Chapter 1: Algebra and Equations

Section 1.1 The Real Numbers

1. True. This statement is true, since every integer can be written as the ratio of the integer and 1.
For example, $5 = \frac{5}{1}$.

2. False. For example, 5 is a real number, and $5 = \frac{10}{2}$ which is not an irrational number.

3. Answers vary with the calculator, but $\frac{2,508,429,787}{798,458,000}$ is the best.

4. $0 + (-7) = (-7) + 0$
This illustrates the commutative property of addition.

5. $6(t + 4) = 6t + 6 \cdot 4$
This illustrates the distributive property.

6. $3 + (-3) = (-3) + 3$
This illustrates the commutative property of addition.

7. $(-5) + 0 = -5$
This illustrates the identity property of addition.

8. $(-4)\bullet(\frac{-1}{4}) = 1$
This illustrates the multiplicative inverse property.

9. $8 + (12 + 6) = (8 + 12) + 6$
This illustrates the associative property of addition.

10. $1\bullet(-20) = -20$
This illustrates the identity property of multiplication.

11. Answers vary. One possible answer: The sum of a number and its additive inverse is the additive identity. The product of a number and its multiplicative inverse is the multiplicative identity.

12. Answers vary. One possible answer: When using the commutative property, the order of the addends or multipliers are changed, while the grouping of the addends or multipliers is changed when using the associative property.

For Exercises 13–16, let $p = -2$, $q = 3$ and $r = -5$.

13. $-3(p + 5q) = -3[-2 + 5(3)] = -3[-2 + 15]$
$= -3(13) = -39$

14. $2(q - r) = 2(3 + 5) = 2(8) = 16$

15. $\frac{q + r}{q + p} = \frac{3 + (-5)}{3 + (-2)} = \frac{-2}{1} = -2$

16. $\frac{3q}{3p - 2r} = \frac{3(3)}{3(-2) - 2(-5)} = \frac{9}{-6 + 10} = \frac{9}{4}$

17. Let $r = 3.8$.
$\text{APR} = 12r = 12(3.8) = 45.6\%$

18. Let $r = 0.8$.
$\text{APR} = 12r = 12(0.8) = 9.6\%$

19. Let $APR = 11$.
$APR = 12r$
$11 = 12r$
$\frac{11}{12} = r$
$r \approx .9167\%$

20. Let $APR = 13.2$.
$APR = 12r$
$13.2 = 12r$
$\frac{13.2}{12} = r$
$r = 1.1\%$

21. $3 - 4 \cdot 5 + 5 = 3 - 20 + 5 = -17 + 5 = -12$

22. $8 - (-4)^2 - (-12)$
Take powers first.
$8 - 16 - (-12)$
Then add and subtract in order from left to right.
$8 - 16 + 12 = -8 + 12 = 4$

23. $(4 - 5) \cdot 6 + 6 = -1 \cdot 6 + 6 = -6 + 6 = 0$

24. $\dfrac{2(3-7)+4(8)}{4(-3)+(-3)(-2)}$

Work above and below fraction bar. Do multiplications and work inside parentheses.

$$=\frac{2(-4)+32}{-12+6}=\frac{-8+32}{-12+6}=\frac{24}{-6}=-4$$

25. $8-4^2-(-12)$

Take powers first.

$8 - 16 - (-12)$

Then add and subtract in order from left to right.

$8 - 16 + 12 = -8 + 12 = 4$

26. $-(3-5)-\left[2-\left(3^2-13\right)\right]$

Take powers first.

$-(3 - 5) - [2 - (9 - 13)]$

Work inside brackets and parentheses.

$$-(-2)-[2-(-4)]=2-[2+4]$$
$$=2-6=-4$$

27. $\dfrac{2(-3)+\frac{3}{(-2)}-\frac{2}{\left(-\sqrt{16}\right)}}{\sqrt{64}-1}$

Work above and below fraction bar. Take roots.

$$\frac{2(-3)+\frac{3}{(-2)}-\frac{2}{(-4)}}{8-1}$$

Do multiplications and divisions.

$$\frac{-6-\frac{3}{2}+\frac{1}{2}}{8-1}$$

Add and subtract.

$$\frac{-\frac{12}{2}-\frac{3}{2}+\frac{1}{2}}{7}=\frac{-\frac{14}{2}}{7}=\frac{-7}{7}=-1$$

28. $\dfrac{6^2-3\sqrt{25}}{\sqrt{6^2+13}}$

Take powers and roots.

$$\frac{36-3(5)}{\sqrt{36+13}}=\frac{36-15}{\sqrt{49}}=\frac{21}{7}=3$$

29. $\dfrac{2040}{523}, \dfrac{189}{37}, \sqrt{27}, \dfrac{4587}{691}, 6.735, \sqrt{47}$

30. $\dfrac{187}{63}, 2.9884, \sqrt{\sqrt{85}}, \pi, \sqrt{10}, \dfrac{385}{117}$

31. 12 is less than 18.5.

$12 < 18.5$

32. –2 is greater than –20.

$-2 > -20$

33. x is greater than or equal to 5.7.

$x \geq 5.7$

34. y is less than or equal to –5.

$y \leq -5$

35. z is at most 7.5.

$z \leq 7.5$

36. w is negative.

$w < 0$

37. $-6 < -2$

38. $3/4 = .75$

39. $3.14 < \pi$

40. $1/3 > .33$

41. a lies to the right of b or is equal to b.

42. $b + c = a$

43. $c < a < b$

44. a lies to the right of 0

45. $(-8, -1)$

This represents all real numbers between –8 and –1, not including –8 and –1. Draw parentheses at –8 and –1 and a heavy line between them. The parentheses at –8 and –1 show that neither of these points belongs to the graph.

46. $[-1, 10]$

This represents all real numbers between –1 and 10, including –1 and 10. Draw brackets at –1 and 10 and a heavy line between them.

47. $(-2, 3]$

All real numbers x such that $-2 < x \leq 3$

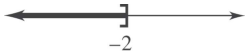
Draw a heavy line from –2 to 3. Use a parenthesis at –2 since it is not part of the graph. Use a bracket at 3 since it is part of the graph.

48. [–2, 2)
This represents all real numbers between –2 and 2, including –2, but not including 2.
Draw a bracket at –2, a parenthesis at 2, and a heavy line between them.

49. $(-2, \infty)$

All real numbers x such that $x > -2$
Start at –2 and draw a heavy line to the right.
Use a parenthesis at –2 since it is not part of the graph.

50. (–∞, –2]
This represents all real numbers less than or equal to –2. Draw a bracket at –2 and a heavy line to the left.



51. $|-9| - |-12| = 9 - (12) = -3$

52. $|8| - |-4| = 8 - (4) = 4$

53. $-|-4| - |-1 - 14| = -(4) - |-15|$
$= -(4) - 15 = -19$

54. $-|6| - |-12 - 4| = -(6) - |-16| = -6 - (16) = -22$

55. $|5| \_\_\_ |-5|$
$5 \_\_ 5$
$5 = 5$

56. $-|-4| \_\_\_ |4|$
$-4 \_\_ 4$
$-4 < 4$

57. $|10 - 3| \_\_\_ |3 - 10|$
$|7| \_\_\_ |-7|$
$7 \_\_\_ 7$
$7 = 7$

58. $|6 - (-4)| \_\_\_ |-4 - 6|$
$|10| \_\_\_ |-10|$
$10 \_\_\_ 10$
$10 = 10$

59. $|-2 + 8| \_\_\_ |2 - 8|$
$|6| \_\_\_ |-6|$
$6 \_\_\_ 6$
$6 = 6$

60. $|3| \cdot |-5| \_\_\_ |3(-5)|$
$|3| \cdot |-5| \_\_\_ |-15|$
$3 \cdot 5 \_\_\_ 15$
$15 = 15$

61. $|3 - 5| \_\_\_ |3| - |5|$
$|-2| \_\_\_ 3 - 5$
$2 \_\_\_ -2$
$2 > -2$

62. $|-5 + 1| \_\_\_ |-5| + |1|$
$|-4| \_\_\_ 5 + 1$
$4 \_\_\_ 6$
$4 < 6$

63. When $a < 7$, $a - 7$ is negative.
So $|a - 7| = -(a - 7) = 7 - a$.

64. When $b \geq c$, $b - c$ is positive.
So $|b - c| = b - c$.

Answers will vary for exercises 65–67. Sample answers are given.

65. No, it is not always true that $|a + b| = |a| + |b|$. For example, let $a = 1$ and $b = -1$. Then,
$|a + b| = |1 + (-1)| = |0| = 0$, but
$|a| + |b| = |1| + |(-1)| = 1 + 1 = 2$.

66. Yes, if a and b are any two real numbers, it is always true that $|a - b| = |b - a|$. In general, $a - b = -(b - a)$. When we take the absolute value of each side, we get
$|a - b| = |-(b - a)| = |b - a|$.

67. $|2 - b| = |2 + b|$ only when $b = 0$. Then each side of the equation is equal to 2. If b is any other value, subtracting it from 2 and adding it to 2 will produce two different values.

68. For females: $|x - 63.5| \leq 8.4$; for males: $|x - 68.9| \leq 9.3$