PROBLEM 3-1

GIVEN: An equimolar mixture of O_2 and N_2 at $T = 400K$, $P = 1atm$

FIND: The mixture density and molar concentration

ASSUMPTIONS: Ideal gas behavior

APPROACH: Determine the mixture MW and then use the ideal gas equation of state to determine the mixture density and molar concentration

mixture molecular weight: $MW_{mix} = X_{O_2} MW_{O_2} + X_{N_2} MW_{N_2}$

$$X_{O_2} = \frac{N_{O_2}}{N_{TOT}} = \frac{1}{2} \quad , \quad X_{N_2} = 1 - X_{O_2} = \frac{1}{2}$$

$$MW_{mix} = 0.5(32) + 0.5(28.013) = 30.007 \; kg/kmole$$

mixture density: $PV = mRT \longrightarrow \rho = \frac{m}{V} = \frac{P}{RT}$

$$R = \bar{R}/MW = \frac{8.315}{30.007} = 0.277$$

$$\boxed{\rho = \frac{101.325 \; kPa}{(0.277)(400K)} = 0.914 \; kg/m^3}$$

molar concentration: $PV = N\bar{R}T \longrightarrow C = \frac{N}{V} = \frac{P}{\bar{R}T}$

$$\boxed{C = \frac{101.325 \; kPa}{(8.315)(400)} = 0.0305 \; \frac{kmole}{m^3}}$$

COMMENTS: The molar concentration could also have been determined by dividing the mixture density by the MW_{mix}:

$$C = \frac{\rho}{MW} = \frac{0.914}{30.007} = 0.0305 \; \frac{KMOLE}{m^3}$$

GIVEN: An equimolar mixture of O_2 and N_2 at $T = 400 K$ and $P = 1\ atm$

FIND: The O_2 and N_2 mass fractions

ASSUMPTIONS: Ideal gas behavior

APPROACH: Calculate the mixture MW, X_{O_2} and X_{N_2} and then relate X_i to Y_i

mixture MW: $MW_{mix} = X_{O_2}(MW_{O_2}) + X_{N_2}(MW_{N_2})$

$$X_{O_2} = \frac{N_{O_2}}{N_{TOT}} = \frac{1}{2} \quad , \quad X_{N_2} = 1 - X_{O_2} = \frac{1}{2}$$

$$MW_{mix} = 0.5(32) + 0.5(28.013) = 30.007$$

O_2 and N_2 mass fractions:

$$Y_{O_2} = X_{O_2} \frac{MW_{O_2}}{MW_{mix}} \qquad\qquad Y_{N_2} = X_{N_2} \frac{MW_{N_2}}{MW_{mix}}$$

$$Y_{O_2} = (0.5)\frac{32}{30.007} \qquad\qquad Y_{N_2} = 0.5\frac{32}{30.007}$$

$$Y_{O_2} = 0.533 \qquad\qquad Y_{N_2} = 0.467$$

$$\Sigma Y_i = 1.0$$

COMMENTS: Note that $\Sigma Y_i = 1.0$ as expected and that mixture X_i and Y_i are independent of T and P for a fixed composition.

GIVEN: n-octane in air at $T = 400 K$ and $P = 3.5$ atm

FIND: a) The binary diffusivity, $\mathcal{D}$, utilizing the value given in Appendix D.
   b) Compare the change in diffusivity in going from the reference conditions (Appendix D) to $T = 400 K$, $P = 3.5$ atm, to the change in $\rho\mathcal{D}$

ASSUMPTIONS: Ideal gas behavior, binary diffusion

APPROACH: Using Eqn. 3.18a and the ideal gas E.O.S. we get $\mathcal{D} \propto T^{3/2} P^{-1}$. Given $\mathcal{D}$ at the reference conditions $(T = 273 K, P = 1 atm)$ and T and P at the current conditions we can find $\mathcal{D}$. Similarly, we find that $\rho\mathcal{D} \propto T^{1/2}$ using the ideal gas E.O.S.

a) Diffusivity:
$$\mathcal{D} = \mathcal{D}_{ref} \left(\frac{T}{T_{ref}}\right)^{3/2} \left(\frac{P}{P_{ref}}\right)^{-1}$$

$$\mathcal{D} = (0.505 \times 10^{-5}\ m^2/s) \left(\frac{400}{273}\right)^{3/2} \left(\frac{3.5\ atm}{1\ atm}\right)^{-1} = 2.559 \times 10^{-6}$$

b) using above relationship
$$\frac{\mathcal{D}}{\mathcal{D}_{ref}} = \left(\frac{T}{T_{ref}}\right)^{3/2} \left(\frac{P}{P_{ref}}\right)^{-1} = 0.507$$

and
$$\frac{\rho\mathcal{D}}{(\rho\mathcal{D})_{ref}} = \left(\frac{T}{T_{ref}}\right)^{1/2} = 1.21$$

COMMENTS: Note that the quantity $\rho\mathcal{D}$ changes much less than the diffusivity, $\mathcal{D}$, and that the change in $\mathcal{D}$ may have been even more drastic if the pressure were not greater than the reference pressure. For constant $P = 1$ atm:
$$\frac{\mathcal{D}}{\mathcal{D}_{ref}} = \left(\frac{T}{T_{ref}}\right)^{3/2} = \left(\frac{400}{273}\right)^{1/2} = 1.77$$

Increases in T and P affect the diffusivity in opposite manners.

GIVEN: Equation 3.18a, which was derived assuming that A and B are approximately the same size and mass, and the following relations which permit Eqn 3.18a to be generalized to the case where $M_a \neq M_b$ and $\sigma_A \neq \sigma_B$:

$$m = \frac{m_A m_B}{\frac{1}{2}(m_A + m_B)} \qquad\qquad \sigma = \frac{1}{2}(\sigma_A + \sigma_B)$$

FIND: The binary diffusivity of O_2 in N_2 at 273K for $\sigma_{O_2} = 3.467$ Å and $\sigma_{N_2} = 3.798$ Å. Does this estimate agree with the published value of $\mathcal{D} = 1.8 \times 10^{-5}$ m²/s. Is good agreement expected? Why or why not?

ASSUMPTIONS: Molecules are rigid and non-attracting, ideal gas behavior, kinetic theory holds

APPROACH: Knowing the MW of O_2 and N_2, determine m_A and m_B using Avagadro's number. Calculate m and σ and substitute into 3.18a.

mass of O_2 and N_2 molecules

O_2: $m_{O_2} = \frac{MW_{O_2}}{N} = \frac{31.999 \text{ Kg/Kmol}}{6.022 \times 10^{26} \text{ molec/Kmol}} = 53.137 \times 10^{-27}$ Kg/molec

N_2: $m_{N_2} = \frac{MW_{N_2}}{N} = \frac{28.013}{6.022 \times 10^{26}} = 49.520 \times 10^{-27}$ Kg/molec

$$m = \frac{m_{O_2} m_{N_2}}{\frac{1}{2}(m_{O_2} + m_{N_2})} = 49.607 \times 10^{-27} \text{ Kg/molec}$$

$$\sigma = \frac{1}{2}(\sigma_{O_2} + \sigma_{N_2}) = \frac{1}{2}(3.467\text{Å} + 3.798\text{ Å}) = 363.3 \times 10^{-12} \text{ m}$$

at 273 K $\mathcal{D} = \frac{2}{3}\left(\frac{K_B^3 T}{\pi^3 m}\right)^{1/2} \frac{T}{\sigma^2 P} = \frac{2}{3}\left[\frac{(1.381 \times 10^{-23} \text{ J/k})^3 (273\text{K})}{\pi^3 (49.607 \times 10^{-27} \text{ Kg})}\right]^{1/2} \frac{273 \text{ K}}{(363.3 \times 10^{-12} \text{ m})^2 (101325 \text{ Pa})}$

$$\boxed{\mathcal{D} = 9.31 \times 10^{-6} \text{ m}^2/\text{s}}$$

COMMENTS: Compared with the published value of $\mathcal{D} = 1.8 \times 10^{-5}$ m²/s, the value calculated above is inaccurate but considering that the value calculated above is based on a rigid sphere model, the results are not bad.

GIVEN: $\dot{m}_{C_6H_{14}} = 8.2 \cdot 10^{-8}$ kg/s

$D = 50\,mm = 0.05\,m$

$L = 20\,cm = 0.20\,m$

$Y_{C_6H_{14},i} = 0.482$

$\mathscr{D}_{C_6H_{14}-air} = 8.0 \cdot 10^{-4}\,m^2/s$

A. FIND: $\dot{m}''_{C_6H_{14}}$

SOLUTION: Apply definition of $\dot{m}''_A$ (Eqn. 3.2):

$$\dot{m}''_{C_6H_{14}} = \frac{\dot{m}_{C_6H_{14}}}{A_{x-sec}} = \frac{\dot{m}_{C_6H_{14}}}{\pi D^2/4} = \frac{8.2 \cdot 10^{-8}}{\pi (.05)^2/4}\,\frac{kg}{s}\,\frac{1}{m^2}$$

$$\boxed{\dot{m}''_{C_6H_{14}} = 4.176 \cdot 10^{-5}\,kg/s\text{-}m^2}$$

B. FIND: Bulk flow of C_6H_{14}

SOLUTION: Apply definition for binary $(C_6H_{14}-air)$ mixture (Eqn. 3.3a):

Bulk flow $\equiv \dot{m}\,Y_{C_6H_{14},i}$ where $\dot{m} = \dot{m}_{C_6H_{14}}$ for Stefan problem

$= 8.2 \cdot 10^{-8} \cdot 0.482 = \boxed{3.952 \cdot 10^{-8}\,kg/s}$

C. FIND: C_6H_{14} diffusional flux

SOLUTION: Apply definition for binary mixture (Eqn. 3.3b):

Diffusional flux $\equiv -\rho \mathscr{D} \dfrac{dY_{C_6H_{14}}}{dx}$; Apply Fick's law (Eqn. 3.1):

$$-\rho \mathscr{D} \frac{dY}{dx} = \dot{m}''_{C_6H_{14}} - \dot{m}''_{C_6H_{14}} Y_{C_6H_{14},i} = (1 - Y_{C_6H_{14},i})\,\dot{m}''_{C_6H_{14},i}$$

$$= (1 - 0.482) \cdot 4.176 \cdot 10^{-5} = \boxed{2.163 \cdot 10^{-5}\,kg/s\text{-}m^2}$$

GIVEN: Water in a 25-mm-diameter test tube evaporating into dry air at one atmosphere. The distance from the water-air interface to the top of the tube is $L = 15$ cm. The mass fraction of the water vapor at the water-air interface is 0.0235 and the binary diffusivity for water vapor in air is 2.6×10^{-5} m²/s.

FIND: a) the mass evaporation rate of the water
b) the water vapor mass fraction at $x = L/2$
c) the fraction of the water mass flowrate due to bulk flow and the fraction due to diffusion at $x = L/2$
d) part c at $x = 0$ and $x = L$. Graph the results of part c and d. Discuss

ASSUMPTIONS: liquid level in tube is constant, air is insoluble in water, ideal gas

APPROACH: This is the Stefan problem in which water vapor is transported through a stagnant layer of air in the column. Equation 3.40 is applied with appropriate boundary conditions.

Equation 3.40:
$$\dot{m}''_{H_2O} = \frac{\dot{m}_{H_2O}}{A_{TUBE}} = \frac{\bar{\rho}\, \mathcal{D}_{H_2O-AIR}}{L} \ln\left[\frac{1 - Y_{H_2O,\infty}}{1 - Y_{H_2O,i}}\right]$$

to evaluate $\bar{\rho}$: at interface $X_{H_2O} = Y_{H_2O}\dfrac{MW_{mix}}{MW_{H_2O}}$

$$MW_{mix} = X_{H_2O}\, MW_{H_2O} + (1 - X_{H_2O})\, MW_{air}$$

So,
$$X_{H_2O} = \frac{Y_{H_2O}(MW_{air}/MW_{H_2O})}{1 - Y_{H_2O} + Y_{H_2O}(MW_{air}/MW_{H_2O})}$$

$$X_{H_2O} = \frac{0.0235\,(28.85/18.016)}{1 - 0.0235 + 0.0235(28.85/18.016)} = 0.0371$$

$$MW_{mix} = 0.0371\,(18.016) + (1 - 0.0317)(28.85) = 28.60 \frac{kg}{kmol}$$

$$\rho = \frac{m}{V} = \frac{P}{RT} \qquad \text{where} \qquad R = \frac{\bar{R}}{MW} = \frac{8.315}{28.60} = 0.2907$$

$$\rho = \frac{101.325}{(0.2907)(298)} = 1.17 \frac{kg}{m^3} \qquad \text{at the interface}$$

at the top of the tube: $Y_{H_2O} = 0$, "mixture" is only air

$$\rho = \frac{P}{RT} \qquad R = \frac{\bar{R}}{MW_{air}} = \frac{8.315}{28.85} = 0.288$$

$$\rho = \frac{101.325}{(0.288)(298)} = 1.18 \frac{kg}{m^3}$$

$$\bar{\rho} = \frac{1}{2}(1.17 + 1.18) = 1.175 \; kg/m^3$$

solving for mass flux

$$\dot{m}_{H_2O}'' = \frac{\bar{\rho}\mathcal{D}}{L} \ln\left[\frac{1-Y_{H_2O,\infty}}{1-Y_{H_2O,i}}\right] = \frac{(1.175)(2.6\times10^{-5})}{0.15} \ln\left[\frac{1-0}{1-0.0235}\right]$$

$$\dot{m}_{H_2O}'' = 4.843 \times 10^{-6} \; kg/m^2\text{-}s$$

$$\dot{m}_{H_2O} = \dot{m}_{H_2O}'' A_{TUBE} = \dot{m}_{H_2O}'' \pi \frac{D^2}{4} = (4.843\times10^{-6})\pi \frac{(0.025)^2}{4} = 2.38\times10^{-9} \; kg/s$$

$$\boxed{\dot{m}_{H_2O} = 2.38 \times 10^{-9} \; kg/s}$$

b) water vapor mass fraction at $x = \frac{L}{2} = 0.075 \; m$
   using Eqtn. 3.39

$$Y_A(x) = 1 - (1 - Y_{A,i}) \exp\left[\frac{\dot{m}_A'' x}{\rho\mathcal{D}}\right]$$

$$Y_{H_2O}(0.075) = 1 - (1 - 0.0235) \exp\left[\frac{(4.843\times10^{-6})(0.075)}{(1.175)(2.6\times10^{-5})}\right] = 0.0118$$

$$\boxed{Y_{H_2O}(0.075) = 0.0118}$$

c) from Eqtn. 3.35 :

$$\dot{m}_A'' = \underset{\uparrow}{Y_A \dot{m}_A''} - \underset{\uparrow}{\rho\mathcal{D}_{AB} \frac{dY_A}{dx}} = CONST$$

$$ BULK \; FLOW \qquad DIFFUSION$$

for conservation of mass $\quad -\rho\mathcal{D}_{AB}\frac{dY_A}{dx} = (1-Y_A)\dot{m}_A''$

fraction of water mass flowrate due to bulk flow

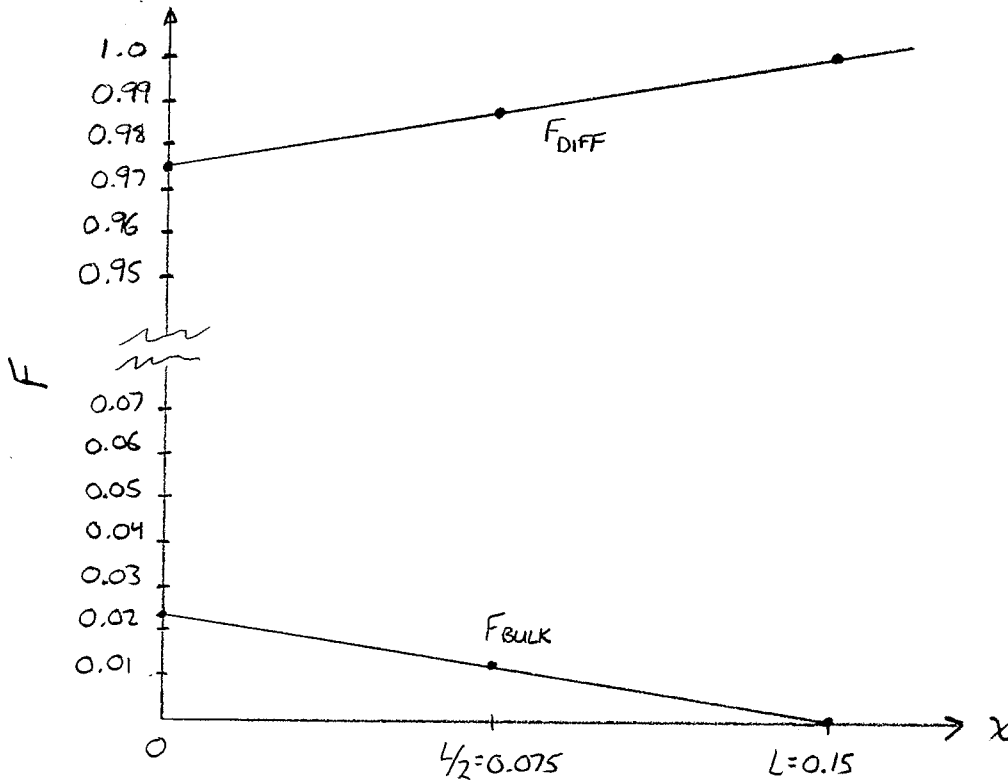$$F_{BULK} = \frac{Y_A \dot{m}_A''}{\dot{m}_A''} \qquad\qquad\qquad \text{(continued)}$$

fraction of water mass flowrate due to diffusion

$$F_{DIFF} = \frac{(1-Y_A)\dot{m}_A''}{\dot{m}_A''} = 1-Y_A$$

The following table contains the results for parts C and d

x (m)	Y_A	F_{BULK}	F_{DIFF}
0	0.0235	0.0235	0.9765
0.075	0.0118	0.0118	0.9882
0.15	~0	~0	1

$x \equiv$ distance above water-air interface

COMMENTS AND DISCUSSION: From the above graph we can see that this problem is clearly diffusion dominated and that the fraction of mass flowrate due to diffusion increases in a linear manner as x increases and that the fraction of mass flow due to the bulk flow decreases in a similar manner.

GIVEN: Water in a 25-mm-diameter test tube evaporating into dry air at 1 atm. The distance from the water-air interface to the top of the test tube is $L = 0.15$ m. The binary diffusivity of water vapor in air is 2.6×10^{-5} m²/s. The water temperature is 21°C and the free-stream air is dry.

FIND: The water mass evaporation rate

ASSUMPTIONS: water-air interface is stationary, air is insoluble in water, ideal gas behavior, equilibrium exists at interface $(P_{H_2O}(x=0) = P_{sat})$

APPROACH: Use standard thermo steam tables to determine P_{sat}. Based on ideal gas assumption $X_{H_2O,i} = P_{H_2O}/P$ and Eqtn. 3.40 can be applied.

from Van Wylen & Sonntag, <u>Fundamentals</u> <u>of</u> <u>Classical</u> <u>Thermodynamics</u>:

$$P_{H_2O} = P_{sat} = 2.505 \text{ KPa} \quad \text{at} \quad T = 21°C$$

$$X_{H_2O} = P_{H_2O}/P = (2.505 \text{ KPa})/(101.325 \text{ KPa}) = 0.0247 \longrightarrow Y_{H_2O} = 0.0156$$

$$MW_{mix} = X_{H_2O}(MW_{H_2O}) + X_{air}(MW_{air})$$

at interface: $MW_{mix} = 0.0247(18.016) + (1 - 0.0247)(28.85) = 28.58$ Kg/Kmole

in free stream: $MW_{mix} = 0(18.016) + 1(28.85) = 28.85$ Kg/Kmole

$$\rho = \frac{P}{RT} = \frac{MW_{mix} P}{\bar{R} T}$$

at interface: $\rho_i = \dfrac{(28.58)(101.325)}{(8.315)(294)} = 1.185$ Kg/m³

in free stream: $\rho_f = \dfrac{(28.85)(101.325)}{(8.315)(294)} = 1.196$ Kg/m³

$\bar{\rho} = \frac{1}{2}(\rho_i + \rho_f) = 1.19$ Kg/m³

Eqtn 3.40

$$\dot{m} = A_{TUBE}\, \dot{m}''_{H_2O} = \frac{\pi d^2}{4} \frac{\bar{\rho}\mathscr{D}}{L} \ln\left[\frac{1 - Y_{H_2O,\infty}}{1 - Y_{H_2O,i}}\right] = \frac{\pi(0.025)^2}{4} \frac{(1.19)(2.6\times10^{-5})}{0.15} \ln\left[\frac{1 - 0}{1 - 0.0156}\right]$$

$$\boxed{\dot{m} = 1.59 \times 10^{-9} \text{ Kg/s}}$$

COMMENTS: See problem 3-7

GIVEN: Water in a 25-mm-diameter test tube evaporating into air with a relative humidity of 50%. The distance from the water-air interface to the top of the test tube is $L=0.15$ m. The binary diffusivity of water vapor in air is 2.6×10^{-5} m^2/s. The water temperature is 21 °C

FIND: The water mass evaporation rate and the rate of heat transfer required to maintain the liquid water at 21 °C

ASSUMPTIONS: water-air interface is stationary, air is insoluble in water, ideal gas behavior, equilibrium exists at interface $(P_{H_2O} = P_{sat}$ @ $x=0)$

APPROACH: Use steam tables to determine P_{sat} and $X_{H_2O,i} = P_{sat}/P$. Using mass fractions of H_2O vapor at interface and in ambient air, calculate the mass flowrate. Then using a first law analysis evaluate the heat transfer required to maintain a water temperature of 21 °C

at $T = 21$ °C $\qquad$ $P_{H_2O} = P_{sat} = 2.505$ KPa $\longrightarrow$ $X_{H_2O} = 0.0247$, $Y_{H_2O} = 0.0156$
see problem 3-6

$MW_{mix} = X_{H_2O} MW_{H_2O} + (1 - X_{H_2O}) MW_{AIR} = (0.0247)(18.016) + (1 - 0.0247)(28.85) = 28.58$ $Kg/Kmole$
at interface

in ambient air : Relative Humidity, $\phi = \dfrac{P_{H_2O}}{P_{sat}} = 0.5$

$\qquad$ using P_{sat} @ 21 °C $\qquad$ $P_{H_2O} = 0.5(2.505$ KPa$) = 1.253$ KPa

$\qquad$ $X_{H_2O,\infty} = \dfrac{P_{H_2O}}{P} = \dfrac{1.253}{101.325} = 0.0124 \longrightarrow Y_{H_2O,\infty} = 0.00778$

$MW_{mix,\infty} = (0.0124)(18.016) + (1 - 0.0124)(28.85) = 28.72$ $Kg/Kmole$

at interface : $\rho_i = \dfrac{MW_{mix} P}{RT} = \dfrac{(28.58)(101.325)}{(8.315)(294)} = 1.18$ Kg/m^3 $\left.\begin{array}{c} \bar{\rho} = \frac{1}{2}(\rho_i + \rho_f) \\ \\ \bar{\rho} = 1.185 \ Kg/m^3 \end{array}\right\}$

in ambient : $\rho_f = \dfrac{(28.72)(101.325)}{(8.315)(294)} = 1.19$ Kg/m^3

$\dot{m}_{H_2O} = A_{TUBE} \dot{m}''_{H_2O} = \dfrac{\pi d^2}{4} \cdot \dfrac{\bar{\rho} \mathscr{D}}{L} \ln\left[\dfrac{1 - Y_{H_2O,\infty}}{1 - Y_{H_2O,i}}\right] = \dfrac{\pi(.025)^2}{4} \cdot \dfrac{(1.185)(2.6 \times 10^{-5})}{0.15} \ln\left[\dfrac{1 - 0.00778}{1 - 0.0156}\right]$

$\boxed{\dot{m}_{H_2O} = 798 \times 10^{-12} \ Kg/s}$

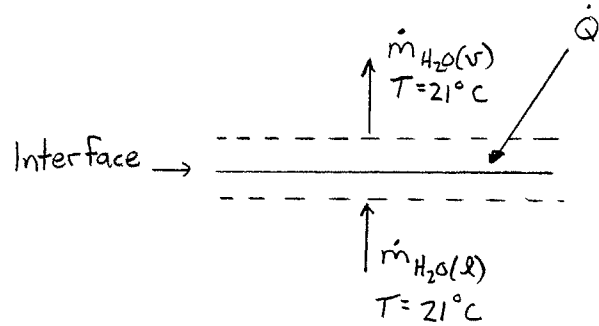b) Heat transfer required to maintain water at 21°C

first law:

$$\dot{Q} - \cancel{\dot{W}}^{=0} = \dot{m}(h_{out} - h_{in})$$

$$\dot{Q} = \dot{m}(h_{H_2O(v)} - h_{H_2O(\ell)})$$

assuming vapor and liquid H_2O are at $T = 21°C$ means that

$$h_{H_2O(v)} - h_{H_2O(\ell)} = h_{fg} \longrightarrow h_{fg} = \frac{44011}{18} \, ^{KJ}/_{kg} \qquad \text{Appendix A}$$

so,

$$\dot{Q} = \dot{m} h_{fg} = (798 \times 10^{-12} \, ^{kg}/_s)(44011 \, ^{KJ}/_{kg}) = \frac{35.1 \times 10^{-6}}{18} \, ^{KJ}/_s$$

$$\boxed{\dot{Q} = 0.00195 \, W}$$

as a point of comparison, the heat transfer required if the ambient air is dry (Problem 3-6) is:

$$\dot{Q} = (1.59 \times 10^{-9} \, ^{kg}/_s)(\frac{44011}{18} \, ^{KJ}/_{kg}) = \frac{70}{18} \times 10^{-6} \, ^{KJ}/_s = 0.0039 \, W$$

COMMENTS: Compare the mass flowrates calculated in problems 3-6 and 3-7. Note how the presence of water vapor in the ambient air inhibits the mass flowrate. Also note how the ambient water vapor affects the heat transfer required to maintain the water at a constant temperature.

GIVEN: Liquid n-hexane in a 50-mm diameter graduated cylinder with air blowing across the top. The distance from the liquid-air interface to the cylinder top is 20 cm and the hexane is at a temperature of 25°C. The diffusivity of hexane in air is $8.8 \times 10^{-6} \, m^2/s$

FIND: the evaporation rate of n-hexane

ASSUMPTIONS: hexane-air interface is stationary, air is insoluble in hexane, ideal gas behavior, Clausius Clapeyron equation applies

APPROACH: Use Clausius-Clapeyron equation to determine the hexane mole fraction at the liquid-air interface. Then use equation 3-40 to determine the hexane mass flux

Clausius-Clapeyron :   reference state : $T_{boil} = 342 \, K$, $P_{Boil} = 1 \, atm$
$h_{fg} = 335 \, KJ/kg$, $MW = 86.177 \, kg/kmol$

$$\frac{P_{sat}}{P_{boil}} = \exp\left[\frac{-h_{fg}}{(R_u/MW)}\left(\frac{1}{T_{sat}} - \frac{1}{T_{boil}}\right)\right] = \exp\left[\frac{-335}{(8.315/86.177)}\left(\frac{1}{298} - \frac{1}{342}\right)\right] = 0.223$$

$P_{sat} = 0.223 \, atm$ $X_{C_6H_{14}} = 0.233$, $Y_{C_6H_6} = X_{C_6H_{14}} \frac{MW_{C_6H_{14}}}{MW_{mix}} = 0.233 \frac{(86.177)}{41.63} = 0.482$

$MW_{mix} = X_{C_6H_{14}} MW_{C_6H_{14}} + (1-X_{C_6H_{14}}) MW_{air} = 0.223(86.177) + (0.777)(28.85) = 41.63 \, kg/kmol$

in ambient :
   $MW_\infty = MW_{air} = 28.85$, $Y_{C_6H_{14},\infty} = 0$

at interface: $\rho_i = \frac{P \cdot MW_{mix}}{R_u T} = \frac{101.325(41.63)}{(8.315)(298)} = 1.70 \, kg/m^3$ $\quad\left.\right\}$ $\bar\rho = \frac{1}{2}(\rho_i + \rho_a)$

in ambient: $\rho = \frac{P MW_{air}}{R_u T} = \frac{(101.325)(28.85)}{(8.315)(298)} = 1.18 \, kg/m^3$ $\quad = 1.44 \, kg/m^3$

$$\boxed{\dot m} = A_{tube}\, \dot m'' = \frac{\pi d^2}{4} \frac{\rho \mathcal{D}}{L} \ln\left[\frac{1-Y_\infty}{1-Y_i}\right] = \frac{\pi (0.05)^2}{4} \frac{(1.44)(8.8\times10^{-6})}{0.2} \ln\left[\frac{1-0}{1-0.482}\right] = \boxed{81.83 \times 10^{-9} \, kg/s}$$

COMMENTS: if liquid were fed into the cylinder to keep the liquid surface fixed then it would take 2.2 hrs to evaporate 1cc of hexane at this evaporation rate.

$m = \rho V = 659(1\times10^{-6}) = 659 \times 10^{-6} \, kg \quad \rightarrow \quad t = \frac{m}{\dot m} = \frac{659\times10^{-6}}{81.83 \times 10^{-9}} = 8053 \, sec$

GIVEN: A 1-mm-diameter water droplet evaporating into dry air at 500 K and 1 atm

FIND: The evaporation rate constant

ASSUMPTIONS: Quasi-steady, uniform droplet temperature, constant thermophysical properties, spherically symmetric, ideal gas behavior, air is insoluble in water, droplet temperature = 75°C (see problem 3-10)

APPROACH: This is simply the Stefan problem in spherical coordinates. The water vapor mole and mass fractions at the liquid-air interface can be determined from the steam tables and the thermophysical properties can be evaluated at an average temperature. The evaporation constant is defined in Eqtn. 3.58

Water vapor at interface: $T = 75°C$ $P_{sat} = 38.58$ KPa

$$\chi_{H_2O,s} = \frac{P_{sat}}{P} = \frac{38.58}{101.325} = 0.381 \qquad Y_{H_2O,s} = 0.278$$

$$MW_{mix} = \chi_{H_2O} MW_{H_2O} + (1-\chi_{H_2O}) MW_{air} = 0.381(18.016) + 0.619(28.85) = 24.72 \; Kg/Kmole$$

$$\rho_s = \frac{MW_{mix} \, P}{R_u T} = \frac{24.72 \, (101.325)}{8.315 \, (348 \, K)} = 0.866 \; Kg/m^3$$

in ambient air:

$$MW = MW_{air} = 28.85 \longrightarrow \rho_\infty = \frac{(28.85)(101.325)}{(8.315)(500 \, K)} = 0.703 \; Kg/m^3$$

$$\bar{\rho} = \frac{\rho_s + \rho_\infty}{2} = \frac{0.866 + 0.703}{2} = 0.784 \; Kg/m^3$$

The average temperature of the air-vapor is $\bar{T} = \dfrac{T_s + T_\infty}{2} = \dfrac{348 \, K + 500 \, K}{2}$

$$\bar{T} = 424 \; K$$

from Appendix D $\mathcal{D} = 2.2 \times 10^{-5} \; m^2/s$ at $T = 273 \, K \longrightarrow \mathcal{D} = \mathcal{D}_{ref} \left(\dfrac{T}{T_{ref}}\right)^{3/2}$

$$= (2.2 \times 10^{-5}) \left(\frac{424}{273}\right)^{3/2} = 4.2 \times 10^{-5} \; m^2/s$$

the evaporation rate constant:

$$\boxed{K} = \frac{8 \bar{\rho} \mathcal{D}}{\rho \ell} \ln\left[1 + \frac{Y_s - Y_\infty}{1 - Y_s}\right] = \frac{8(.784)(4.2 \times 10^{-5})}{1000} \ln\left[1 + \frac{0.278 - 0}{1 - 0.278}\right] = \boxed{85.8 \times 10^{-9} \; m^2/s}$$

GIVEN: 50 μm-diameter water droplets evaporating in air at 1 atm. The droplet temperature is 75°C and the mean air temperature is 200°C

FIND: The lifetimes of the droplets for ambient H_2O mole fractions of 0.1, 0.2, and 0.3

ASSUMPTIONS: Quasi-steady, uniform droplet temperature, constant thermophysical properties, spherically symmetric, ideal gas behavior, air is insoluble in water

APPROACH: This is the Stefan problem in spherical coordinates. Use the steam tables to evaluate the H_2O mass fraction at the droplet surface and Eqtn. 3-60 to evaluate the lifetime.

at the droplet surface: $P_{H_2O} = P_{sat} = 38.58 \, kPa$ at $T = 75°C$

$$X_{H_2O,s} = \frac{P_{H_2O}}{P} = \frac{38.58}{101.325} = 0.381 \quad , \quad Y_{H_2O,s} = X_{H_2O} \frac{MW_{H_2O}}{MW_{mix,s}} = 0.381 \frac{(18.016)}{(24.72)} = 0.278$$

$$MW_{mix,s} = X_{H_2O} MW_{H_2O} + (1-X_{H_2O}) MW_{air} = 0.381(18.016) + 0.619(28.85) = 24.72 \, kg/kmole$$

in ambient air:

$X_{H_2O_\infty} = 0.1 \qquad MW_{mix,\infty} = 0.1(18.016) + (1-0.1) 28.85 = 27.76 \, kg/m^3$

$X_{H_2O_\infty} = 0.2 \qquad MW_{mix,\infty} = 0.2(18.016) + (1-0.2) 28.85 = 26.68 \, kg/m^3$

$X_{H_2O_\infty} = 0.3 \qquad MW_{mix,\infty} = 0.3(18.016) + (1-0.3) 28.85 = 25.6 \, kg/m^3$

using $\quad \bar{\rho} = \frac{\overline{MW} P}{R_u \bar{T}} \quad$ where $\quad \overline{MW} = \frac{1}{2}(MW_{mix,s} + MW_{mix,\infty})$

$X = 0.1 \quad \bar{\rho} = \frac{(26.24)(101.325)}{(8.315)(473 K)} = 0.676 \, kg/m^3$

$X = 0.2 \quad \bar{\rho} = \frac{(25.7)(101.325)}{(8.315)(473 K)} = 0.662 \, kg/m^3$

$X = 0.3 \quad \bar{\rho} = \frac{(25.16)(101.325)}{(8.315)(473 K)} = 0.648 \, kg/m^3$

evaluating the diffusivity $\quad \mathcal{D} = \mathcal{D}_{ref} \left(\frac{\bar{T}}{T_{ref}}\right)^{3/2} \quad \longrightarrow$ Appendix D for $\mathcal{D}_{ref}$ and T_{ref}

$$\mathcal{D} = (2.2 \times 10^{-5}) \left(\frac{473}{273}\right)^{3/2} = 5.02 \times 10^{-5} \, m^2/s$$

(continued)

Water vapor mass fraction in ambient air

$$Y_{H_2O,\infty} = X_{H_2O,\infty} \frac{MW_{H_2O}}{MW_{mix,\infty}}$$

$X_{H_2O,\infty} = 0.1$ $Y_{H_2O,\infty} = 0.1 \frac{(18.016)}{(27.76)} = 0.0649$

$X_{H_2O,\infty} = 0.2$ $Y_{H_2O,\infty} = 0.2 \frac{(18.016)}{(26.68)} = 0.135$

$X_{H_2O,\infty} = 0.3$ $Y_{H_2O,\infty} = 0.3 \frac{(18.016)}{(25.6)} = 0.211$

Using   Eqtn. 3-60

$$t = D_0^2 / K \qquad \text{where} \qquad K = \frac{8\rho\mathcal{D}}{\rho_l} \ln\left(1 + \frac{Y_s - Y_\infty}{1 - Y_s}\right)$$

$D_0 = 50 \times 10^{-6} m$, $\rho_l = 1000 \, kg/m^3$

$X_{H_2O,\infty} = 0.1$ $K = \frac{8(0.676)(5.02 \times 10^{-5})}{1000} \ln\left[1 + \frac{0.278 - 0.0649}{1 - 0.278}\right] = 70.21 \times 10^{-9} \, m^2/s$

$$\boxed{t = \frac{(50 \times 10^{-6} m)^2}{70.21 \times 10^{-9}} = 35.6 \, ms}$$

$X_{H_2O,\infty} = 0.2$ $K = \frac{8(0.662)(5.02 \times 10^{-5})}{1000} \ln\left[1 + \frac{0.278 - 0.135}{1 - 0.278}\right] = 48.04 \times 10^{-9} \, m^2/s$

$$\boxed{t = \frac{(50 \times 10^{-6})^2}{48.04 \times 10^{-9}} = 52.0 \, ms}$$

$X_{H_2O,\infty} = 0.3$ $K = \frac{8(0.648)(5.02 \times 10^{-5})}{1000} \ln\left[1 + \frac{0.278 - 0.211}{1 - 0.278}\right] = 23.09 \times 10^{-9} \, m^2/s$

$$\boxed{t = \frac{(50 \times 10^{-6})^2}{23.09 \times 10^{-9}} = 108.3 \, ms}$$

COMMENTS: Increasing $Y_{H_2O,\infty}$ suppresses evaporation in a nonlinear fashion. For the conditions of this problem, doubling $Y_{H_2O,\infty}$ results in an evaporation time 1.46 times greater, while tripling $Y_{H_2O,\infty}$ effectively triples the evaporation time.
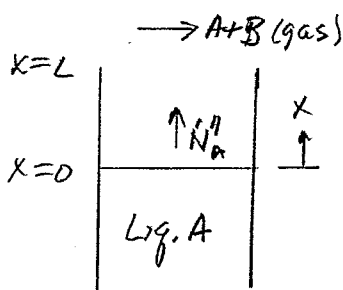
PROBLEM 3-12

GIVEN: Molar flux form of Fick's law (Eqn. 3.6)

FIND: Stefan problem solution

ASSUMPTIONS: Ideal gas, binary diffusion, 1-D

SOLUTION:

Diagram (left): A+B (gas), $x=L$, $x=0$, $\uparrow \dot{N}_A''$, Liq. A

Eqn. 3.6:

$$\dot{N}_A'' = \chi_A (\dot{N}_A'' + \dot{N}_B'')^{\,0} - c\mathscr{D}_{AB}\frac{d\chi_A}{dx}$$

$$\dot{N}_A''(1-\chi_A) = -c\mathscr{D}_{AB}\frac{d\chi_A}{dx}$$

separate variables: $\quad -\dfrac{\dot{N}_A''}{c\mathscr{D}_{AB}}\,dx = \dfrac{d\chi_A}{1-\chi_A}$

Note that $c\ (=P/R_u T)$ is constant (unlike ρ); thus, $-\dot{N}_A''/c\mathscr{D}_{AB}$ is constant:

$$\int -\frac{\dot{N}_A''}{c\mathscr{D}_{AB}}\,dx = -\frac{\dot{N}_A''}{c\mathscr{D}_{AB}}\int dx = \int \frac{d\chi_A}{1-\chi_A}$$

Apply limits ($\chi_A = \chi_{A,i}$ @ $x=0$; $\chi_A = \chi_{A,\infty}$ @ $x=L$):

$$\frac{\dot{N}_A''}{c\mathscr{D}_{AB}}L = \ln\left[\frac{1-\chi_{A,\infty}}{1-\chi_{A,i}}\right] \quad \text{or} \quad \boxed{\dot{N}_A'' = \frac{c\mathscr{D}_{AB}}{L}\ln\frac{1-\chi_{A,\infty}}{1-\chi_{A,i}}}$$

COMMENT: Note that our result is exact, unlike Eqn. 3.40, where ρ was assumed to be constant. The density varies with position (x) because MW_{mix} varies with x.

GIVEN: Result from problem 3-12; data from Ex. 3.1.

FIND: $\dot{m}_{C_6H_6}$

ASSUMPTIONS: Binary Stefan flow, ideal gas

SOLUTION: As most of the work has already been done in Ex. 3.1, the solution is straightforward:

$$x_{C_6H_6,i} = \frac{P_{sat}}{P} = \frac{0.145}{1} = 0.145$$

$$x_{C_6H_6,\infty} = 0$$

$$c = \frac{P}{R_u T} = \frac{101,325}{8315(298)} = 0.04089 \ kmol/m^3$$

Apply problem 3-12 result to find $\dot{N}''_{C_6H_6}$:

$$\dot{N}''_{C_6H_6} = \frac{c \mathcal{D}_{AB}}{L} \ln\left[\frac{1 - x_{C_6H_6,\infty}}{1 - x_{C_6H_6,i}}\right]$$

$$= \frac{0.04089 \,(0.88 \cdot 10^{-5})}{0.10} \ln\left[\frac{1 - 0}{1 - 0.145}\right] \frac{kmol}{m^3} \frac{m^2}{s} \frac{1}{m}$$

$$= 5.637 \cdot 10^{-7} \ kmol/s\text{-}m^2$$

Now $\dot{m}''_{C_6H_6} = \dot{N}''_{C_6H_6} MW_{C_6H_6} = 5.637 \cdot 10^{-7} (78.108)$

$$= 4.403 \cdot 10^{-5} \ kg/s\text{-}m^2$$

		$\dot{m}''$ $(kg/s\text{-}m^2)$
Ex. 3.1	Approx.	$4.409 \cdot 10^{-5}$
	Exact	$4.403 \cdot 10^{-5}$

COMMENT: We see that, for the particular conditions of Ex. 3.1, the approximate solution does an excellent job. This is because $p(x)$ is almost linear; for situations where $p(x)$ is highly nonlinear, less good agreement should be expected.