2.1 Classify the uncertainty associated with following items as either aleatory or epistemic and explain your reason for your classification: average wind speed over a 30 day period, location of a certain applied load, change in strength of a soil caused by sampling method, capacity determined by a certain analysis method, magnitude of live load caused by vehicles travelling on a bridge, soil shear strength as measured by a certain method.

Solution

- Uncertainty of the average wind speed is aleatory. This is a random process that we cannot affect.
- Uncertainty of location of an applied load is mostly aleatory. There is a certain accuracy with which a structure can be built and the designer had little or no control over this accuracy. In theory there is some epistemic uncertainty in that could be reduced with better construction techniques, but from a practical standpoint this uncertainty is aleatory.
- Uncertainty in the change in strength of a soil caused by sampling method is an epistemic uncertainty. Improved sampling techniques can reduce this uncertainty.
- Uncertainty in the capacity determined by a certain design method is generally epistemic. With improved analytical tools we can reduce this uncertainty.
- Uncertainty in magnitude of live load caused by vehicles travelling on a bridge is inherently aleatory. This is a random process which we cannot affect.
- The uncertainty in the soil shear strength as measured by a certain method is a combination of epistemic and aleatory uncertainty. The uncertainty caused by the quality of the equipment used and the care of the technician making the measurement is epistemic and can be reduced by the use of more precise equipment and better training of the technician. However, there is aleatory uncertainty in the soil strength inherent in the natural processes that created the soil.

2.2 Figure 2.1 shows the PDF for a normal distribution determined from the unconfined compression tests shown in the histogram. Does the mean and standard deviation of this PDF represent aleatory or epistemic uncertainty? Explain.

Solution

The mean and standard deviation of this PDF contain both aleatory and epistemic uncertainty. The mean of 20.8 and standard deviation of 7.30 are estimate valued of the true mean and standard deviation of the unconfined compressive strength of this sandstone. The epistemic uncertainty is associated with the number of samples used to estimate the parameters. If we had taken more samples, we would have better estimates. However, this particular sample obviously contains a large number of measurements. Therefore the estimated standard deviation is probably very close to the aleatory uncertainty and testing more specimens is unlike to reduce the uncertainty significantly.

2.3 List three sources of epistemic uncertainty associated with determining the soil strength at a given site and describe how you might reduce these uncertainties.

Solution

Source	How do reduce
Small sample size	Take and test more samples
Sloppy laboratory techniques	Improve laboratory methods
Old or poor quality testing equipment	Acquire improved testing equipment
Disturbance of soil samples before or during testing	Use better sampling and testing methods
Mixing up results from different samples	Improve documentation methods to eliminate mixing up samples

2.4 Using a random number generator create a sample of 4 relative densities using the PDF presented in Figure 2.2. Repeat the exercise to create 3 different sample sets. Compute the mean and standard deviation of your sample. Compute the mean and standard deviation of each sample set. Compare the means and standard deviations of your samples with each other and with the mean and standard deviation of the original distribution. Discuss the differences among the sample sets and the original distribution, including the type of uncertainties you are dealing with. How many samples do you think are needed to reliably determine the mean and standard deviation of the relative density of this particular soil?

Solution

There are an infinite number of solutions to this problem. The table below shows Excel spreadsheet formula that can be used to generate the random sample sets.

	A	B	C	D
1	μ	σ	N	Z
2	94.9	5.7	=NORM.S.INV(RAND())	=\$A\$2+\$B\$2*C2
3			=NORM.S.INV(RAND())	=\$A\$2+\$B\$2*C3
4			=NORM.S.INV(RAND())	=\$A\$2+\$B\$2*C4
5			=NORM.S.INV(RAND())	=\$A\$2+\$B\$2*C5
6			μ	=AVERAGE(D2:D5)
			σ	=STDEV.S(D2:D5)

The table below shows three sample sets generated with the Excel spreadsheet shown above. Note that the average of the samples ranges from 6.5 below the distribution mean to 6.8 above it. Also one estimate of the standard deviation is nearly twice that of the original distribution. It is possible, using sampling theory, to determine the number of sample required to have a certain confidence level in the estimated parameters. However, this is well beyond the scope of this text. Students should note that increasing the sample size to 3 to 7, significantly reduces the variability of the estimated mean and standard deviation.

Sample #	Trial 1	Trial 2	Trial 3
1	107.34	92.44	95.79
2	98.75	78.47	83.10
3	101.50	100.55	83.95
4	99.02	102.07	90.80
Sample mean	101.65	93.38	88.41
Sample Standard Deviation	3.99	10.80	6.01

2.5 A certain column will carry a dead load estimated to be 400 k with a COV of 0.1 and a live load of 200 k with a COV of 0.25. What is the mean and standard deviation of the total column load? What is the probability that this load will exceed 750 k?

Solution

First we must compute the standard deviation of each random variable from their mean and COV using Equation 2.10.

$$\sigma_D = 0.1(400) = 40$$
$$\sigma_L = 0.25(200) = 50$$

Then we compute the mean and standard deviation of the total column load using Equations 2.17 and 2.18

$$\mu_{Total} = 400 + 200 = 600$$
$$\sigma_{Total} = \sqrt{40^2 + 50^2} = 64$$

Then using Equation 2.15 we compute the probability that the load exceeds 750 and 1 minus the probability that it is less than 750

$$P\left(\text{Load} > 750\right) = 1 - \Phi\left(\frac{750 - 600}{64}\right) = 1 - \Phi\left(2.34\right) = 9.5 \times 10^{-3}$$

2.6 A simply supported beam has a length of 3 m and carries a distributed load with a mean of 5 kN/m and a COV of 0.2. What is the mean and standard deviation of the maximum moment in the beam? What is the probability the maximum moment will exceed 7 kN-m?

Solution

The equation for the maximum moment in a simply supported beam subject to a distributed load is

$$M_{\max} = \frac{wl^2}{8} = w\frac{3^2}{8} = 1.125w$$

Using Equations 2.10, 2.17 and 2.18 the mean and standard deviation of $M_{\max}$ is

$$\mu_{M_{\max}} = 1.125w = 5.62$$
$$\sigma_{M_{\max}} = 1.125\sigma_{M_{\max}} = 1.125\left(\mathrm{COV}_w \cdot \mu_w\right) = 1.125\left(0.2 \cdot 5\right) = 1.125$$

Then using Equation 2.15 we compute the probability that the load exceeds 750 and 1 minus the probability that it is less than 750

$$P\left(M_{\max} > 7\right) = 1 - \Phi\left(\frac{7 - 5.62}{1.125}\right) = 1 - \Phi\left(1.23\right) = 0.11$$

2.7 Using the data shown in Figure 2.5, determine the probability that tangent of the friction angle for the mudstone at the Confederation Bridge site is less than 0.25.

Solution

The data in Figure 2.5 is lognormally distributed with μ = -1.09 and σ = 0.270. Using Equation 2.16

$$P\left(\tan\phi < 0.25\right) = \Phi\left(\frac{\ln 0.25-(-1.09)}{0.270}\right) = \Phi\left(-1.097\right) = 0.136$$

Or there is a 13.6% chance that $\tan\phi$ will be less than 0.25.

2.8 The capacity for a certain foundation system is estimated to be 620 kN with a COV of 0.3. The demand on the foundation is estimated to be 150 kN with a COV of 0.15. Compute the mean factor of safety of this foundation and its probability of failure.

Solution

The mean factor of safety is

$$\mu_F = \frac{620}{150} = 4.1$$

The standard deviation of demand and capacity are computed using Equation 2.10

$$\sigma_D = 0.15(150) = 22.5$$
$$\sigma_C = 0.3(620) = 186$$

The mean and standard deviation of the safety margin, *m*, are computed using Equations 2.17 and 2.18

$$\mu_m = 620 - 150 = 470$$

$$\sigma_m = \sqrt{186^2 + 22.5^2} = 187$$

And the probability that $m < 0$ is computed using Equation 2.15

$$P(m \le 0) = \Phi\left(\frac{0-470}{187}\right) = \Phi(-2.51) = 6\times10^{-3}$$

2.9 We wish to design a shallow foundation with a probability of failure of 10^{-3}. The footing supports a column carrying a dead load with a mean of 30 k and COV of 0.05 and a live load with a mean of 10 k and COV of 0.15. Based on the uncertainty of soil properties and our analysis method, we estimate the COV of the foundation capacity to be 0.2. For what mean capacity does the foundation need to be designed?

Solution

We want to select a value of μ_C such that $P_f = 10^{-3}$ or $\Phi\left(\frac{\mu_C - \mu_m}{\sigma_m}\right) = 10^{-3}$. From Equations 2.10, 2.17, and 2.18

$$\mu_m = \mu_C - \mu_D - \mu_L$$

$$\sigma_C = \sqrt{\sigma_C^2 + \sigma_D^2 + \sigma_L^2} = \sqrt{(\mathrm{COV}_C\mu_C)^2 + (\mathrm{COV}_D\mu_D)^2 + (\mathrm{COV}_D\mu_L)^2}$$

And

$$\Phi\left(\frac{\mu_C - (\mu_D + \mu_L)}{\sqrt{(\mathrm{COV}_C\mu_C)^2 + (\mathrm{COV}_D\mu_D)^2 + (\mathrm{COV}_D\mu_L)^2}}\right) = 10^{-3}$$

Substituting know values of for the COVs and means

$$\Phi\left(\frac{\mu_C - (30 + 10)}{\sqrt{(0.2\mu_C)^2 + (0.05 \cdot 30)^2 + (0.15 \cdot 10)^2}}\right) = 10^{-3}$$

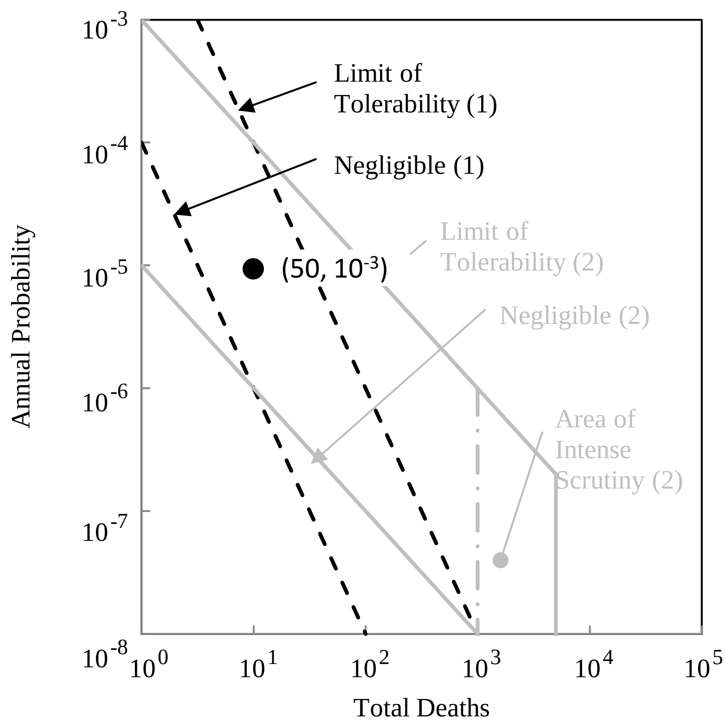
Solving this equation iteratively using Excel we get

μ_C = 106 k

2.10 Assume the foundation in Problem 2.9 was to support a high voltage transmission line near the Danish city of Århus. If the transmission line fails it will potentially kill 50 people. If the computed probability of failure is for a design life of 100 years, is risk associated with the failure of design acceptable based on the Danish guidance in Figure 2.8? Explain.

Solution

The probability of failure in Problem 2.9 was set to 10^{-3}. If this is the total probability of failure over 100 years, then the annual probability of failure is approximately $10^{-3}/100 = 10^{-5}$. The point with 50 deaths and an probability of 10^{-5} is plotted on Figure 2.8 below. This point lies between the negligible line and limit of tolerability for the Danish code. In this zone the project must include mitigations to make the risk "as low as reasonably practicable" or the probability of failure must be reduced to an annual probability of 10^{-6}.



2.11 For the footing in Example 2.2, compute factor of safety required for a probability of failure of 5×10^{-4} assuming the COV of the demand is 0.15

Solution

From Example 2.2 we know that the mean capacity is 11,910 lb/ft^2 with a standard deviation of 2,280 lb/ft^2. The question is what is the greatest mean demand that will give us a probability of failure of 5×10^{-4}. To compute this we must compute the mean and standard deviation of the safety margin, *m*, as a function of the mean and standard deviation of the demand, *D*.

$$\mu_m = \mu_C - \mu_D = 11{,}910 - \mu_D$$
$$\sigma_m = \sqrt{\sigma_C^2 + \sigma_D^2} = \sqrt{2{,}280 + \mathrm{COV}_D \mu_D}\sqrt{2{,}280 + 0.15\mu_D}$$

And

$$5\times10^{-4} = \Phi\left(\frac{\mu_D - \mu_m}{\sigma_m}\right) = \Phi\left(\frac{\mu_D - (11{,}910 - \mu_D)}{\sqrt{2{,}280 + 0.15\mu_D}}\right)$$
$$5\times10^{-4} = \Phi\left(\frac{2\mu_D - 11{,}910}{\sqrt{2{,}280 + 0.15\mu_D}}\right)$$

Solving the above equation iteratively using Excel, we compute

$$\mu_D = 4135$$

And the mean factor of safety, *F*, is then

$$F = \frac{11{,}910}{4{,}135} = 2.9$$

2.12 If the ASD design method has work satisfactorily for over 50 years, what's the value in changing to LRFD method?

Solution

There are two major advantages to LRFD when compared to ASD. First, since LRFD uses multiple partial factors of safety, it is more flexible and produces designs with more consistent probabilities of failure for different load combinations, and different material property variability. Second, the partial safety factors in LRFD are selected based on an optimization process that uses probability theory explicitly include the variability of the loads, material properties, and analysis methods.