

1/10

$$\begin{aligned}
 y(x) &= \frac{r}{h} x \\
 y(x+\Delta x) &= \frac{r}{h} (x+\Delta x) \\
 \Delta V &= V(x+\Delta x) - V(x) \\
 &= \frac{1}{3}\pi [y(x+\Delta x)]^2 [x+\Delta x] \\
 &\quad - \frac{1}{3}\pi [y(x)]^2 [x] \\
 &= \frac{1}{3}\pi \left[\frac{r}{h}(x+\Delta x) \right]^2 [x+\Delta x] - \frac{1}{3}\pi \left[\frac{r}{h}x \right]^2 [x] \\
 &= \frac{1}{3}\pi \frac{r^2}{h^2} \left[(x+\Delta x)^3 - x^3 \right] \\
 &= \frac{1}{3}\pi \frac{r^2}{h^2} \left[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3 \right] \\
 &= \frac{\pi r^2}{h^2} \left[x^2\Delta x + x(\Delta x)^2 + \frac{1}{3}(\Delta x)^3 \right]
 \end{aligned}$$

In the limit as $\Delta x \rightarrow dx$, the higher-order terms drop out.