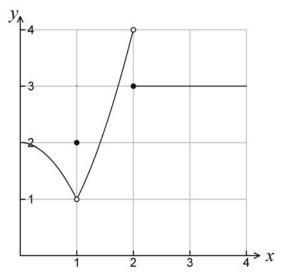
1. The function f and its graph are shown below:

$$f(x) = \begin{cases} -x^2 + 2 & 0 \le x < 1 \\ 2 & x = 1 \\ x^2 & 1 < x < 2 \\ 3 & x \ge 2 \end{cases}$$

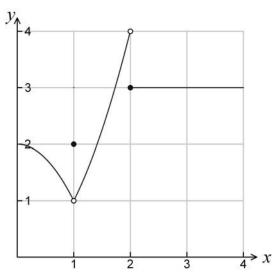


- (a) Calculate  $\lim_{x\to 2^-} f(x)$
- (b) Which value is greater  $\lim_{x\to 1} f(x)$  or f(1)? Justify your conclusion.
- (c) At what value(s) of c on the interval [0,4] does  $\lim_{x\to c} f(x)$  not exist? Justify your conclusion.

Chapter 1

1. The function f and its graph are shown below:

$$f(x) = \begin{cases} -x^2 + 2 & 0 \le x < 1 \\ 2 & x = 1 \\ x^2 & 1 < x < 2 \\ 3 & x \ge 2 \end{cases}$$



(a) Calculate  $\lim_{x\to 2^-} f(x)$ 

As we approach x = 2 from the left, the graph of f(x) nears 4 so  $\lim_{x \to 2^{-}} f(x) = 4$ .

- $+1 \lim_{x \to 2^{-}} f(x) = 4$
- (b) Which value is greater  $\lim_{x\to 1} f(x)$  or f(1)? Justify your conclusion.

Since  $\lim_{x \to 1^-} f(x) = 1$  and  $\lim_{x \to 1^+} f(x) = 1$ ,  $\lim_{x \to 1} f(x) = 1$ . From the function equation and graph, we see f(1) = 2. Therefore,  $f(1) > \lim_{x \to 1} f(x)$ .

- +1  $\lim_{x \to 1} f(x) = 1$ +1 f(1) = 2+2  $f(1) > \lim_{x \to 1} f(x)$
- (c) At what value(s) of c on the interval [0,4] does  $\lim_{x\to c} f(x)$  not exist? Justify your conclusion.

If the function f is continuous at x = c, then  $\lim_{x \to c} f(x)$  exists and  $\lim_{x \to c} f(x) = f(c)$ . We find the discontinuities in the graph of f(x). Discontinuities occur at x = 1 and x = 2.

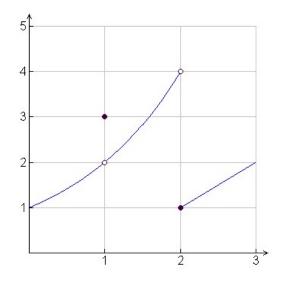
Consider  $\lim_{x\to 1} f(x)$  and  $\lim_{x\to 2} f(x)$ . In part (b), we showed  $\lim_{x\to 1} f(x) = 1$  so the limit exists at x = 1.

Consider  $\lim_{x\to 2} f(x)$ .  $\lim_{x\to 2^-} f(x) = 4$  and  $\lim_{x\to 2^+} f(x) = 3$ . Since the left and right hand limits are not equal,  $\lim_{x\to 2} f(x)$  does not exist.

- +1 If f is continuous at x = c, then  $\lim_{x \to c} f(x)$  exists.
- $+1 \lim_{x \to 1} f(x) = 1$
- +1  $\lim_{x \to 2^{-}} f(x) = 4$  and  $\lim_{x \to 2^{+}} f(x) = 3$
- +1 At x = 2,  $\lim_{x \to c} f(x)$  does not exist.

1. The function f and its graph are shown below:

$$f(x) = \begin{cases} 2^{x} & 0 \le x < 1\\ 3 & x = 1\\ 2^{x} & 1 < x < 2\\ x - 1 & 2 \le x \end{cases}$$



- (a) Calculate  $\lim_{x\to 2^+} f(x)$
- (b) Which value is greater  $\lim_{x\to 1} f(x)$  or f(2)? Justify your conclusion.
- (c) At what value(s) of c on the interval [0,3] does  $\lim_{x\to c} f(x)$  not exist? Justify your conclusion.