) \\
&= P(A \cap B) + [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] \\
&= P(A) + P(B) - P(A \cap B).
\end{aligned}$$

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

Result 4:

$$P(A \cap B^c) = P(A) - P(A \cap B) = P(A) - P(B).$$

Since

$$P(A \cap B^c) = P(A) - P(B) \geq 0,$$
$$P(A) \geq P(B).$$

2.4

$$\begin{aligned} P(A \cap B) &= 1 - P\left((A \cap B)^c\right) \\ &= 1 - P\left(A^c \cup B^c\right) \\ &\geq 1 - [P(A^c) + P(B^c)] \\ &\geq 1 - [1 - P(A) + 1 - P(B)] \\ &\geq P(A) + P(B) - 1. \end{aligned}$$

2.5 (a) $\binom{52}{2} = 1326.$

(b) $\binom{4}{2} = 6.$

(c) $\binom{4}{1}\binom{48}{1} + \binom{4}{2} = 198.$

2.6 Out of the 54 numbers, 6 of them will be chosen for the lottery and 48 will not. So for the grand prize, the probability of winning is

$$\frac{\binom{6}{6}\binom{48}{0}}{\binom{54}{6}} = \frac{1}{25,827,165}.$$

For the second prize, the probability of winning is

$$\frac{\binom{6}{5}\binom{48}{1}}{\binom{54}{6}} = \frac{288}{25,827,165}.$$

For the third prize, the probability of winning is

$$\frac{\binom{6}{4}\binom{48}{2}}{\binom{54}{6}} = \frac{16,920}{25,827,165}.$$

2.7 The total number of moves is $m+n$. On each move, the path can either go right or up. So the number of paths between (0,0) and (m,n) is the same as the number of ways to fill $m+n$ moves with m rights (and the rest ups), or $\binom{m+n}{m}$.

2.8 First split the n objects into two groups, one with n_1 and the other with n_2. Then we can get r total objects by adding up the combinations selecting i from group 1 and $r-i$ from group 2, so that

$$\binom{n}{r} = \sum_i \binom{n_1}{i}\binom{n_2}{r-i}.$$

If $r < n_1$ then we can only select up to n_1 objects from the first group and the upper limit of the sum is n_1. Otherwise, it is r, which yields $\min(n_1, r)$. Similarly, if $r > n_2$ then we can only select up to n_2 objects from the second group, so $r - i \leq n_2$ and the lower limit is $r - n_2$. Otherwise, we can select r objects yielding a lower limit on i of 0. So the final result is

$$\binom{n}{r} = \sum_{i=\max(0,r-n_2)}^{\min(n_1,r)} \binom{n_1}{i}\binom{n_2}{r-i}.$$

2.9

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

$$
\begin{aligned}
\binom{n-1}{r-1}+\binom{n-1}{r} &= \frac{(n-1)!}{(r-1)!(n-r)!}+\frac{(n-1)!}{r!(n-r)!} \\
&= \frac{n!}{r!(n-r)!}\left(\frac{r}{n}+\frac{n-r}{n}\right) \\
&= \binom{n}{r}.
\end{aligned}
$$

n	Coefficients
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

2.10 (a) $\binom{7}{2}=21$.

(b) $\binom{7}{2}(2)^2(-3)^5=-20,412$.

(c) $\frac{7!}{2!2!3!}=210$.

2.11 (a)

$$\frac{\binom{12}{3}}{\binom{34}{3}}=\frac{220}{5984}.$$

(b)

$$\frac{\binom{2}{2}\binom{32}{1}}{\binom{34}{3}}=\frac{32}{5984}.$$

(c)

$$\frac{\binom{2}{2}\binom{32}{1}}{\binom{34}{3}}+\frac{\binom{12}{2}\binom{22}{1}}{\binom{34}{3}}+\frac{\binom{20}{2}\binom{14}{1}}{\binom{34}{3}}=\frac{4144}{5984}.$$

(d)

$$\frac{\binom{2}{2}\binom{32}{1}}{\binom{34}{3}}+\frac{\binom{12}{2}\binom{22}{1}}{\binom{34}{3}}+\frac{\binom{20}{2}\binom{14}{1}}{\binom{34}{3}}+\frac{\binom{12}{3}\binom{22}{0}}{\binom{34}{3}}+\frac{\binom{20}{3}\binom{14}{0}}{\binom{34}{3}}=\frac{5504}{5984}.$$

2.12 (a)

$$\frac{25}{30}\times\frac{5}{29}=0.144.$$

(b)

$$\frac{25}{30}\times\frac{5}{29}+\frac{5}{30}\times\frac{4}{29}=0.167.$$

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

(c)

$$\frac{5}{30} \times \frac{4}{29} \times \frac{3}{28} = 0.002.$$

2.13 (a)

$$P(T \cap N^c) = P(T) - P(T \cap N) = 0.77 - 0.45 = 0.32.$$

(b)

$$P((T \cup R)^c) = 1 - P(T \cup R) = 1 - P(T) - P(R) + P(T \cap R) = 1 - .77 - .47 + .29 = 0.05.$$

(c)

$$\begin{aligned} P((T \cup R \cup N)^c) &= 1 - P(N) - P(T \cup R) + P(N \cap (T \cup R)) \\ &= 1 - 0.63 - 0.95 + P((N \cap T) \cup (N \cap R)) \\ &= -0.58 + P(N \cap T) + P(N \cap R) - P(N \cap T \cap R) \\ &= -0.58 + 0.45 + 0.21 - 0.06 = 0.02. \end{aligned}$$

2.14 If we order the 12 kids by team, then there are 12! ways to assign the performance ranks. However, within each team, order is irrelevant, so we need to divide out the 3! ways of ordering the three kids per team. So the final number of ways of ranking the teams is

$$\frac{12!}{3!3!3!3!} = 369,600.$$

2.15 For all four suits to be represented, one suit must have 2 cards and the other suits must have 1 card each. There are $\binom{4}{1}$ ways to choose the suit with 2 cards, and there are $\binom{13}{2}$ ways to select the 2 cards from that suit. For the remaining suits, there are $\binom{13}{1}$ ways to select the 1 card from that suit. So the final probability is

$$\frac{\binom{4}{1}\binom{13}{2}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{5}} = \frac{685,464}{25,827,165}.$$

Solutions to Section 2.2

2.16

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.6} = 0.5.$$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.3}{0.6} = 0.5.$$

$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{0.5 - 0.3}{1 - 0.6} = 0.5.$$

$$P(B^c|A \cap C) = \frac{P(A \cap B^c \cap C)}{P(A \cap C)} = \frac{P(A \cap C) - P(A \cap B \cap C)}{P(A \cap C)} = \frac{0.2 - 0.1}{0.2} = 0.5.$$

2.17 (a) For A and B to be mutually exclusive, $P(A \cap B) = 0$ or $P(A \cup B) = P(A) + P(B)$. Then

$$0.8 = 0.4 + p,$$

which means that $p = 0.4$.

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

(b) For A and B to be independent, $P(A \cap B) = P(A)P(B)$. Then

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.4 + p - 0.8 = P(A)P(B) = 0.4p,$$

which means that $p = 2/3$.

2.18 (a)

$$P(N|R) = \frac{P(N \cap R)}{P(R)} = \frac{0.21}{0.47} = 0.447.$$

(b)

$$P(R^c|T) = \frac{P(R^c \cap T)}{P(T)} = \frac{P(T) - P(R \cap T)}{P(T)} = \frac{0.77 - 0.29}{0.77} = 0.623.$$

(c)

$$P(T^c|N \cap R) = \frac{P(T^c \cap N \cap R)}{P(N \cap R)} = \frac{P(N \cap R) - P(N \cap R \cap T)}{P(N \cap R)} = \frac{0.21 - 0.06}{0.21} = 0.714.$$

2.19 (a) Let E_1 and F_1 denote whether the first return has an error or is flagged, respectively. Then

$$P(E_1 \cap F_1) = P(F_1|E_1)P(E_1) = 0.9 \times \frac{5}{30} = 0.15.$$

(b)

$$P(F_1) = P(F_1|E_1)P(E_1) + P(F_1|E_1^c)P(E_1^c) = 0.15 + 0.02 \times \frac{25}{30} = 0.167.$$

(c) Let E_2 and F_2 denote whether the second return has an error or is flagged, respectively. Then

$$P(E_2) = P(E_2|E_1)P(E_1) + P(E_2|E_1^c)P(E_1^c) = \frac{4}{29} \times \frac{5}{30} + \frac{5}{29} \times \frac{25}{30} = 0.167$$

and

$$P(F_2 \cap E_2) = P(F_2|E_2)P(E_2) = 0.9 \times 0.167 = 0.15.$$

2.20 Let A_i and P_i denote the events that an ace or a face card is drawn on the ith draw, respectively. Then

$$\begin{aligned} P(\text{ ace before face card}) &= P(A_1) + P((A_1 \cup P_1)^c)P(A_2) + \ldots \\ &= \frac{4}{52} + \left(\frac{36}{52}\right)\frac{4}{52} + \left(\frac{36}{52}\right)^2 \frac{4}{52} + \ldots \\ &= \frac{4}{52}\sum_{i=0}^{\infty}\left(\frac{36}{52}\right)^i \\ &= \frac{4}{52}\left(\frac{1}{1-\frac{36}{52}}\right) = 0.25. \end{aligned}$$

2.21

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

(a) Let F and M denote the genetic contributions from the father and mother respectively. Then the probabilities that a father contributes A or a are, respectively,

$$P(F=A)=P(F=A|AA)P(AA)+P(F=A|Aa)P(Aa)=p_0+q_0/2.$$

and

$$P(F=a)=P(F=a|aa)P(aa)+P(F=a|Aa)P(Aa)=r_0+q_0/2.$$

Then the probability that the first generation is AA is

$$p_1=P(F=A,M=A)=P(F=A)P(M=A)=[P(F=A)]^2=(p_0+q_0/2)^2.$$

Similarly, the probability that the first generation is Aa is

$$\begin{aligned}
q_1 &= P(F=A,M=a)+P(F=a,M=A)\\
&= P(F=A)P(M=a)+P(F=a)P(M=A)\\
&= 2P(F=A)P(F=a)=2(p_0+q_0/2)(r_0+q_0/2).
\end{aligned}$$

Finally, the probability that the first generation is aa is

$$r_1=P(F=a,M=a)=P(F=a)P(M=a)=[P(F=a)]^2=(r_0+q_0/2)^2.$$

(b) Similar to (a),

$$p_2=(p_1+q_1/2)^2,\ q_2=2(p_1+q_1/2)(r_1+q_1/2), \text{ and } r_2=(r_1+q_1/2)^2.$$

Then

$$\begin{aligned}
p_2 &= \left[(p_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right]^2\\
&= [(p_0+q_0/2)(p_0+q_0/2+r_0+q_0/2)]^2\\
&= (p_0+q_0/2)^2\,(1)^2,\\
q_2 &= 2\left[(p_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right]\\
&\quad\times\left[(r_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right]\\
&= 2\,(p_0+q_0/2)\,(r_0+q_0/2)\,,\\
r_2 &= \left[(r_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right]^2\\
&= [(r_0+q_0/2)(r_0+q_0/2+p_0+q_0/2)]^2\\
&= (r_0+q_0/2)^2\,(1)^2.
\end{aligned}$$

For the recursive proof, assume that this set of equations is true for n. Then

$$\begin{aligned}
p_{n+1} &= (p_n+q_n/2)^2\\
&= \left[(p_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right]^2\\
&= [(p_0+q_0/2)(p_0+q_0/2+r_0+q_0/2)]^2\\
&= (p_0+q_0/2)^2\,,\\
q_{n+1} &= 2(p_n+q_n/2)(r_n+q_n/2)\\
&= 2\left[(p_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right]
\end{aligned}$$

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

$$
\begin{aligned}
& \times\left[(r_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right] \\
= \; & 2\,(p_0+q_0/2)\,(r_0+q_0/2)\,, \\
r_{n+1} = \; & (r_n+q_n/2)^2 \\
= \; & \left[(r_0+q_0/2)^2+2(p_0+q_0/2)(r_0+q_0/2)/2\right]^2 \\
= \; & [(r_0+q_0/2)(r_0+q_0/2+p_0+q_0/2)]^2 \\
= \; & (r_0+q_0/2)^2\,.
\end{aligned}
$$

2.22 Place of residence and opinion on a tax increase are not independent, since

$$P(\text{ Yes and City}) = \frac{100}{1000} = 0.1 \neq P(\text{Yes})P(\text{City}) = \frac{400}{1000} \times \frac{400}{1000} = 0.16.$$

2.23

$$
\begin{aligned}
P(A_1 \cup A_2 \cup \ldots \cup A_n) &= 1 - P((A_1 \cup A_2 \cup \ldots \cup A_n)^c) \\
&= 1 - P(A_1^c \cap A_2^c \cap \ldots \cap A_n^c) \\
&= 1 - P(A_1^c)P(A_2^c)\cdots P(A_n^c) \\
&= 1-(1-p)^n.
\end{aligned}
$$

Using $p = 0.9$, for $n = 2$ the reliability is 0.99, for $n = 3$, the reliability is 0.999, and for $n = 4$, the reliability is 0.9999. As n gets larger, the reliability approaches 1.

2.24 (a) The event that there is current from A to C is

$$(R_1 \cap R_2) \cup (R_3) \cup (R_4 \cap R_5).$$

(b) The probability that there is current from A to C is

$$
\begin{aligned}
P(\text{Current}) &= P((R_1 \cap R_2) \cup (R_3) \cup (R_4 \cap R_5)) \\
&= P(R_1 \cap R_2) + P(R_3) + P(R_4 \cap R_5) \\
&\quad -P(R_1 \cap R_2 \cap R_3) - P(R_1 \cap R_2 \cap R_4 \cap R_5) - P(R_3 \cap R_4 \cap R_5) \\
&\quad +P(R_1 \cap R_2 \cap R_3 \cap R_4 \cap R_5) \\
&= (0.9)^2 + 0.9 + (0.9)^2 - (0.9)^3 - (0.9)^4 - (0.9)^3 + (0.9)^5 \\
&= 0.996.
\end{aligned}
$$

2.25 (a) Let D be the event that an appliance is defective. Then

$$P(B \cap D) = P(D|B)P(B) = 0.08 \times 0.37 = 0.0296.$$

(b)

$$P(D) = P(A \cap D) + P(B \cap D) = P(D|A)P(A) + 0.0296 = 0.04 \times 0.63 + 0.0296 = 0.0548.$$

(c)

$$P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{0.0296}{0.0548} = 0.5401.$$

2.26

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

(a) Let E be the event that a tax return contains an error, and let F be the event that a tax return is flagged. Then

$$P(E \cap F) = P(F|E)P(E) = 0.85 \times 0.15 = 0.1275.$$

(b)

$$P(F) = P(E \cap F) + P(E^c \cap F) = 0.1275 + P(F|E^c)P(E^c) = 0.1275 + 0.05 \times 0.85 = 0.17.$$

(c)

$$P(E^c|F^c) = \frac{P(E^c \cap F^c)}{P(F^c)} = \frac{P(F^c|E^c)P(E^c)}{1 - P(F)} = \frac{0.95 \times 0.85}{1 - 0.17} = 0.973.$$

2.27 (a) Let D and ND refer to the events where a person has or doesn't have the disease, respectively. Then

$$P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|ND)P(ND)} = \frac{0.99 \times 0.1}{0.99 \times 0.1 + 0.02 \times 0.9} = 0.846.$$

(b)

$$P(ND|-) = \frac{P(-|ND)P(ND)}{P(-|ND)P(ND) + P(-|D)P(D)} = \frac{0.98 \times 0.9}{0.98 \times 0.9 + 0.01 \times 0.1} = 0.999.$$

The diagnostic test appears pretty reliable, although it is less reliable in identifying true positives than true negatives.

(c)

$$P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|ND)P(ND)} = \frac{0.99 \times 0.001}{0.99 \times 0.001 + 0.02 \times 0.999} = 0.047.$$

(d) For rare diseases, too many false positives would appear in the screening program, and it would not be very effective in identifying people with the disease.

Solutions to Section 2.3

2.28 (a) For this to be a p.m.f.,

$$\sum_x f(x) = c(1/2) + c(1/4) + c(1/8) + c(1/16) = 1,$$

or $c = 16/15 = 1.067$.

(b) The c.d.f. is

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1.067/2 = 0.533 & \text{if } 1 \le x < 2 \\ 0.533 + 1.067/4 = 0.8 & \text{if } 2 \le x < 3 \\ 0.8 + 1.067/8 = 0.933 & \text{if } 3 \le x < 4 \\ 1 & \text{if } x \ge 4. \end{cases}$$

2.29 (a)

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

x	$f(x)$	$F(y)$, for $x \le y < x+1$
0	6/36	6/36
1	10/36	16/36
2	8/36	24/36
3	6/36	30/36
4	4/36	34/36
5	2/36	1

(b)

$$P(0 < x \le 3) = F(3) - F(0) = 24/36.$$

$$P(1 \le x < 3) = F(2) - F(0) = 18/36.$$

2.30 (a) For this to be a p.m.f.,

$$\sum_x f(x) = \sum_{x=1}^{\infty} \frac{1}{n(n+1)} = 1.$$

Since

$$\sum_{x=1}^{n} \frac{1}{n(n+1)} = \frac{n}{n+1},$$

then

$$\sum_{x=1}^{\infty} = \lim_{n\to\infty} \frac{n}{n+1} = 1,$$

and $f(x)$ is a p.m.f.

(b) From (a) the c.d.f. is

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{i}{i+1} & \text{if } i \le x < i+1 \end{cases}$$

2.31 For x between n and N, the random variable $X = x$ when the largest chip is x and the remaining $n-1$ chips are smaller than x. Out of the $x-1$ chips smaller than x, we want to choose $n-1$, so there are

$$\binom{x-1}{n-1}$$

ways to do this. The total number of ways to choose n chips is

$$\binom{N}{n},$$

so the p.m.f. is

$$P(X = x) = \frac{\binom{x-1}{n-1}}{\binom{N}{n}}$$

2.32 (a) For this to be a p.d.f.,

$$\int_x f(x) = \int_0^1 0.5\, dx + \int_1^3 (0.5 + c(x-1))\, dx = 1.$$

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

Since

$$\begin{aligned}\int_x f(x) &= .5x|_0^1 + .5x|_1^3 + \frac{c}{2}(x-1)^2|_1^3 \\ &= 1.5 + \frac{c}{2}(4-0) = 1.5 + 2c = 1,\end{aligned}$$

then c must be -0.25.

(b) Using the value of c found above, the new p.d.f. for $1 \le X < 3$ is

$$f(x) = 0.5 - 0.25(x-1) = 0.75 - 0.25x.$$

The c.d.f. for x between 0 and 1 is

$$F(x) = \int_0^x 0.5\,dx = 0.5x.$$

At $x = 1$, $F(1) = 0.5$, so the c.d.f. for x between 1 and 3 is

$$F(x) = F(1) + \int_1^x (0.75 - 0.25x)\,dx = 0.5 + (0.75x - 0.125x^2)_1^x = -.125x^2 + 0.75x - .125.$$

Then the final c.d.f. is given by

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.5x & \text{if } 0 \le x < 1 \\ -.125x^2 + .75x - .125 & \text{if } 1 \le x < 3 \\ 1 & \text{if } x \ge 3. \end{cases}$$

2.33 (a) Continuous.

(b)

$$P(1 \le X \le 3) = F(3) - F(1) = 0.8 - 0.4 = 0.4.$$

(c)

$$P(X \ge 1) = 1 - F(1) = 1 - 0.4 = 0.6.$$

2.34 (a) Discrete.

(b)

$$P(1 \le X < 2) = P(X = 1) = 0.8 - 0.4 = 0.4.$$

(c)

$$P(X \ge 1) = 1 - F(0) = 1 - 0.4 = 0.6.$$

Solutions to Section 2.4

2.35 (a) The p.d.f. is

$$f(x) = \begin{cases} \frac{1}{N} & \text{if } x = 1, 2, \ldots, N \\ 0 & \text{otherwise .} \end{cases}$$

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.

(b) The mean is

$$E(X) = \sum_x x f(x) = \sum_{x=1}^{N} x \times \frac{1}{N} = \frac{1}{N} \times \frac{N(N+1)}{2} = \frac{N+1}{2}.$$

The variance is

$$\begin{aligned}
\text{Var}(X) &= \sum_x x^2 f(x) - \left(\frac{N+1}{2}\right)^2 \\
&= \frac{1}{N}\sum_{x=1}^{N} x^2 - \left(\frac{N+1}{2}\right)^2 \\
&= \frac{N(N+1)(2N+1)}{6N} - \left(\frac{N+1}{2}\right)^2 \\
&= \frac{N+1}{2}\left[\frac{2N+1}{3} - \frac{N+1}{2}\right] \\
&= \frac{(N+1)(N-1)}{12}.
\end{aligned}$$

(c) For a single die, $N = 6$, so that $E(X) = 7/2 = 3.5$ and $\text{Var}(X) = (7)(5)/12 = 2.917$.

2.36 (a) The mean is

$$E(X) = \sum_x x f(x) = 0 \times 0.1 + 1 \times 0.2 + \ldots + 8 \times 0.02 = 2.57.$$

The variance is

$$\text{Var}(X) = \sum_x x^2 f(x) - E(X)^2 = 0^2 \times 0.1 + 1^2 \times 0.2 + \ldots + 8^2 \times 0.02 - 2.57^2 = 3.545.$$

(b) The skewness is

$$\beta_3 = \frac{E[(X-\mu)^3]}{\sigma^3} = \frac{(0-2.57)^3 \times 0.1 + \ldots + (8-2.57)^3 \times 0.02}{(3.545)^{3/2}} = 0.948.$$

The distribution is positively skewed.

2.37 (a)

$$\begin{aligned}
E(X) &= \sum_x x f(x) = \sum_{x=n}^{N} x \frac{\binom{x-1}{n-1}}{\binom{N}{n}} \\
&= \frac{1}{\binom{N}{n}} \sum_{x=n}^{N} \frac{x(x-1)!}{(n-1)!(x-n)!} \\
&= \frac{n}{\binom{N}{n}} \sum_{x=n}^{N} \binom{x}{n} \\
&= \frac{n}{\binom{N}{n}} \binom{N+1}{n+1} \\
&= \frac{n(n!(N-n)!)}{N!} \frac{(N+1)!}{(n+1)!(N-n)!} \\
&= \frac{n(N+1)}{n+1}.
\end{aligned}$$

Copyright © 2000 Pearson Education, Inc. Publishing as Prentice Hall.