Chapter One: Management Science

PROBLEM SUMMARY

1. Total cost, revenue, profit, and break-even
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4. Break-even volume
5. Graphical analysis (1−2)
6. Graphical analysis (1−4)
7. Break-even sales volume
8. Break-even volume as a percentage of capacity (1−2)
9. Break-even volume as a percentage of capacity (1−3)
10. Break-even volume as a percentage of capacity (1−4)
11. Effect of price change (1−2)
12. Effect of price change (1−4)
13. Effect of variable cost change (1−12)
14. Effect of fixed cost change (1−13)
15. Break-even analysis
16. Effect of fixed cost change (1−7)
17. Effect of variable cost change (1−7)
18. Break-even analysis
19. Break-even analysis
20. Break-even analysis
21. Break-even analysis; volume and price analysis
22. Break-even analysis; profit analysis
23. Break-even analysis
24. Break-even analysis; profit analysis
25. Break-even analysis; price and volume analysis
26. Break-even analysis; profit analysis
27. Break-even analysis; profit analysis
28. Break-even analysis; profit analysis
29. Linear programming
30. Linear programming
31. Linear programming
32. Linear programming
33. Forecasting/statistics
34. Linear programming
35. Waiting lines
36. Shortest route

PROBLEM SOLUTIONS

1. a) $v = 300,\ c_f = \$8{,}000,$

$c_v = \$65$ per table, $p = \$180;$

$\text{TC} = c_f + vc_v = \$8{,}000 + (300)(65) = \$27{,}500;$

$\text{TR} = vp = (300)(180) = \$54{,}000;$

$Z = \$54{,}000 - 27{,}500 = \$26{,}500$ per month

b) $v = \dfrac{c_f}{p - c_v} = \dfrac{8{,}000}{180 - 65} = 69.56$ tables per month

2. a) $v = 12{,}000,\ c_f = \$60{,}000,\ c_v = \$9,$

$p = \$25;\ \text{TC} = c_f + vc_v$

$= 60{,}000 + (12{,}000)(9)$

$= \$168{,}000;$

$\text{TR} = vp = (12{,}000)(\$25) = \$300{,}000;$

$Z = \$300{,}000 - 168{,}000 = \$132{,}000$ per year

b) $v = \dfrac{c_f}{p - c_v} = \dfrac{60{,}000}{25 - 9} = 3{,}750$ tires per year

3. a) $v = 18{,}000,\ c_f = \$21{,}000,\ c_v = \$.45,$

$p = \$1.30;$

$\text{TC} = c_f + vc_v = \$21{,}000 + (18{,}000)(.45) = \$29{,}100;$

$\text{TR} = vp = (18{,}000)(1.30) = \$23{,}400;$

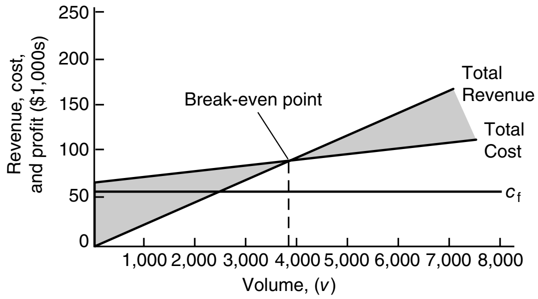
$Z = \$23{,}400 - 29{,}100 = -\$5{,}700$ (loss)

b) $v = \dfrac{c_f}{p - c_v} = \dfrac{21{,}000}{1.30 - .45} = 24{,}705.88$ yd per month

4. $c_f = \$25{,}000,\ p = \$.40,\ c_v = \$.15,$

$v = \dfrac{c_f}{p - c_v} = \dfrac{25{,}000}{.40 - .15} = 100{,}000$ lb per month

5.



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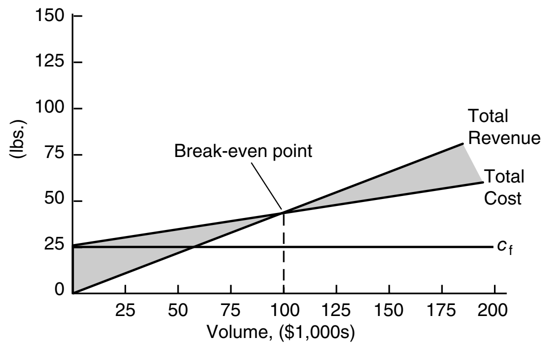


7. $v = \dfrac{c_f}{p - c_v} = \dfrac{\$25{,}000}{30 - 10} = 1{,}250$ dolls

8. Break-even volume as percentage of capacity

$= \dfrac{v}{k} = \dfrac{3{,}750}{8{,}000} = .469 = 46.9\%$

9. Break-even volume as percentage of capacity

$= \dfrac{v}{k} = \dfrac{24{,}750.88}{25{,}000} = .988 = 98.8\%$

10. Break-even volume as percentage of

capacity $= \dfrac{v}{k} = \dfrac{100{,}000}{120{,}000} = .833 = 83.3\%$

11. $v = \dfrac{c_f}{p - c_v} = \dfrac{60{,}000}{31 - 9} = 2{,}727.3$ tires

per year; it reduces the break-even volume from 3,750 tires to 2,727.3 tires per year.

12. $v = \dfrac{c_f}{p - c_v} = \dfrac{25{,}000}{.60 - .15} = 55{,}555.55$ lb

per month; it reduces the break-even volume from 100,000 lb per month to 55,555.55 lb.

13. $v = \dfrac{c_f}{p - c_v} = \dfrac{25{,}000}{.60 - .22} = 65{,}789.47$ lb

per month; it increases the break-even volume from 55,555.55 lb per month to 65,789.47 lb per month.

14. $v = \dfrac{c_f}{p - c_v} = \dfrac{39{,}000}{.60 - .22} = 102{,}613.57$ lb

per month; it increases the break-even volume from 65,789.47 lb per month to 102,631.57 lb per month.

15. Initial profit: $Z = vp - c_f - vc_v = (9{,}000)(.75) - 4{,}000 - (9{,}000)(.21) = 6{,}750 - 4{,}000 - 1{,}890 =$ \$860 per month; increase in price: $Z = vp - c_f - vc_v = (5{,}700)(.95) - 4{,}000 - (5{,}700)(.21) = 5{,}415 - 4{,}000 - 1{,}197 =$ \$218 per month; the dairy should not raise its price.

16. $v = \dfrac{c_f}{p - c_v} = \dfrac{35{,}000}{30 – 10} = 1{,}750$

The increase in fixed cost from \$25,000 to \$35,000 will increase the break-even point from 1,250 to 1,750 or 500 dolls; thus, he should not spend the extra \$10,000 for advertising.

17. Original break-even point (from problem 7) = 1,250

New break-even point:

$v = \dfrac{c_f}{p - c_v} = \dfrac{17{,}000}{30 - 14} = 1{,}062.5$

Reduces BE point by 187.5 dolls.

18. a) $v = \dfrac{c_f}{p - c_v} = \dfrac{\$27{,}000}{8.95 - 3.75} = 5{,}192.30$ pizzas

b) $\dfrac{5{,}192.3}{20} = 259.6$ days

c) Revenue for the first 30 days $= 30(pv - vc_v)$

$= 30[(8.95)(20) - (20)(3.75)]$

$= \$3{,}120$

\$27,000 − 3,120 = \$23,880, portion of fixed cost not recouped after 30 days.

New $v = \dfrac{c_f}{p - c_v} = \dfrac{\$23{,}880}{7.95 - 3.75} = 5{,}685.7$ pizzas

Total break-even volume = 600 + 5,685.7 = 6,285.7 pizzas

$$\text{Total time to break-even} = 30 + \frac{5{,}685.7}{20}$$
$$= 314.3 \text{ days}$$

19. a) Cost of Regular plan = \$55 + (.33)(260 minutes)
= \$140.80

Cost of Executive plan = \$100 + (.25)(60 minutes)
= \$115

Select Executive plan.

b) $55 + (x - 1{,}000)(.33) = 100 + (x - 1{,}200)(.25)$

$-275 + .33x = .25x - 200$

$x = 937.50$ minutes per month or 15.63 hrs.

20. a) $$14{,}000 = \frac{7{,}500}{p - .35}$$

p = \$0.89 to break even

b) If the team did not perform as well as expected the crowds could be smaller; bad weather could reduce crowds and/or affect what fans eat at the game; the price she charges could affect demand.

c) This will be a subjective answer, but \$1.25 seems to be a reasonable price.

$Z = vp - c_f - vc_v$

$Z = (14{,}000)(1.25) - 7{,}500 - (14{,}000)(0.35)$

$= 17{,}500 - 12{,}400$

= \$5,100

21. a) c_f = \$1,700

c_v = \$12 per pupil

p = \$75

$$v = \frac{1{,}700}{75 - 12}$$

= 26.98 or 27 pupils

b) $Z = vp - c_f - vc_v$

$\$5{,}000 = v(75) - \$1{,}700 - v(12)$

$63v = 6{,}700$

$v = 106.3$ pupils

c) $Z = vp - c_f - vc_v$

$\$5{,}000 = 60p - \$1{,}700 - 60(12)$

$60p = 7{,}420$

$p = \$123.67$

22. a) c_f = \$350,000

c_v = \$12,000

p = \$18,000

$$v = \frac{c_f}{p - c_v}$$

$$= \frac{350{,}000}{18{,}000 - 12{,}000}$$

= 58.33 or 59 students

b) $Z = (75)(18{,}000) - 350{,}000 - (75)(12{,}000)$

= \$100,000

c) $Z = (35)(25{,}000) - 350{,}000 - (35)(12{,}000)$

= 105,000

This is approximately the same as the profit for 75 students and a lower tuition in part (b).

23. p = \$400

c_f = \$8,000

c_v = \$75

Z = \$60,000

$$v = \frac{Z + c_f}{p - c_v}$$

$$v = \frac{60{,}000 + 8{,}000}{400 - 75}$$

$v = 209.23$ teams

24. Fixed cost (c_f) = 875,000

Variable cost (c_v) = \$200

Price (p) = (225)(12) = \$2,700

$v = c_f/(p - c_v) = 875{,}000/(2{,}700 - 200)$

$= 350$

With volume doubled to 700:

Profit (Z) = (2,700)(700) – 875,000 – (700)(200)

= \$875,000

25. Fixed cost (c_f) = 100,000

Variable cost (c_v) = \$(.50)(.35) + (.35)(.50) + (.15)(2.30)

= \$0.695

Price (p) = \$6

Profit (Z) = (6)(45,000) – 100,000 – (45,000)(0.695)

= \$138,725

This is not the financial profit goal of \$150,000.

The price to achieve the goal of \$150,000 is,

$p = (Z + c_f + vc_v)/v$

= (150,000 + 100,000 + (45,000)(.695))/45,000

= \$6.25

The volume to achieve the goal of $150,000 is,

$v = (Z + c_f)/(p - c_v)$

$= (150{,}000 + 100{,}000)/(6 - .695)$

$= 47{,}125$

26. a) Monthly fixed cost (c_f) = cost of van/60 months + labor (driver)/month

= (21,500/60) + (30.42 days/month)($8/hr)(5 hr/day)

= 358.33 + 1,216.80

= $1,575.13

Variable cost (c_v) = $1.35 + 15.00

= $16.35

Price (p) = $34

$v = c_f/(p - v_c)$

$= (1{,}575.13)/(34 - 16.35)$

$v = 89.24$ orders/month

b) 89.24/30.42 = 2.93 orders/day − Monday through Thursday

Double for weekend = 5.86 orders/day − Friday through Sunday

Orders per month = approximately (18 days)(2.93 orders) + (12.4 days)(5.86 orders)

= 125.4 delivery orders per month

Profit = total revenue − total cost

$= vp - (c_f + vc_v)$

$= (125.4)(34) - 1{,}575.13 - (125.4)(16.35)$

$= 638.18$

27. a) $v = \dfrac{c_f}{p - c_v} = \dfrac{500}{30 - 14}$

$v = 31.25$ jobs

b) (8 weeks)(6 days/week)(3 lawns/day) = 144 lawns

$Z = (144)(30) - 500 - (144)(14)$

$Z = \$1{,}804$

c) (8 weeks)(6 days/week)(4 lawns/day) = 192 lawns

$Z = (192)(25) - 500 - (192)(14)$

$Z = \$1{,}612$

No, she would make less money than (b)

28. a) $v = \dfrac{c_f}{p - c_v} = \dfrac{700}{35 - 3}$

$v = 21.88$ jobs

b) (6 snows)(2 days/snow)(10 jobs/day) = 120 jobs

$Z = (120)(35) - 700 - (120)(3)$

$Z = \$3{,}140$

c) (6 snows)(2 days/snow)(4 jobs/day) = 48 jobs

$Z = (48)(150) - 1800 - (48)(28)$

$Z = \$4{,}056$

Yes, better than (b)

d) $Z = (120)(35) - 700 - (120)(18)$

$Z = \$1{,}340$

Yes, still a profit with one more person

29. There are two possible answers, or solution points:

$x = 25$, $y = 0$ or $x = 0$, $y = 50$

Substituting these values in the objective function:

$Z = 15(25) + 10(0) = 375$

$Z = 15(0) + 10(50) = 500$

Thus, the solution is $x = 0$ and $y = 50$

This is a simple linear programming model, the subject of the next several chapters. The student should recognize that there are only two possible solutions, which are the corner points of the feasible solution space, only one of which is optimal.

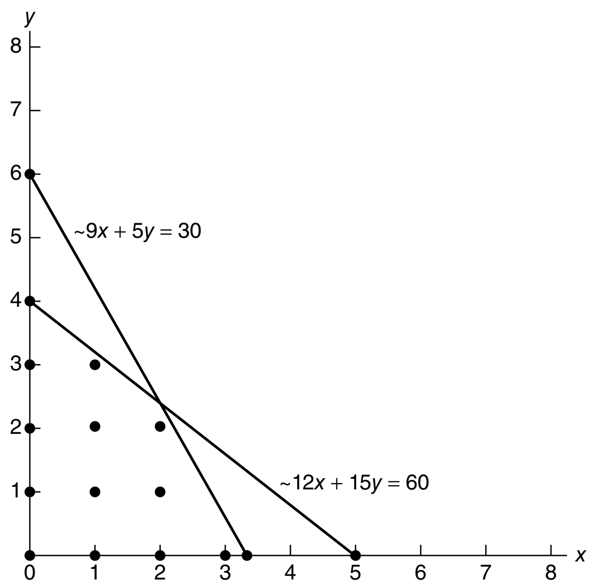
30. The solution is computed by solving simultaneous equations,

$x = 30$, $y = 10$, $Z = \$1{,}400$

It is the only, i.e., “optimal” solution because there is only one set of values for x and y that satisfy both constraints simultaneously.

31.

# bowls	# mugs	Labor usage $12x + 15y <= 60$	Clay usage $9x+5y <= 30$	Profit $300x + 250y$	Possible solution?
0	1	15	5	250	yes
1	0	12	9	300	yes
1	1	27	14	550	yes
0	2	30	10	500	yes
2	0	24	18	600	yes
1	2	42	19	800	yes
2	1	39	23	850	yes
2	2	54	28	1100	yes, best solution
0	3	45	15	750	yes
3	0	36	27	900	yes
1	3	57	24	1050	yes
3	1	51	32	1150	no
2	3	69	33	1350	no
3	2	66	37	1400	no
3	3	81	42	1650	no
4	0	48	36	1200	no
0	4	60	20	1000	yes
1	4	72	29	1300	no
4	1	63	41	1450	no
2	4	84	38	1600	no
4	2	78	46	1700	no



32. Maximize $Z = \$30x_{AN} + 70x_{AJ} + 40x_{BN} + 60x_{BJ}$
subject to

$$x_{AN} + x_{AJ} = 400$$
$$x_{BN} + x_{BJ} = 400$$
$$x_{AN} + x_{BN} = 500$$
$$x_{AJ} + x_{BJ} = 300$$

The solution is $x_{AN} = 400$, $x_{BN} = 100$, $x_{BJ} = 300$, and $Z = 34{,}000$

This problem can be solved by allocating as much as possible to the lowest cost variable, $x_{AN} = 400$, then repeating this step until all the demand has been met. This is a similar logic to the minimum cell cost method.

33. This is virtually a straight linear relationship between time and site visits; thus, a simple linear graph would result in a forecast of approximately 34,500 site visits.

34. Determine logical solutions:

	Cakes	Bread	Total Sales
1.	0	2	$12
2.	1	2	$22
3.	3	1	$36
4.	4	0	$40

Each solution must be checked to see if it violates the constraints for baking time and flour. Some possible solutions can be logically discarded because they are obviously inferior. For example, 0 cakes and 1 loaf of bread is clearly inferior to 0 cakes and 2 loaves of bread. 0 cakes and 3 loaves of bread is not possible because there is not enough flour for 3 loaves of bread.

Using this logic, there are four possible solutions as shown. The best one, 4 cakes and 0 loaves of bread, results in the highest total sales of $40.

35. This problem demonstrates the cost trade-off inherent in queuing analysis, the topic of Chapter 13. In this problem the cost of service, i.e., the cost of staffing registers, is added to the cost of customers waiting, i.e., the cost of lost sales and ill will, as shown in the following table.

Registers staffed	1	2	3	4	5	6	7	8
Waiting time (mins)	20	14	9	4	1.7	1	0.5	0.1
Cost of service ($)	60	120	180	240	300	360	420	480
Cost of waiting ($)	850	550	300	50	0	0	0	0
Total cost ($)	910	670	480	290	300	360	420	480

The total minimum cost of $290 occurs with 4 registers staffed

36. The shortest route problem is one of the topics of Chapter 7. At this point, the most logical "trial and error" way that most students will probably approach this problem is to identify all the feasible routes and compute the total distance for each, as follows:

1-2-6-9 = 228
1-2-5-9 = 213
1-3-5-9 = 211
1-3-8-9 = 276
1-4-7-8-9 = 275

Obviously inferior routes like 1-3-4-7-8-9 and 1-2-5-8-9 that include additional segments to the routes listed above can be logically eliminated from consideration. As a result, the route 1-3-5-9 is the shortest.

An additional aspect to this problem could be to have the students look at these routes on a real map and indicate which they think might "practically" be the best route. In this case, 1-2-5-9 would likely be a better route, because even though it's two miles farther it is Interstate highway the whole way, whereas 1-3-5-9 encompasses U.S. 4-lane highways and state roads.

CASE SOLUTION: CLEAN CLOTHES CORNER LAUNDRY

a) $v = \dfrac{c_f}{p - c_v} = \dfrac{1{,}700}{1.10 - .25} = 2{,}000$ items per month

b) Solution depends on number of months; 36 used here. \$16,200 ÷ 36 = \$450 per month, thus monthly fixed cost is \$2,150

$$v = \frac{c_f}{p - c_v} = \frac{2{,}150}{1.10 - .25} = 2{,}529.4 \text{ items per month}$$

529.4 additional items per month

c) $Z = vp - c_f - vc_v$

$= 4{,}300(1.10) - 2{,}150 - 4{,}300(.25)$

$= \$1{,}505$ per month

After 3 years, $Z = \$1{,}955$ per month

d) $v = \dfrac{c_f}{p - c_v} = \dfrac{1{,}700}{.99 - .25} = 2{,}297.3$

$Z = vp - c_f - vc_v$

$= 3{,}800(.99) - 1{,}700 - 3{,}800(.25)$

$= \$1{,}112$ per month

e) With both options:

$Z = vp - c_f - vc_v$

$= 4{,}700(.99) - 2{,}150 - 4{,}700(.25)$

$= \$1{,}328$

She should purchase the new equipment but not decrease prices.

CASE SOLUTION: OCOBEE RIVER RAFTING COMPANY

Alternative 1: $c_f = \$3{,}000$

$p = \$20$

$c_v = \$12$

$$v_1 = \frac{c_f}{p - c_v} = \frac{3{,}000}{20 - 12} = 375 \text{ rafts}$$

Alternative 2: $c_f = \$10{,}000$

$p = \$20$

$c_v = \$8$

$$v_2 = \frac{c_f}{p - c_v} = \frac{10{,}000}{20 - 8} = 833.37$$

If demand is less than 375 rafts, the students should not start the business.

If demand is less than 833 rafts, alternative 2 should not be selected, and alternative 1 should be used if demand is expected to be between 375 and 833.33 rafts.

If demand is greater than 833.33 rafts, which alternative is best? To determine the answer, equate the two cost functions.

$$3{,}000 + 12v = 10{,}000 + 8v$$
$$4v = 7{,}000$$
$$v = 1{,}750$$

This is referred to as the point of indifference between the two alternatives. In general, for demand lower than this point (1,750) the alternative should be selected with the lowest fixed cost; for demand greater than this point the alternative with the lowest variable cost should be selected. (This general relationship can be observed by graphing the two cost equations and seeing where they intersect.)

Thus, for the Ocobee River Rafting Company, the following guidelines should be used:

demand < 375, do not start business; 375 < demand < 1,750, select alternative 1; demand > 1,750, select alternative 2

Since Penny estimates demand will be approximately 1,000 rafts, alternative 1 should be selected.

$Z = v_p - c_f - vc_v$

$= (1{,}000)(20) - 3{,}000 - (1{,}000)(12)$

$Z = \$5{,}000$

CASE SOLUTION: CONSTRUCTING A DOWNTOWN PARKING LOT IN DRAPER

a) The annual capital recovery payment for a capital expenditure of \$4.5 million over 30 years at 8% is,

$(4{,}500{,}000)[0.08(1 + .08)^{30}] / (1 + .08)^{30} - 1$

$= \$399{,}723.45$

This is part of the annual fixed cost. The other part of the fixed cost is the employee annual salaries of \$140,000. Thus, total fixed costs are,

\$399,723.45 + 140,000 = \$539,723.45

$$v = \frac{c_f}{p - c_v}$$
$$= \frac{539{,}723.45}{3.20 - 0.60}$$
$$= 207{,}585.94 \text{ parked cars per year}$$

b) If 365 days per year are used, then the daily usage is,

$$\frac{207{,}585.94}{365} = 568.7 \text{ or approximately 569 cars per day}$$

This seems like a reachable goal given the size of the town and the student population.