# Chapter P
# Fundamental Concepts of Algebra

**Section P.1**

**Check Point Exercises**

**1.** $8+6(x-3)^2 = 8+6(13-3)^2$
$= 8+6(10)^2$
$= 8+6(100)$
$= 8+600$
$= 608$

**2.** Since 2015 is 15 years after 2000, substitute 15 for $x$.
$T = 4x^2+341x+3194$
$= 4(15)^2+341(15)+3194$
$= 9209$
If trends continue, the tuition and fees will be \$9209.

**3.** The elements common to {3, 4, 5, 6, 7} and {3, 7, 8, 9} are 3 and 7.
$\{3,4,5,6,7\}\cap\{3,7,8,9\} = \{3,7\}$

**4.** The union is the set containing all the elements of either set.
$\{3,4,5,6,7\}\cup\{3,7,8,9\} = \{3,4,5,6,7,8,9\}$

**5.** $\left\{-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

**a.** Natural numbers: $\sqrt{9}$ because $\sqrt{9}=3$

**b.** Whole numbers: $0,\ \sqrt{9}$

**c.** Integers: $-9,\ 0,\ \sqrt{9}$

**d.** Rational numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \sqrt{9}$

**e.** Irrational numbers: $\frac{\pi}{2},\ \sqrt{10}$

**f.** Real numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}$

**6.** **a.** $\left|1-\sqrt{2}\right|$

Because $\sqrt{2}\approx 1.4,$ the number inside the absolute value bars is negative. The absolute value of $x$ when $x<0$ is $-x$. Thus,
$\left|1-\sqrt{2}\right| = -\left(1-\sqrt{2}\right) = \sqrt{2}-1$

**b.** $|\pi-3|$

Because $\pi\approx 3.14,$ the number inside the absolute value bars is positive. The absolute value of a positive number is the number itself. Thus,
$|\pi-3| = \pi-3.$

**c.** $\frac{|x|}{x}$

Because $x>0,\ \ |x| = x.$

Thus, $\frac{|x|}{x} = \frac{x}{x} = 1$

**7.** $|-4-(5)| = |-9| = 9$
The distance between –4 and 5 is 9.

**8.** $7(4x^2+3x)+2(5x^2+x)$
$= 7(4x^2+3x)+2(5x^2+x)$
$= 28x^2+21x+10x^2+2x$
$= 38x^2+23x$

**9.** $6+4[7-(x-2)]$
$= 6+4[7-x+2)]$
$= 6+4[9-x]$
$= 6+36-4x$
$= 42-4x$

**Concept and Vocabulary Check P.1**

**1.** expression

**2.** $b$ to the $n$th power; base; exponent

**3.** formula; modeling; models

**4.** intersection; $A\cap B$

**5.** union; $A\cup B$

**6.** natural

**7.** whole

**8.** integers

**9.** rational

**10.** irrational

**11.** rational; irrational

**12.** absolute value; $x$, $-x$

**13.** $b+a$ ; $ba$

**14.** $a+(b+c)$ ; $(ab)c$

**15.** $ab+ac$

**16.** 0; inverse; 0; identity

**17.** inverse; 1; identity

**18.** simplified

**19.** $a$

**Exercise Set P.1**

**1.** $7+5(10)=7+50=57$

**2.** $8+6(5)=8+30=38$

**3.** $6(3)-8=18-8=10$

**4.** $8(3)-4=24-4=20$

**5.** $8^2+3(8)=64+24=88$

**6.** $6^2+5(6)=36+30=66$

**7.** $7^2-6(7)+3=49-42+3=7+3=10$

**8.** $8^2-7(8)+4=64-56+4=8+4=12$

**9.** $4+5(9-7)^3=4+5(2)^3$
$=4+5(8)=4+40=44$

**10.** $6+5(8-6)^3=6+5(2)^3$
$=6+5(8)$
$=6+40=46$

**11.** $8^2-3(8-2)=64-3(6)$
$=64-18=46$

**12.** $8^2-4(8-3)=64-4(5)=64-20=44$

**13.** $\dfrac{5(x+2)}{2x-14}=\dfrac{5(10+2)}{2(10)-14}$
$=\dfrac{5(12)}{6}$
$=5\cdot 2$
$=10$

**14.** $\dfrac{7(x-3)}{2x-16}=\dfrac{7(9-3)}{2(9)-16}=\dfrac{7(6)}{2}=7\cdot 3=21$

**15.** $\dfrac{2x+3y}{x+1}; x=-2, y=4$
$=\dfrac{2(-2)+3(4)}{-2+1}=\dfrac{-4+12}{-1}=\dfrac{8}{-1}=-8$

**16.** $\dfrac{2x+y}{xy-2x}$ ; $x=-2$ and $y=4$
$\dfrac{2(-2)+4}{(-2)(4)-2(-2)}=\dfrac{-4+4}{-8+4}=\dfrac{0}{4}=0$

**17.** $C=\dfrac{5}{9}(50-32)=\dfrac{5}{9}(18)=10$
$50^\circ$F is equivalent to $10^\circ$C.

**18.** $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(86-32)=\dfrac{5}{9}(54)=30$
$86^\circ$F is equivalent to $30^\circ$C.

**19.** $h=4+60t-16t^2=4+60(2)-16(2)^2$
$=4+120-16(4)=4+120-64$
$=124-64=60$
Two seconds after it is kicked, the ball's height is 60 feet.

**20.** $h=4+60t-16t^2$
$=4+60(3)-16(3)^2$
$=4+180-16(9)$
$=4+180-144$
$=184-144=40$
Three seconds after it is kicked, the ball's height is 40 feet.

**21.** $\{1,2,3,4\}\cap\{2,4,5\}=\{2,4\}$

**22.** $\{1,3,7\}\cap\{2,3,8\}=\{3\}$

**23.** $\{s,e,t\}\cap\{t,e,s\}=\{s,e,t\}$

**24.** $\{r,e,a,l\}\cap\{l,e,a,r\}=\{r,e,a,l\}$

**25.** $\{1,3,5,7\}\cap\{2,4,6,8,10\}=\{\ \}$
The empty set is also denoted by $\varnothing$.

**26.** $\{1,3,5,7\}\cap\{-5,-3,-1\}=\{\ \}$ or $\varnothing$

**27.** $\{a,b,c,d\}\cap\varnothing=\varnothing$

**28.** $\{w,y,z\}\cap\varnothing=\varnothing$

**29.** $\{1,2,3,4\}\cup\{2,4,5\}=\{1,2,3,4,5\}$

**30.** $\{1,3,7,8\}\cup\{2,3,8\}=\{1,2,3,7,8\}$

**31.** $\{1,3,5,7\}\cup\{2,4,6,8,10\}$
$=\{1,2,3,4,5,6,7,8,10\}$

**32.** $\{0,1,3,5\}\cup\{2,4,6\}=\{0,1,2,3,4,5,6\}$

**33.** $\{a,e,i,o,u\}\cup\varnothing=\{a,e,i,o,u\}$

**34.** $\{e,m,p,t,y\}\cup\varnothing=\{e,m,p,t,y\}$

**35.** **a.** $\sqrt{100}$

**b.** $0,\sqrt{100}$

**c.** $-9,0,\sqrt{100}$

**d.** $-9,-\frac{4}{5},0,0.25,9.2,\sqrt{100}$

**e.** $\sqrt{3}$

**f.** $-9,-\frac{4}{5},0,0.25,\sqrt{3},9.2,\sqrt{100}$

**36.** **a.** $\sqrt{49}$

**b.** $0,\sqrt{49}$

**c.** $-7,0,\sqrt{49}$

**d.** $-7,-0.\overline{6},0,\sqrt{49}$

**e.** $\sqrt{50}$

**f.** $-7,-0.\overline{6},0,\sqrt{49},\sqrt{50}$

**37.** **a.** $\sqrt{64}$

**b.** $0,\sqrt{64}$

**c.** $-11,0,\sqrt{64}$

**d.** $-11,-\frac{5}{6},0,0.75,\sqrt{64}$

**e.** $\sqrt{5},\pi$

**f.** $-11,-\frac{5}{6},0,0.75,\sqrt{5},\pi,\sqrt{64}$

**38.** **a.** $\sqrt{4}$

**b.** $0,\sqrt{4}$

**c.** $-5,0,\sqrt{4}$

**d.** $-5,-0.\overline{3},0,\sqrt{4}$

**e.** $\sqrt{2}$

**f.** $-5,-0.\overline{3},0,\sqrt{2},\sqrt{4}$

**39.** 0

**40.** Answers will vary. An example is $\frac{1}{2}$.

**41.** Answers will vary. An example is 2.

**42.** Answers will vary. An example is $-2$.

**43.** true; $-13$ is to the left of $-2$ on the number line.

**44.** false; $-6$ is to the left of 2 on the number line.

**45.** true; 4 is to the right of $-7$ on the number line.

**46.** true; $-13$ is to the left of $-5$ on the number line.

**47.** true; $-\pi=-\pi$

**48.** true; $-3$ is to the right of $-13$ on the number line.

**49.** true; 0 is to the right of $-6$ on the number line.

**50.** true; 0 is to the right of $-13$ on the number line.

**51.** $|300|=300$

**52.** $|-203|=203$

**53.** $|12-\pi| = 12-\pi$

**54.** $|7-\pi| = 7-\pi$

**55.** $|\sqrt{2}-5| = 5-\sqrt{2}$

**56.** $|\sqrt{5}-13| = 13-\sqrt{5}$

**57.** $\dfrac{-3}{|-3|} = \dfrac{-3}{3} = -1$

**58.** $\dfrac{-7}{|-7|} = \dfrac{-7}{7} = -1$

**59.** $\big||-3|-|-7|\big| = |3-7| = |-4| = 4$

**60.** $\big||-5|-|-13|\big| = |5-13| = |-8| = 8$

**61.** $|x+y| = |2+(-5)| = |-3| = 3$

**62.** $|x-y| = |2-(-5)| = |7| = 7$

**63.** $|x|+|y| = |2|+|-5| = 2+5 = 7$

**64.** $|x|-|y| = |2|-|-5| = 2-5 = -3$

**65.** $\dfrac{y}{|y|} = \dfrac{-5}{|-5|} = \dfrac{-5}{5} = -1$

**66.** $\dfrac{|x|}{x}+\dfrac{|y|}{y} = \dfrac{|2|}{2}+\dfrac{|-5|}{-5} = \dfrac{2}{2}+\dfrac{5}{-5} = 1+(-1) = 0$

**67.** The distance is $|2-17| = |-15| = 15$.

**68.** The distance is $|4-15| = |-11| = 11$.

**69.** The distance is $|-2-5| = |-7| = 7$.

**70.** The distance is $|-6-8| = |-14| = 14$.

**71.** The distance is $|-19-(-4)| = |-19+4| = |-15| = 15$.

**72.** The distance is $|-26-(-3)| = |-26+3| = |-23| = 23$.

**73.** The distance is
$|-3.6-(-1.4)| = |-3.6+1.4| = |-2.2| = 2.2$.

**74.** The distance is
$|-5.4-(-1.2)| = |-5.4+1.2| = |-4.2| = 4.2$.

**75.** $6 + (-4) = (-4) + 6$;
commutative property of addition

**76.** $11\cdot(7+4) = 11\cdot 7 + 11\cdot 4$;
distributive property of multiplication over addition

**77.** $6 + (2 + 7) = (6 + 2) + 7$;
associative property of addition

**78.** $6\cdot(2\cdot 3) = 6\cdot(3\cdot 2)$;
commutative property of multiplication

**79.** $(2 + 3) + (4 + 5) = (4 + 5) + (2 + 3)$;
commutative property of addition

**80.** $7\cdot(11\cdot 8) = (11\cdot 8)\cdot 7$;
commutative property of multiplication

**81.** $2(-8 + 6) = -16 + 12$;
distributive property of multiplication over addition

**82.** $-8(3+11) = -24 + (-88)$;
distributive property of multiplication over addition

**83.** $\dfrac{1}{x+3}(x+3) = 1; x \neq -3$,
inverse property of multiplication

**84.** $(x+4)+\big[-(x+4)\big] = 0$;
inverse property of addition

**85.** $\begin{aligned} 5(3x+4)-4 &= 5\cdot 3x + 5\cdot 4 - 4 \\ &= 15x + 20 - 4 \\ &= 15x + 16 \end{aligned}$

**86.** $\begin{aligned} 2(5x+4)-3 &= 2\cdot 5x + 2\cdot 4 - 3 \\ &= 10x + 8 - 3 \\ &= 10x + 5 \end{aligned}$

**87.** $\begin{aligned} 5(3x-2)+12x &= 5\cdot 3x - 5\cdot 2 + 12x \\ &= 15x - 10 + 12x \\ &= 27x - 10 \end{aligned}$

**88.** $\begin{aligned} 2(5x-1)+14x &= 2\cdot 5x - 2\cdot 1 + 14x \\ &= 10x - 2 + 14x \\ &= 24x - 2 \end{aligned}$