

Chapter 2

Polynomial and Rational Functions

Section 2.1

Check Point Exercises

1. a. $(5 - 2i) + (3 + 3i)$
 $= 5 - 2i + 3 + 3i$
 $= (5 + 3) + (-2 + 3)i$
 $= 8 + i$

b. $(2 + 6i) - (12 - i)$
 $= 2 + 6i - 12 + i$
 $= (2 - 12) + (6 + 1)i$
 $= -10 + 7i$

2. a. $7i(2 - 9i) = 7i(2) - 7i(9i)$
 $= 14i - 63i^2$
 $= 14i - 63(-1)$
 $= 63 + 14i$

b. $(5 + 4i)(6 - 7i) = 30 - 35i + 24i - 28i^2$
 $= 30 - 35i + 24i - 28(-1)$
 $= 30 + 28 - 35i + 24i$
 $= 58 - 11i$

3. $\frac{5 + 4i}{4 - i} = \frac{5 + 4i}{4 - i} \cdot \frac{4 + i}{4 + i}$
 $= \frac{20 + 5i + 16i + 4i^2}{20 + 2i - 4}$
 $= \frac{16 + 4i - 4i - i^2}{20 + 2i - 4}$
 $= \frac{16 + 1}{16 + 2i}$
 $= \frac{17}{16 + 2i}$
 $= \frac{16}{17} + \frac{21}{17}i$

4. a. $\sqrt{-27} + \sqrt{-48} = i\sqrt{27} + i\sqrt{48}$
 $= i\sqrt{9 \cdot 3} + i\sqrt{16 \cdot 3}$
 $= 3i\sqrt{3} + 4i\sqrt{3}$
 $= 7i\sqrt{3}$

b. $(-2 + \sqrt{-3})^2 = (-2 + i\sqrt{3})^2$
 $= (-2)^2 + 2(-2)(i\sqrt{3}) + (i\sqrt{3})^2$
 $= 4 - 4i\sqrt{3} + 3i^2$
 $= 4 - 4i\sqrt{3} + 3(-1)$
 $= 1 - 4i\sqrt{3}$

c. $\frac{-14 + \sqrt{-12}}{2} = \frac{-14 + i\sqrt{12}}{2}$
 $= \frac{-14 + 2i\sqrt{3}}{2}$
 $= \frac{-14}{2} + \frac{2i\sqrt{3}}{2}$
 $= -7 + i\sqrt{3}$

5. $x^2 - 2x + 2 = 0$
 $a = 1, b = -2, c = 2$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$
 $x = \frac{2 \pm \sqrt{4 - 8}}{2}$
 $x = \frac{2 \pm \sqrt{-4}}{2}$
 $x = \frac{2 \pm 2i}{2}$
 $x = 1 \pm i$

The solution set is $\{1 + i, 1 - i\}$.

Concept and Vocabulary Check 2.1

1. $\sqrt{-1}$; -1
2. complex; imaginary; real
3. $-6i$
4. $14i$
5. 18 ; $-15i$; $12i$; $-10i^2$; 10
6. $2 + 9i$
7. $2 + 5i$
8. i ; i ; $2i\sqrt{5}$
9. $-1 \pm i \frac{\sqrt{6}}{2}$

Exercise Set 2.1

1. $(7 + 2i) + (1 - 4i) = 7 + 2i + 1 - 4i$
 $= 7 + 1 + 2i - 4i$
 $= 8 - 2i$
2. $(-2 + 6i) + (4 - i)$
 $= -2 + 6i + 4 - i$
 $= -2 + 4 + 6i - i$
 $= 2 + 5i$
3. $(3 + 2i) - (5 - 7i) = 3 - 5 + 2i + 7i$
 $= 3 + 2i - 5 + 7i$
 $= -2 + 9i$
4. $(-7 + 5i) - (-9 - 11i) = -7 + 5i + 9 + 11i$
 $= -7 + 9 + 5i + 11i$
 $= 2 + 16i$
5. $6 - (-5 + 4i) - (-13 - i) = 6 + 5 - 4i + 13 + i$
 $= 24 - 3i$
6. $7 - (-9 + 2i) - (-17 - i) = 7 + 9 - 2i + 17 + i$
 $= 33 - i$
7. $8i - (14 - 9i) = 8i - 14 + 9i$
 $= -14 + 8i + 9i$
 $= -14 + 17i$
8. $15i - (12 - 11i) = 15i - 12 + 11i$
 $= -12 + 15i + 11i$
 $= -12 + 26i$
9. $-3i(7i - 5) = -21i^2 + 15i$
 $= -21(-1) + 15i$
 $= 21 + 15i$
10. $-8i(2i - 7) = -16i^2 + 56i = -16(-1) + 56i$
 $= 9 - 25i^2 = 9 + 25 = 34 = 16 + 56i$
11. $(-5 + 4i)(3 + i) = -15 - 5i + 12i + 4i^2$
 $= -15 + 7i - 4$
 $= -19 + 7i$
12. $(-4 - 8i)(3 + i) = -12 - 4i - 24i - 8i^2$
 $= -12 - 28i + 8$
 $= -4 - 28i$
13. $(7 - 5i)(-2 - 3i) = -14 - 21i + 10i + 15i^2$
 $= -14 - 15 - 11i$
 $= -29 - 11i$
14. $(8 - 4i)(-3 + 9i) = -24 + 72i + 12i - 36i^2$
 $= -24 + 36 + 84i$
 $= 12 + 84i$
15. $(3 + 5i)(3 - 5i) = 9 - 15i + 15i - 25i^2$
 $= 9 + 25$
 $= 34$
16. $(2 + 7i)(2 - 7i) = 4 - 49i^2 = 4 + 49 = 53$
17. $(-5 + i)(-5 - i) = 25 + 5i - 5i - i^2$
 $= 25 + 1$
 $= 26$
18. $(-7 + i)(-7 - i) = 49 + 7i - 7i - i^2$
 $= 49 + 1$
 $= 50$
19. $(2 + 3i)^2 = 4 + 12i + 9i^2$
 $= 4 + 12i - 9$
 $= -5 + 12i$
20. $(5 - 2i)^2 = 25 - 20i + 4i^2$
 $= 25 - 20i - 4$
 $= 21 - 20i$
21. $\frac{2}{3 - i} = \frac{2}{3 - i} \cdot \frac{3 + i}{3 + i}$
 $= \frac{2(3 + i)}{9 + 1}$
 $= \frac{2(3 + i)}{10}$
 $= \frac{3 + i}{5}$
 $= \frac{3}{5} + \frac{1}{5}i$
22. $\frac{3}{4 + i} = \frac{3}{4 + i} \cdot \frac{4 - i}{4 - i}$
 $= \frac{3(4 - i)}{16 - i^2}$
 $= \frac{3(4 - i)}{17}$
 $= \frac{12}{17} - \frac{3}{17}i$

$$23. \frac{2i}{1+i} = \frac{2i}{1+i} \cdot \frac{1-i}{1-i} = \frac{2i-2i^2}{1+1} = \frac{2+2i}{2} = 1+i$$

$$24. \frac{5i}{2-i} = \frac{5i}{2-i} \cdot \frac{2+i}{2+i} \\ = \frac{10i+5i^2}{4+1} \\ = \frac{-5+10i}{5} \\ = -1+2i$$

$$25. \frac{8i}{4-3i} = \frac{8i}{4-3i} \cdot \frac{4+3i}{4+3i} \\ = \frac{32i+24i^2}{16+9} \\ = \frac{-24+32i}{25} \\ = -\frac{24}{25} + \frac{32}{25}i$$

$$26. \frac{-6i}{3+2i} = \frac{-6i}{3+2i} \cdot \frac{3-2i}{3-2i} = \frac{-18i+12i^2}{9+4} \\ = \frac{-12-18i}{13} = -\frac{12}{13} - \frac{18}{13}i$$

$$27. \frac{2+3i}{2+i} = \frac{2+3i}{2+i} \cdot \frac{2-i}{2-i} \\ = \frac{4+4i-3i^2}{4+1} \\ = \frac{7+4i}{5} \\ = \frac{7}{5} + \frac{4}{5}i$$

$$28. \frac{3-4i}{4+3i} = \frac{3-4i}{4+3i} \cdot \frac{4-3i}{4-3i} \\ = \frac{12-25i+12i^2}{16+9} \\ = \frac{-25i}{25} \\ = -i$$

$$29. \sqrt{-64} - \sqrt{-25} = i\sqrt{64} - i\sqrt{25} \\ = 8i - 5i = 3i$$

$$30. \sqrt{-81} - \sqrt{-144} = i\sqrt{81} - i\sqrt{144} = 9i - 12i \\ = -3i$$

$$31. 5\sqrt{-16} + 3\sqrt{-81} = 5(4i) + 3(9i) \\ = 20i + 27i = 47i$$

$$32. 5\sqrt{-8} + 3\sqrt{-18} \\ = 5i\sqrt{8} + 3i\sqrt{18} = 5i\sqrt{4 \cdot 2} + 3i\sqrt{9 \cdot 2} \\ = 10i\sqrt{2} + 9i\sqrt{2} \\ = 19i\sqrt{2}$$

$$33. (-2 + \sqrt{-4})^2 = (-2 + 2i)^2 \\ = 4 - 8i + 4i^2 \\ = 4 - 8i - 4 \\ = -8i$$

$$34. (-5 - \sqrt{-9})^2 = (-5 - i\sqrt{9})^2 = (-5 - 3i)^2 \\ = 25 + 30i + 9i^2 \\ = 25 + 30i - 9 \\ = 16 + 30i$$

$$35. (-3 - \sqrt{-7})^2 = (-3 - i\sqrt{7})^2 \\ = 9 + 6i\sqrt{7} + i^2(7) \\ = 9 - 7 + 6i\sqrt{7} \\ = 2 + 6i\sqrt{7}$$

$$36. (-2 + \sqrt{-11})^2 = (-2 + i\sqrt{11})^2 \\ = 4 - 4i\sqrt{11} + i^2(11) \\ = 4 - 11 - 4i\sqrt{11} \\ = -7 - 4i\sqrt{11}$$

$$37. \frac{-8 + \sqrt{-32}}{24} = \frac{-8 + i\sqrt{32}}{24} \\ = \frac{-8 + i\sqrt{16 \cdot 2}}{24} \\ = \frac{-8 + 4i\sqrt{2}}{24} \\ = -\frac{1}{3} + \frac{\sqrt{2}}{6}i$$

$$38. \frac{-12 + \sqrt{-28}}{32} = \frac{-12 + i\sqrt{28}}{32} = \frac{-12 + i\sqrt{4 \cdot 7}}{32} \\ = \frac{-12 + 2i\sqrt{7}}{32} = -\frac{3}{8} + \frac{\sqrt{7}}{16}i$$

$$\begin{aligned}
 39. \quad \frac{-6 - \sqrt{-12}}{48} &= \frac{-6 - i\sqrt{12}}{48} \\
 &= \frac{-6 - i\sqrt{4 \cdot 3}}{48} \\
 &= \frac{-6 - 2i\sqrt{3}}{48} \\
 &= -\frac{1}{8} - \frac{\sqrt{3}}{24}i
 \end{aligned}$$

$$\begin{aligned}
 40. \quad \frac{-15 - \sqrt{-18}}{33} &= \frac{-15 - i\sqrt{18}}{33} = \frac{-15 - i\sqrt{9 \cdot 2}}{33} \\
 &= \frac{-15 - 3i\sqrt{2}}{33} = -\frac{5}{11} - \frac{\sqrt{2}}{11}i
 \end{aligned}$$

$$\begin{aligned}
 41. \quad \sqrt{-8}(\sqrt{-3} - \sqrt{5}) &= i\sqrt{8}(i\sqrt{3} - \sqrt{5}) \\
 &= 2i\sqrt{2}(i\sqrt{3} - \sqrt{5}) \\
 &= -2\sqrt{6} - 2i\sqrt{10}
 \end{aligned}$$

$$\begin{aligned}
 42. \quad \sqrt{-12}(\sqrt{-4} - \sqrt{2}) &= i\sqrt{12}(i\sqrt{4} - \sqrt{2}) \\
 &= 2i\sqrt{3}(2i - \sqrt{2}) \\
 &= 4i^2\sqrt{3} - 2i\sqrt{6} \\
 &= -4\sqrt{3} - 2i\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad (3\sqrt{-5})(-4\sqrt{-12}) &= (3i\sqrt{5})(-8i\sqrt{3}) \\
 &= -24i^2\sqrt{15} \\
 &= 24\sqrt{15}
 \end{aligned}$$

$$\begin{aligned}
 44. \quad (3\sqrt{-7})(2\sqrt{-8}) &= (3i\sqrt{7})(2i\sqrt{8}) = (3i\sqrt{7})(2i\sqrt{4 \cdot 2}) \\
 &= (3i\sqrt{7})(4i\sqrt{2}) = 12i^2\sqrt{14} = -12\sqrt{14}
 \end{aligned}$$

$$\begin{aligned}
 45. \quad x^2 - 6x + 10 &= 0 \\
 x &= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)} \\
 x &= \frac{6 \pm \sqrt{36 - 40}}{2} \\
 x &= \frac{6 \pm \sqrt{-4}}{2} \\
 x &= \frac{6 \pm 2i}{2}
 \end{aligned}$$

$x = 3 \pm i$
The solution set is $\{3 + i, 3 - i\}$.

$$\begin{aligned}
 46. \quad x^2 - 2x + 17 &= 0 \\
 x &= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(17)}}{2(1)} \\
 x &= \frac{2 \pm \sqrt{4 - 68}}{2} \\
 x &= \frac{2 \pm \sqrt{-64}}{2} \\
 x &= \frac{2 \pm 8i}{2} \\
 x &= 1 \pm 4i
 \end{aligned}$$

The solution set is $\{1 + 4i, 1 - 4i\}$.

$$\begin{aligned}
 47. \quad 4x^2 + 8x + 13 &= 0 \\
 x &= \frac{-8 \pm \sqrt{8^2 - 4(4)(13)}}{2(4)} \\
 &= \frac{-8 \pm \sqrt{64 - 208}}{8} \\
 &= \frac{-8 \pm \sqrt{-144}}{8} \\
 &= \frac{-8 \pm 12i}{8} \\
 &= \frac{4(-2 \pm 3i)}{8} \\
 &= \frac{-2 \pm 3i}{2} \\
 &= -1 \pm \frac{3}{2}i
 \end{aligned}$$

The solution set is $\left\{-1 + \frac{3}{2}i, -1 - \frac{3}{2}i\right\}$.

48. $2x^2 + 2x + 3 = 0$

$$\begin{aligned}
 x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(3)}}{2(2)} \\
 &= \frac{-2 \pm \sqrt{4 - 24}}{4} \\
 &= \frac{-2 \pm \sqrt{-20}}{4} \\
 &= \frac{-2 \pm 2i\sqrt{5}}{4} \\
 &= \frac{2(-1 \pm i\sqrt{5})}{4} \\
 &= \frac{-1 \pm i\sqrt{5}}{2} \\
 &= -\frac{1}{2} \pm \frac{\sqrt{5}}{2}i
 \end{aligned}$$

The solution set is $\left\{-\frac{1}{2} + \frac{\sqrt{5}}{2}i, -\frac{1}{2} - \frac{\sqrt{5}}{2}i\right\}$.

49. $3x^2 - 8x + 7 = 0$

$$\begin{aligned}
 x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(7)}}{2(3)} \\
 &= \frac{8 \pm \sqrt{64 - 84}}{6} \\
 &= \frac{8 \pm \sqrt{-20}}{6} \\
 &= \frac{8 \pm 2i\sqrt{5}}{6} \\
 &= \frac{2(4 \pm i\sqrt{5})}{6} \\
 &= \frac{4 \pm i\sqrt{5}}{3} \\
 &= \frac{4}{3} \pm \frac{\sqrt{5}}{3}i
 \end{aligned}$$

The solution set is $\left\{\frac{4}{3} + \frac{\sqrt{5}}{3}i, \frac{4}{3} - \frac{\sqrt{5}}{3}i\right\}$.

50. $3x^2 - 4x + 6 = 0$

$$\begin{aligned}
 x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(6)}}{2(3)} \\
 &= \frac{4 \pm \sqrt{16 - 72}}{6} \\
 &= \frac{4 \pm \sqrt{-56}}{6} \\
 &= \frac{4 \pm 2i\sqrt{14}}{6} \\
 &= \frac{2(2 \pm i\sqrt{14})}{6} \\
 &= \frac{2 \pm i\sqrt{14}}{3} \\
 &= \frac{2}{3} \pm \frac{\sqrt{14}}{3}i
 \end{aligned}$$

The solution set is $\left\{\frac{2}{3} + \frac{\sqrt{14}}{3}i, \frac{2}{3} - \frac{\sqrt{14}}{3}i\right\}$.

$$\begin{aligned}
 51. \quad &(2 - 3i)(1 - i) - (3 - i)(3 + i) \\
 &= (2 - 2i - 3i + 3i^2) - (3^2 - i^2) \\
 &= 2 - 5i + 3i^2 - 9 + i^2 \\
 &= -7 - 5i + 4i^2 \\
 &= -7 - 5i + 4(-1) \\
 &= -11 - 5i
 \end{aligned}$$

$$\begin{aligned}
 52. \quad &(8 + 9i)(2 - i) - (1 - i)(1 + i) \\
 &= (16 - 8i + 18i - 9i^2) - (1^2 - i^2) \\
 &= 16 + 10i - 9i^2 - 1 + i^2 \\
 &= 15 + 10i - 8i^2 \\
 &= 15 + 10i - 8(-1) \\
 &= 23 + 10i
 \end{aligned}$$

$$\begin{aligned}
 53. \quad &(2 + i)^2 - (3 - i)^2 \\
 &= (4 + 4i + i^2) - (9 - 6i + i^2) \\
 &= 4 + 4i + i^2 - 9 + 6i - i^2 \\
 &= -5 + 10i
 \end{aligned}$$