CHAPTER 1

Problem 1-1

(a) Write the acceleration as

$$a(t) = \begin{cases} \alpha t, & t \le t_0 \\ 0, & t > t_0 \end{cases}$$

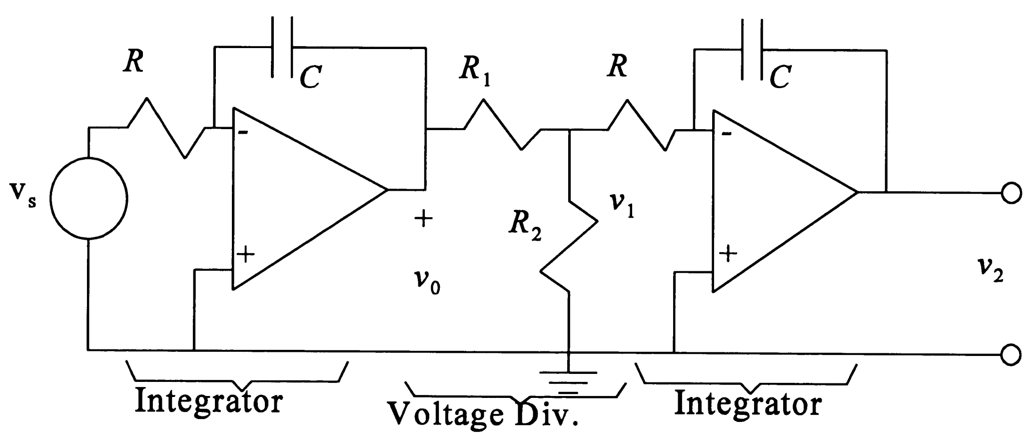
Thus the velocity and position are, respectively, given by

$$v(t) = \int_0^t a(\lambda)d\lambda = \begin{cases} \alpha t^2/2, & t \le t_0 \\ \alpha t_0^2/2, & t > t_0 \end{cases}$$

and

$$x(t) = \int_0^t v(\lambda)d\lambda = \begin{cases} \alpha t^3/6, & t \le t_0 \\ \alpha t_0^3/6 + \alpha t_0^2(t - t_0)/2, & t > t_0 \end{cases}$$

For $t_0 = 72$ s and $\alpha = 5/9$ m/s^2, we have $x(t) = (5/54)t^3$, $t \le 72$ s. At $t = t_0 = 72$ s (burnout), we have $x(t_0) = 35.56$ km.
(b) See the figure below for the integrator.



$$v_2(t) = -\frac{1}{RC}\int_0^t v_1(\lambda)d\lambda$$

Assume that $R_2 << R$. The input impedance to the op-amp integrator is therefore much larger than the output impedance of the previous stage, and

$$v_1(t) = \frac{R_2}{R_1 + R_2} v_0(t)$$

From Example 1-2,

$$v_0(t) = -\frac{1}{RC}\left(\frac{\beta t^2}{2}\right) = -\frac{\beta t^2}{2RC}$$

Therefore,

$$v_2(t) = -\frac{1}{RC}\int_0^t \frac{R_2}{R_1 + R_2}\left(-\frac{\beta}{2RC}\right)\lambda^2 d\lambda$$

Integrating and setting $t = t_0$, we obtain

$$v_2(t_0) = \frac{R_2}{R_1 + R_2}\left(\frac{\beta t_0^2}{2RC}\right)\left(\frac{t_0}{3RC}\right) = 10 \text{ V}$$

The second factor on the right is 10 V because of the maximum output limitation on the first integrator. Thus, we require that

$$\frac{R_2}{R_1 + R_2}\left(\frac{t_0}{3RC}\right) = 1$$

For example, from Example 1-2 we have $RC = 0.36$ s. With $t_0 = 72$ s and $R_1 = 10$ k ohms, we get $R_2 = 152$ ohms.

Problem 1-2

(a) Let $n = 0, 1, 2, 3, ..., N$. Then

$$\begin{aligned} v(T) &= v(0) + Ta(T) \quad \text{(a)} \\ v(2T) &= v(T) + Ta(2T) \quad \text{(b)} \\ &\cdots \\ v(NT) &= v[(N-1)T] + Ta(NT) \quad \text{(c)} \end{aligned}$$

Substitute (a) into (b) and so on until (c is reached. This gives

$$v(NT) = v(0) + T\sum_{n=1}^{N} a(nT)$$

(b) Let $n = 0, 1, 2, 3, ..., N$. Then

$$\begin{aligned} v(T) &= v(0) + (T/2)[a(0) + a(T)] \quad \text{(a)} \\ v(2T) &= v(T) + (T/2)[a(T) + a(2T)] \quad \text{(b)} \\ &\cdots \\ v(NT) &= v[(N-1)T] + (T/2)\{a[(N-1)T] + a(NT)\} \quad \text{(c)} \end{aligned}$$

Substitute (a) into (b) and so on until (c) is reached. The result is as given in the problem statement.

Problem 1-3

(a) A maximum departure of the weight from equilibrium of 1 cm requires a spring constant of

$$K = \frac{Ma_{\max}}{x_{\max}} = \frac{(0.002)(20)}{0.01} = 4 \ \text{kg/s}^2$$

(b) For a minimum increment of 0.5 mm = 0.0005 m, we have

$$\Delta a_{\min} = \frac{K\Delta x_{\min}}{M} = \frac{4(0.0005)}{0.002} = 1 \ \text{m/s}^2$$

(c) The velocity is given by

$$v_r(t) = \int_0^t a(\lambda)d\lambda = \int_0^t 20\,d\lambda = \begin{cases} 20t, & 0 \le 50 \ \text{s} \\ 1000, & t > 50 \ \text{s} \end{cases}$$

Problem 1-4

K is the same as in Example 1-1 because M, x_{max}, and a_{max} are the same. Also, Δa_{min} is the same. The velocity profile is

$$v_r(t) = \begin{cases} \int_0^t 20\, d\lambda = 20t, \ 0 \le t < 10 \\ 200, \ 10 \le t < 20 \\ 200 + \int_{20}^t 20\, d\lambda = 200 + 20(t - 20), \ 20 \le t < 30 \\ 400, \ t > 30 \end{cases}$$

Problem 1-5

From (1-15) and using the $x(t)$ given in the problem, we have

$$\begin{aligned} s(t) &= \cos(\omega_0 t) + \alpha\beta\cos[\omega_0(t - 2\tau)] \\ &= [1 + \alpha\beta\cos(2\omega_0\tau)]\cos(\omega_0 t) + \alpha\beta\sin(2\omega_0\tau)\sin(\omega_0 t) \\ &= A(\tau)\cos[\omega_0 t - \theta(\tau)] \\ &= A(\tau)\cos\theta(\tau)\cos(\omega_0 t) + A(\tau)\sin\theta(\tau)\sin(\omega_0 t) \end{aligned}$$

Set coefficients of like sin/cos terms equal on each side of the identity to obtain

$$\begin{aligned} A(\tau)\cos\theta(\tau) &= 1 + \alpha\beta\cos(2\omega_0\tau) \\ A(\tau)\sin\theta(\tau) &= \alpha\beta\sin(2\omega_0\tau) \end{aligned}$$

Square and add to obtain

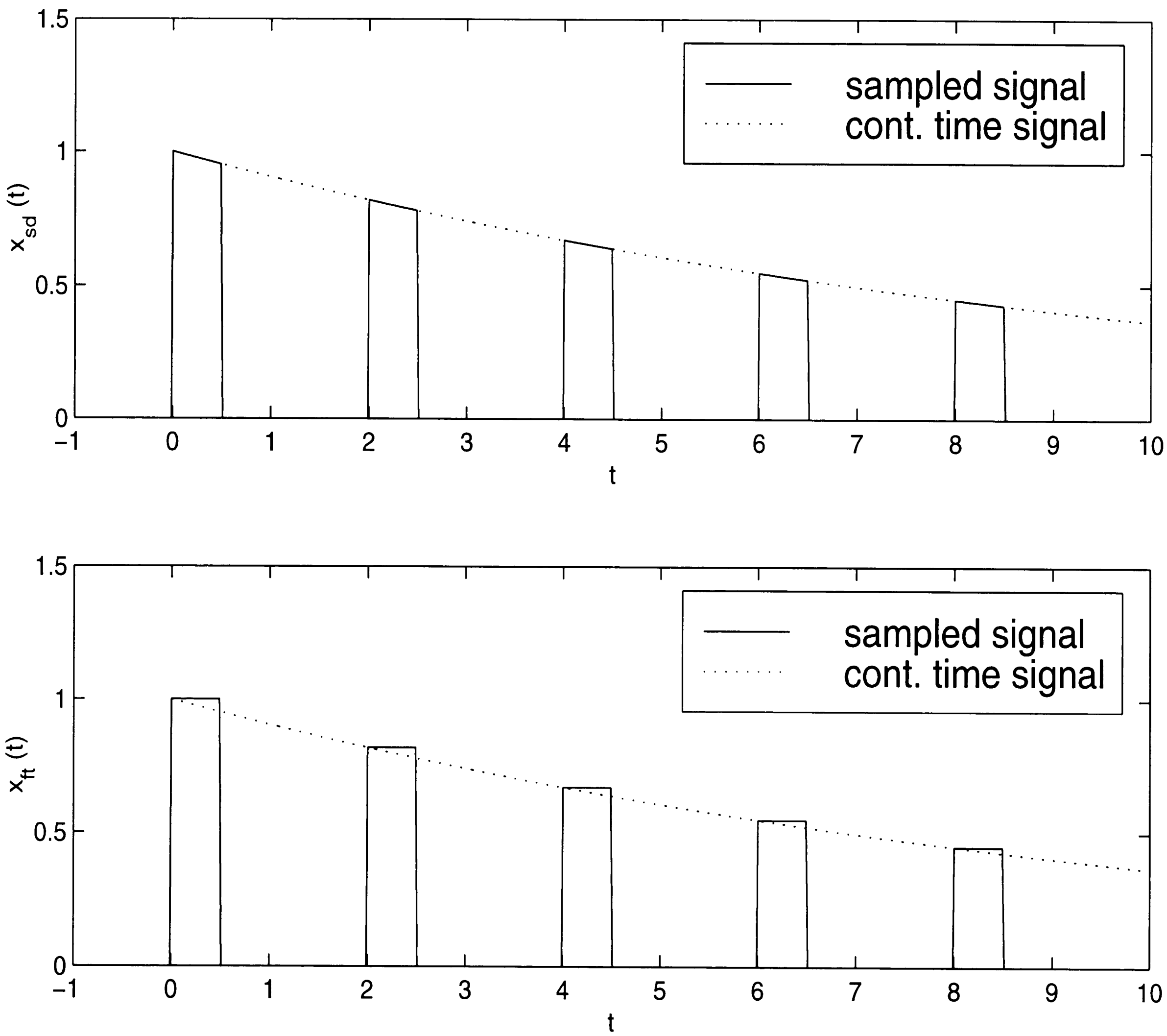
$$A(\tau) = \sqrt{1 + 2\alpha\beta\cos(2\omega_0\tau) + (\alpha\beta)^2}$$

Divide the second equation by the first to obtain

$$\frac{\sin\theta(\tau)}{\cos\theta(\tau)} = \tan\theta(\tau) = \frac{\alpha\beta\sin(2\omega_0\tau)}{1 + \alpha\beta\cos(2\omega_0\tau)}$$

Problem 1-6

Sketches of the analog and sampled signals for both cases are shown below[(a) top and (b) bottom]:



Problem 1-7

(a) The impulse-sampled signal is

$$\begin{aligned} x_{\text{imp. samp}}(t) &= \cos(2\pi t) \sum_{n=-\infty}^{\infty} \delta(t - 0.1n) \\ &= \sum_{n=-\infty}^{\infty} \cos(2\pi t)\,\delta(t - 0.1n) \\ &= \sum_{n=-\infty}^{\infty} \cos(0.2\pi n)\,\delta(t - 0.1n) \end{aligned}$$

where property (1-59) for the unit impulse has been used to get the last result.
(b) The unit-pulse train sampled signal is

$$\begin{aligned} x_{\text{unit pulse samp}}(t) &= \cos(2\pi t) \sum_{n=-\infty}^{\infty} \delta[t - 0.1n] \\ &= \sum_{n=-\infty}^{\infty} \cos(2\pi t)\,\delta[t - 0.1n] \\ &= \sum_{n=-\infty}^{\infty} \cos(0.2\pi n)\,\delta[t - 0.1n] \end{aligned}$$

where the fact that the unit pulse is 1 for its argument 0 and 0 otherwise has been used.

Problem 1-8

(a) The signal can be developed in terms of equations as follows:

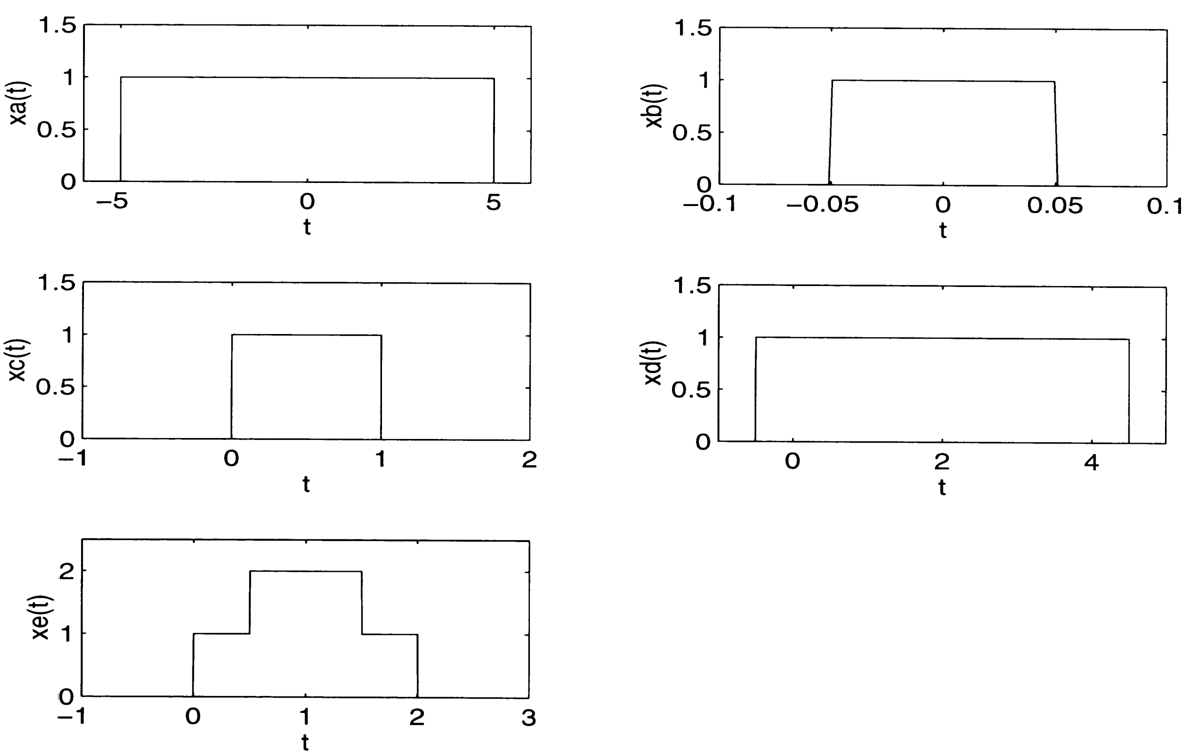
$$\begin{aligned} \Pi(0.1t) &= \begin{cases} 1, & |0.1t| \le 1/2 \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} 1, & |t| \le 10/2 = 5 \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

This is a rectangular pulse of amplitude 1 between -5 and 5 and 0 otherwise. A sketch will be given at the end of the problem solution.
(b) Following a procedure similar to that of (a) one finds that this is a rectangular pulse of amplitude 1 between -0.05 and 0.05 and 0 otherwise. A sketch will be given at the end of the problem solution.
(c) This is a rectangular pulse of amplitude 1 between 0 and 1 and 0 otherwise. A sketch will be given at the end of the problem solution.
(d) This is a rectangular pulse of amplitude 1 between 0.5 and 4.5 and 0 otherwise. A sketch will be given at the end of the problem solution.

(e) The first term of this signal is a rectangular pulse of amplitude 1 between 0 and 2 and 0 otherwise. The second term is a rectangular pulse of amplitude 1 between 0.5 and 1.5 and 0 otherwise. Where both pulses are nonzero, the total amplitude is 2; where only one pulse is nonzero the amplitude is 1. A sketch is provided below.

The MATLAB program below uses the special function given in Section 1-6 (page 32) of the text to provide the plots.

```
%       Sketches for Problem 1-8
%
t = -6:0.0015:6;
xa = pls_fn(0.1*t);
xb = pls_fn(10*t);
xc = pls_fn(t - 0.5);
xd = pls_fn((t - 2)/5);
xe = pls_fn((t - 1)/2) + pls_fn(t - 1);
subplot(3,2,1),plot(t, xa,'-w'), axis([-6 6 0 1.5]),xlabel('t'),ylabel('xa(t)')
subplot(3,2,2),plot(t, xb,'-w'), axis([-.1 .1 0 1.5]),xlabel('t'),ylabel('xb(t)')
subplot(3,2,3),plot(t, xc,'-w'), axis([-1 2 0 1.5]),xlabel('t'),ylabel('xc(t)')
subplot(3,2,4),plot(t, xd,'-w'), axis([-1 5 0 1.5]),xlabel('t'),ylabel('xd(t)')
subplot(3,2,5),plot(t, xe,'-w'), axis([-1 3 0 2.5]),xlabel('t'),ylabel('xe(t)')
```



Problem 1-9

(a) $2\pi f_0 = 50\pi$, so $T_0 = 1/f_0 = 1/25 = 0.04$ s. (b) $2\pi f_0 = 60\pi$, so $T_0 = 1/f_0 = 1/30 = 0.0333$ s. (c) $2\pi f_0 = 70\pi$, so $T_0 = 1/f_0 = 1/35 = 0.0286$ s. (d) We have $50\pi = 2\pi m f_0$ and $60\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 5 Hz with $m = 5$ and $n = 6$. (e) We have $50\pi = 2\pi m f_0$ and $70\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 5 Hz with $m = 5$ and $n = 7$.

Problem 1-10

(a) $|A| = 4.2426$; angle(A) = 0.7854 radians; $B = 5.0 + j\,8.6603$, so Re(B) = 5 and Im(B) = 8.6603. (b) $A+B = 8.0 + j11.6603$. (c) $A - B = -2.0 - j5.6603$. (d) $A*B = -10.9808 + j40.9808$. (e) $A/B = 0.4098 - j0.1098$.

Problem 1-11

(a) $2\pi f_0 = 10\pi$, so $T_0 = 1/f_0 = 1/5 = 0.2$ s. (b) $2\pi f_0 = 17\pi$, so $T_0 = 1/f_0 = 1/8.5 = 0.1176$ s. (c) $2\pi f_0 = 19\pi$, so $T_0 = 1/f_0 = 1/9.5 = 0.1053$ s. (d) We have $10\pi = 2\pi m f_0$ and $17\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 0.5 Hz with $m = 10$ and $n = 17$. (e) We have $10\pi = 2\pi m f_0$ and $19\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 0.5 Hz with $m = 10$ and $n = 19$. (f) We have $17\pi = 2\pi m f_0$ and $19\pi = 2\pi n f_0$, where m and n are integers and f_0 is the largest constant that satisfies these equations. The largest f_0 is 0.5 Hz with $m = 17$ and $n = 19$.

Problem 1-12

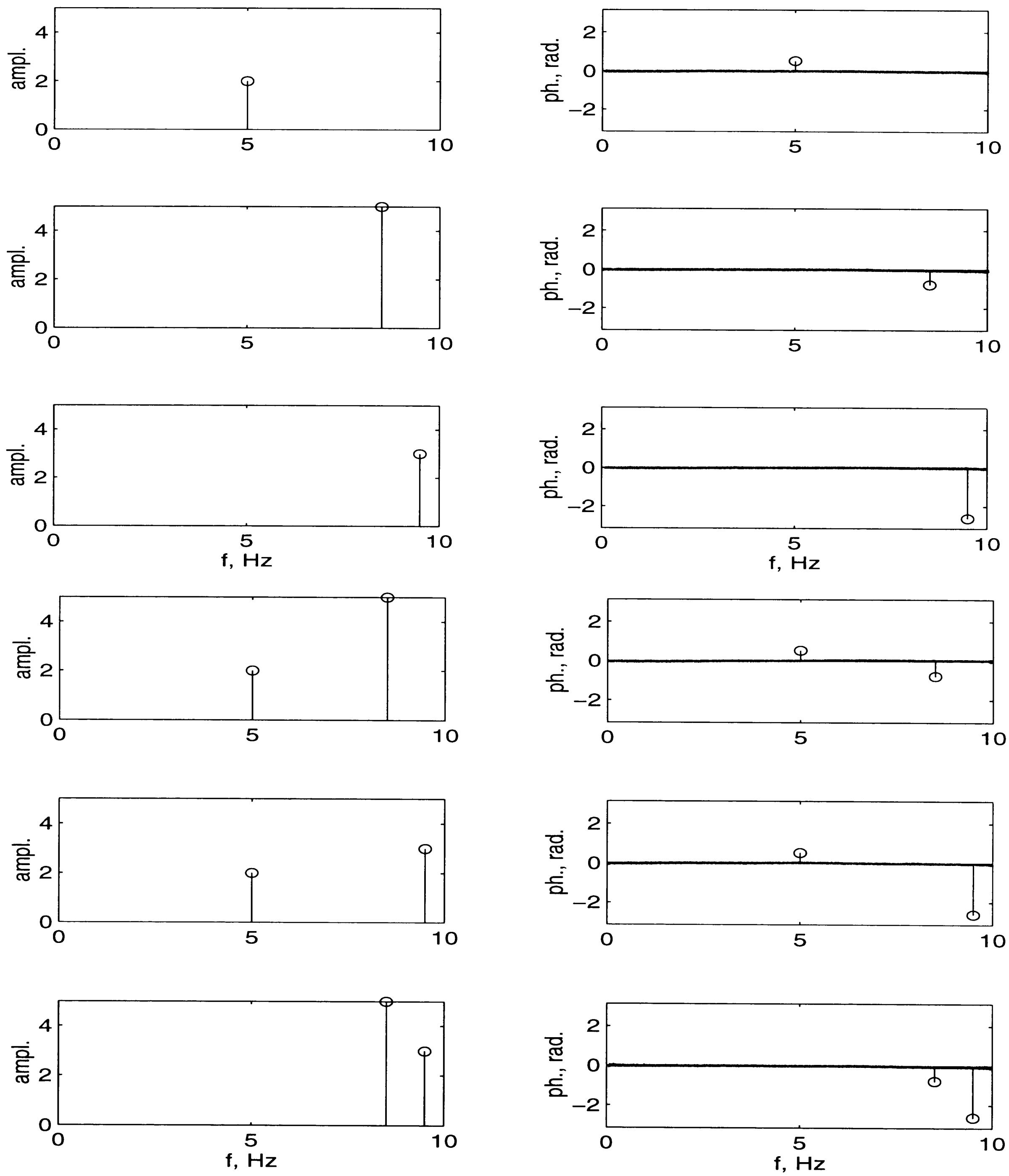
(a) Written as the real part of rotating phasors:

$$
\begin{aligned}
x_a(t) &= \mathrm{Re}[2e^{j(10\pi t + \pi/6)}];\; x_b(t) = \mathrm{Re}[5e^{j(17\pi t - \pi/4)}] \\
x_c(t) &= \mathrm{Re}[3e^{j(10\pi t - \pi/3 - \pi/2)}] = \mathrm{Re}[3e^{j(10\pi t - 5\pi/6)}] \\
x_d(t) &= \mathrm{Re}[2e^{j(10\pi t + \pi/6)} + 5e^{j(17\pi t - \pi/4)}];\; x_e(t) = \mathrm{Re}[2e^{j(10\pi t + \pi/6)} + 3e^{j(10\pi t - 5\pi/6)}] \\
x_f(t) &= \mathrm{Re}[5e^{j(17\pi t - \pi/4)} + 3e^{j(10\pi t - 5\pi/6)}]
\end{aligned}
$$

(b) In terms of counter rotating phasors, the signals are:

$$
\begin{aligned}
x_a(t) &= [e^{j(10\pi t + \pi/6)} + e^{-j(10\pi t + \pi/6)}];\; x_b(t) = [2.5e^{j(17\pi t - \pi/4)} + 2.5e^{j(17\pi t - \pi/4)}] \\
x_c(t) &= [1.5e^{j(10\pi t - 5\pi/6)} + 1.5e^{-j(10\pi t - 5\pi/6)}] \\
x_d(t) &= [e^{j(10\pi t + \pi/6)} + e^{-j(10\pi t + \pi/6)} + 2.5e^{j(17\pi t - \pi/4)} + 2.5e^{-j(17\pi t - \pi/4)}] \\
x_e(t) &= [e^{j(10\pi t + \pi/6)} + e^{-j(10\pi t + \pi/6)} + 1.5e^{j(10\pi t - 5\pi/6)} + 1.5e^{-j(10\pi t - 5\pi/6)}] \\
x_f(t) &= [2.5e^{j(17\pi t - \pi/4)} + 2.5e^{-j(17\pi t - \pi/4)} + 1.5e^{j(10\pi t - 5\pi/6)} + 1.5e^{-j(10\pi t - 5\pi/6)}]
\end{aligned}
$$

(c) Single-sided spectra are plotted below. Double-sided amplitude spectra are obtained by halving the lines and taking mirror image; phase spectra are obtained by taking antisymmetric mirror image.



Problem 1-13

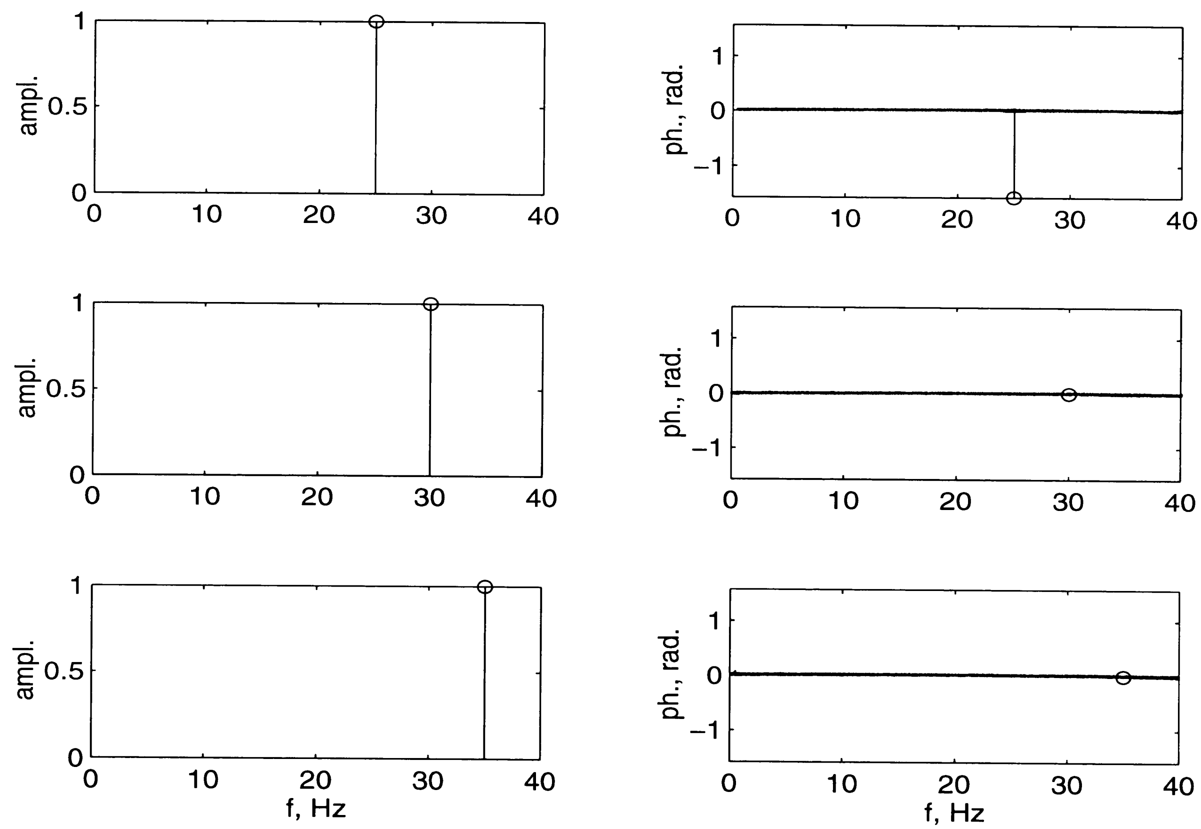
(a) Written as the real part of rotating phasors:

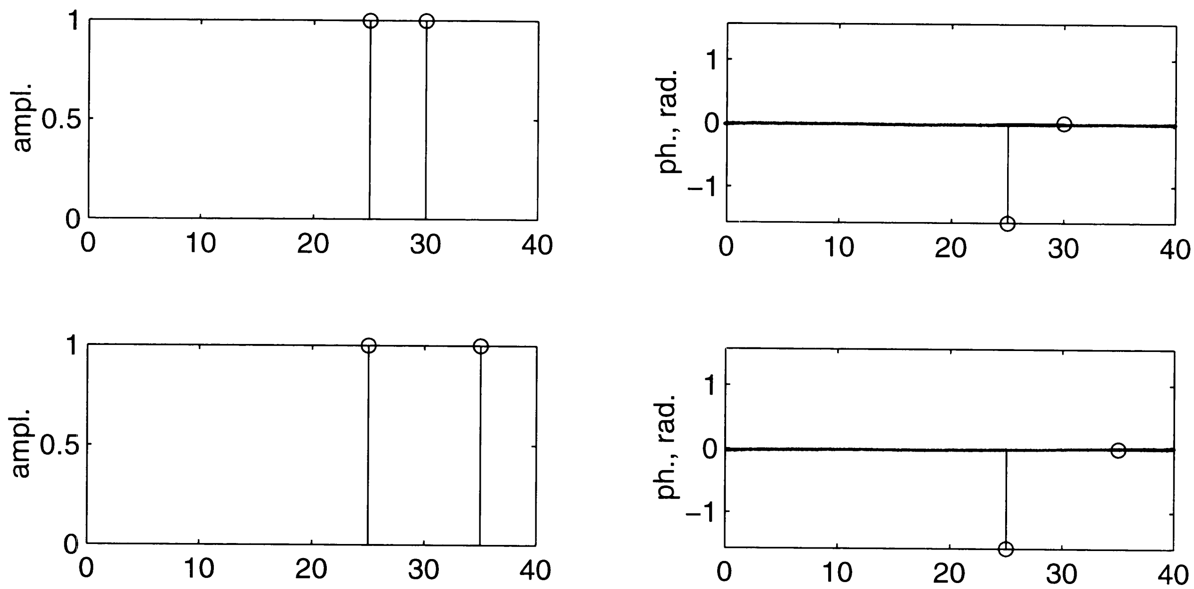
$$x_a(t) = \mathrm{Re}[e^{j(50\pi t - \pi/2)}];\ x_b(t) = \mathrm{Re}[e^{j60\pi t}];\ x_c(t) = \mathrm{Re}[e^{j70\pi t}]$$
$$x_d(t) = \mathrm{Re}[e^{j(50\pi t - \pi/2)} + e^{j60\pi t}];\ x_e(t) = \mathrm{Re}[e^{j(50\pi t - \pi/2)} + e^{j70\pi t}]$$

(b) In terms of counter rotating phasors, the signals are:

$$x_a(t) = \mathrm{Re}[0.5e^{j(50\pi t - \pi/2)} + 0.5e^{-j(50\pi t - \pi/2)}];\ x_b(t) = \mathrm{Re}[0.5e^{j60\pi t} + 0.5e^{-j60\pi t}]$$
$$x_c(t) = \mathrm{Re}[0.5e^{j70\pi t} + 0.5e^{-j70\pi t}]$$
$$x_d(t) = \mathrm{Re}[0.5e^{j(50\pi t - \pi/2)} + 0.5e^{-j(50\pi t - \pi/2)} + 0.5e^{j60\pi t} + 0.5e^{-j60\pi t}]$$
$$x_e(t) = \mathrm{Re}[0.5e^{j(50\pi t - \pi/2)} + 0.5e^{-j(50\pi t - \pi/2)} + 0.5e^{j70\pi t} + \ + 0.5e^{-j70\pi t}]$$

(c) The single-sided amplitude and phase spectra are shown below. See Prob. 1-12c for comments on obtaining double-sided spectra from single-sided spectra.

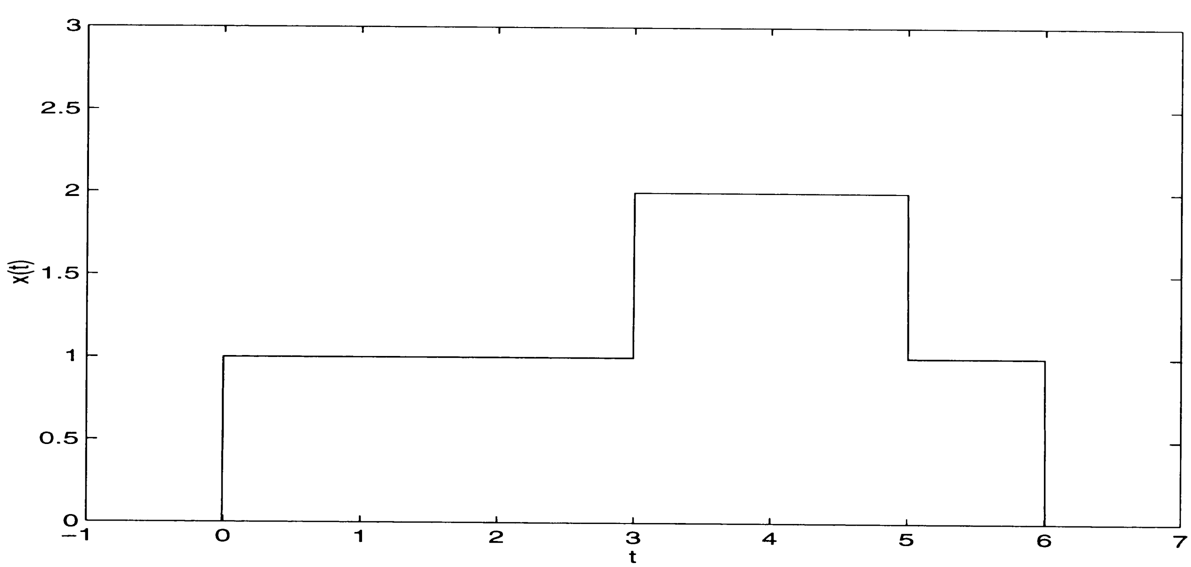




Problem 1-14

(a) A sketch is given below:

From the figure, it is evident that $x(t) = u(t) + u(t-3) - u(t-5) - u(t-6)$.



(b) The derivative of $x(t)$ is $dx(t)/dt = \delta(t) + \delta(t-3) - \delta(t-5) - \delta(t-6)$