CHAPTER 1
Functions, Graphs, and Limits

Section 1.1 The Cartesian Plane and the Distance Formula

Skills Warm Up

1. $\sqrt{(3-6)^2 + [1-(-5)]^2} = \sqrt{(-3)^2 + 6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

2. $\sqrt{(-2-0)^2 + [-7-(-3)]^2} = \sqrt{(-2)^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

3. $\dfrac{5+(-4)}{2} = \dfrac{1}{2}$

4. $\dfrac{-3+(-1)}{2} = \dfrac{-4}{2} = -2$

5. $\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$

6. $\sqrt{8} - \sqrt{18} = 2\sqrt{2} - 3\sqrt{2} = -\sqrt{2}$

7. $\dfrac{x+(-5)}{2} = 7$

$x + (-5) = 14$

$x = 19$

8. $\dfrac{-7+y}{2} = -3$

$-7 + y = -6$

$y = 1$

9. $\sqrt{(3-x)^2 + (7-4)^2} = \sqrt{45}$

$\left(\sqrt{(3-x)^2 + (7-4)^2}\right)^2 = \left(\sqrt{45}\right)^2$

$(3-x)^2 + (7-4)^2 = 45$

$(3-x)^2 + 3^2 = 45$

$(3-x)^2 + 9 = 45$

$(3-x)^2 = 36$

$3 - x = \pm 6$

$-x = -3 \pm 6$

$x = 3 \mp 6$

$x = -3, 9$

10. $\sqrt{(6-2)^2 + (-2-y)^2} = \sqrt{52}$

$\left(\sqrt{(6-2)^2 + (-2-y)^2}\right)^2 = \left(\sqrt{52}\right)^2$

$(6-2)^2 + (-2-y)^2 = 52$

$4^2 + (-2-y)^2 = 52$

$16 + (-2-y)^2 = 52$

$(-2-y)^2 = 36$

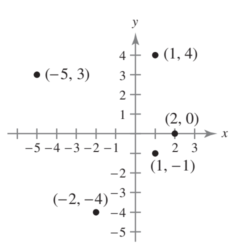
$-2 - y = \pm 6$

$-y = \pm 6 + 2$

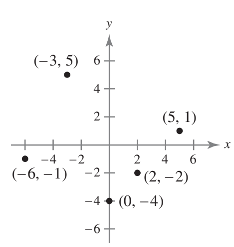
$y = \mp 6 - 2$

$y = -8, 4$

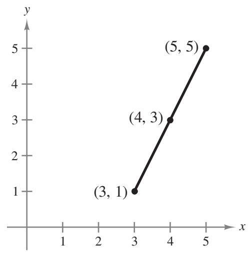
1.



2.



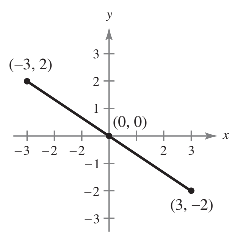
3. (a)



(b) $d = \sqrt{(5-3)^2 + (5-1)^2} = \sqrt{4+16} = 2\sqrt{5}$

(c) Midpoint $= \left(\dfrac{3+5}{2}, \dfrac{1+5}{2}\right) = (4, 3)$

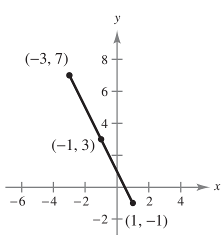
4. (a)



(b) $d = \sqrt{(-3-3)^2 + (2+2)^2} = \sqrt{36+16} = 2\sqrt{13}$

(c) Midpoint $= \left(\dfrac{-3+3}{2}, \dfrac{2+(-2)}{2}\right) = (0, 0)$

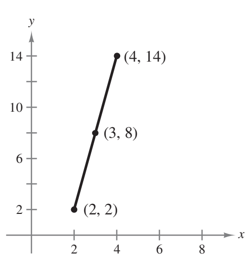
5. (a)



(b) $d = \sqrt{(-3-1)^2 + (7+1)^2} = \sqrt{16+64} = 4\sqrt{5}$

(c) Midpoint $= \left(\dfrac{-3+1}{2}, \dfrac{7-1}{2}\right) = (-1, 3)$

6. (a)



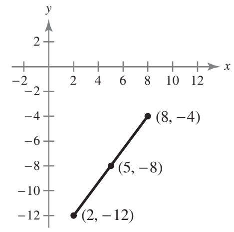
(b) $d = \sqrt{(4-2)^2 + (14-2)^2}$

$= \sqrt{4+144}$

$= 2\sqrt{37}$

(c) Midpoint $= \left(\dfrac{2+4}{2}, \dfrac{2+14}{2}\right) = (3, 8)$

7. (a)



(b) $d = \sqrt{(8-2)^2 + \left(-4-(-12)^2\right)}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36+64}$

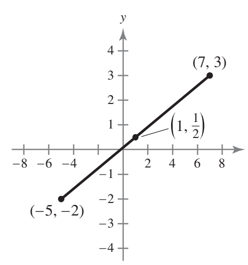
$= \sqrt{100} = 10$

(c) Midpoint $= \left(\dfrac{2+8}{2}, \dfrac{(-12)+(-4)}{2}\right)$

$= \left(\dfrac{10}{2}, \dfrac{-16}{2}\right)$

$= (5, -8)$

8. (a)



(b) $d = \sqrt{(7-(-5))^2 + \left(3-(-2)^2\right)}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144+25}$

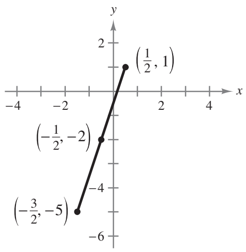
$= \sqrt{169} = 13$

(c) Midpoint $= \left(\dfrac{7+(-5)}{2}, \dfrac{3+(-2)}{2}\right)$

$= \left(\dfrac{2}{2}, \dfrac{1}{2}\right)$

$= \left(1, \dfrac{1}{2}\right)$

9. (a)



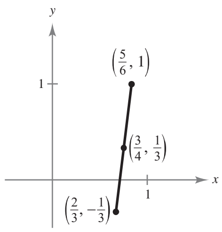
(b) $d = \sqrt{\left[(3/2) - (1/2)\right]^2 + (5 - 1)^2}$

$= \sqrt{4 + 36}$

$= 2\sqrt{10}$

(c) Midpoint $= \left(\dfrac{(1/2) + (-3/2)}{2}, \dfrac{1 + (-5)}{2}\right) = \left(-\dfrac{1}{2}, -2\right)$

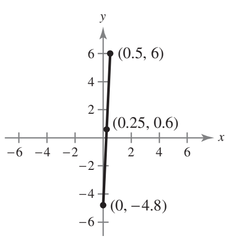
10. (a)



(b) $d = \sqrt{\left(\dfrac{5}{6} - \dfrac{2}{3}\right)^2 + \left(1 + \dfrac{1}{3}\right)^2} = \sqrt{\dfrac{1}{36} + \dfrac{16}{9}} = \dfrac{\sqrt{65}}{6}$

(c) Midpoint $= \left(\dfrac{(5/6) + (2/3)}{2}, \dfrac{1 - (1/3)}{2}\right) = \left(\dfrac{3}{4}, \dfrac{1}{3}\right)$

11. (a)



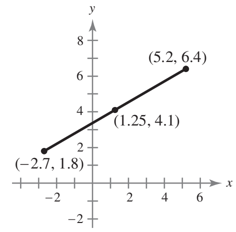
(b) $d = \sqrt{(0.5 - 0)^2 + (6 - (-4.8))^2}$

$= \sqrt{0.25 + 116.64}$

$= \sqrt{116.89}$

(c) Midpoint $= \left(\dfrac{0 + 0.5}{2}, \dfrac{-4.8 + 6}{2}\right) = (0.25, 0.6)$

12. (a)



(b) $d = \sqrt{(-2.7 - 5.2)^2 + (1.8 - 6.4)^2}$

$= \sqrt{62.41 + 21.16}$

$= \sqrt{83.57}$

(c) Midpoint $= \left(\dfrac{5.2 + (-2.7)}{2}, \dfrac{6.4 + 1.8}{2}\right)$

$= (1.25, 4.1)$

13. (a) $a = 4$

$b = 3$

$c = \sqrt{(4 - 0)^2 + (3 - 0)^2} = \sqrt{16 + 9} = 5$

(b) $a^2 + b^2 = 16 + 9 = 25 = c^2$

14. (a) $a = \sqrt{(13 - 1)^2 + (1 - 1)^2} = \sqrt{144 + 0} = 12$

$b = \sqrt{(13 - 13)^2 + (6 - 1)^2} = \sqrt{0 + 25} = 5$

$c = \sqrt{(13 - 1)^2 + (6 - 1)^2} = \sqrt{144 + 25} = 13$

(b) $a^2 + b^2 = 144 + 25 = 169 = c^2$

15. (a) $a = 10$

$b = 3$

$c = \sqrt{(7 + 3)^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}$

(b) $a^2 + b^2 = 100 + 9 = 109 = c^2$

16. (a) $a = \sqrt{(6 - 2)^2 + (-2 + 2)^2} = \sqrt{16 + 0} = 4$

$b = \sqrt{(2 - 2)^2 + (5 + 2)^2} = \sqrt{0 + 49} = 7$

$c = \sqrt{(2 - 6)^2 + (5 + 2)^2} = \sqrt{16 + 49} = \sqrt{65}$

(b) $a^2 + b^2 = 16 + 49 = 65 = c^2$

17. $d_1 = \sqrt{(3-0)^2 + (7-1)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$d_2 = \sqrt{(4-0)^2 + (-1-1)^2}$

$= \sqrt{16 + 4}$

$= \sqrt{20}$

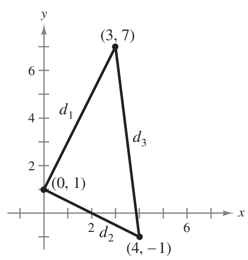
$= 2\sqrt{5}$

$d_3 = \sqrt{(3-4)^2 + [7-(-1)]^2}$

$= \sqrt{1 + 64}$

$= \sqrt{65}$

Because $d_1^2 + d_2^2 = d_3^2$, the figure is a right triangle.



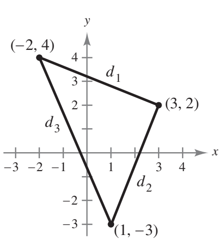
18. $a = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$b = \sqrt{(3-1)^2 + (2+3)^2} = \sqrt{4 + 25} = \sqrt{29}$

$c = \sqrt{(-2-1)^2 + (4+3)^2} = \sqrt{9 + 49} = \sqrt{58}$

Because $a = b$ the figure is an isosceles triangle.

[**Note:** It is also a right triangle since $a^2 + b^2 = c^2$.]



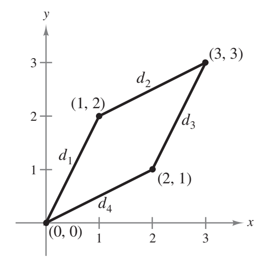
19. $d_1 = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{1 + 4} = \sqrt{5}$

$d_2 = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{4 + 1} = \sqrt{5}$

$d_3 = \sqrt{(2-3)^2 + (1-3)^2} = \sqrt{1 + 4} = \sqrt{5}$

$d_4 = \sqrt{(0-2)^2 + (0-1)^2} = \sqrt{4 + 1} = \sqrt{5}$

Because $d_1 = d_2 = d_3 = d_4$, the figure is a parallelogram.



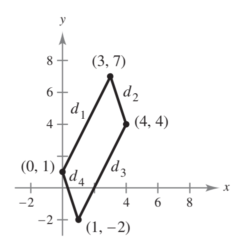
20. $a = \sqrt{(3-0)^2 + (7-1)^2} = \sqrt{9 + 36} = 3\sqrt{5}$

$b = \sqrt{(3-4)^2 + (7-4)^2} = \sqrt{1 + 9} = \sqrt{10}$

$c = \sqrt{(4-1)^2 + (4+2)^2} = \sqrt{9 + 36} = 3\sqrt{5}$

$d = \sqrt{(1-0)^2 + (-2-1)^2} = \sqrt{1 + 9} = \sqrt{10}$

Because $a = c$ and $b = d$, the figure is a parallelogram.



21. $d = \sqrt{(x-1)^2 + (-4-0)^2} = 5$

$\sqrt{x^2 - 2x + 17} = 5$

$x^2 - 2x + 17 = 25$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

22. $d = \sqrt{(x-2)^2 + (2+1)^2} = 5$

$\sqrt{x^2 - 4x + 13} = 5$

$x^2 - 4x + 13 = 25$

$x^2 - 4x - 12 = 0$

$(x+2)(x-6) = 0$

$x = -2, 6$