Chapter 1

Trade in a Model with no Frictions

Exercise 1.1 (a) Equation 1.1 $(Total\ amount\ of\ the\ consumption\ good\ in)_t = N_t y_1 + N_{t-1} y_2$

- Equation 1.2 $(Total\ consumption\ by\ the\ young)_t = N_t c_{1,t}$
- Equation 1.3 $(Total\ consumption\ by\ the\ old)_t = N_{t-1} c_{2,t}$
- Equation 1.4 Feasibility constraint:

  $$N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y_1 + N_{t-1} y_2$$

  With a constant population, we have,

  $$N c_{1,t} + N c_{2,t} \leq N y_1 + N y_2$$

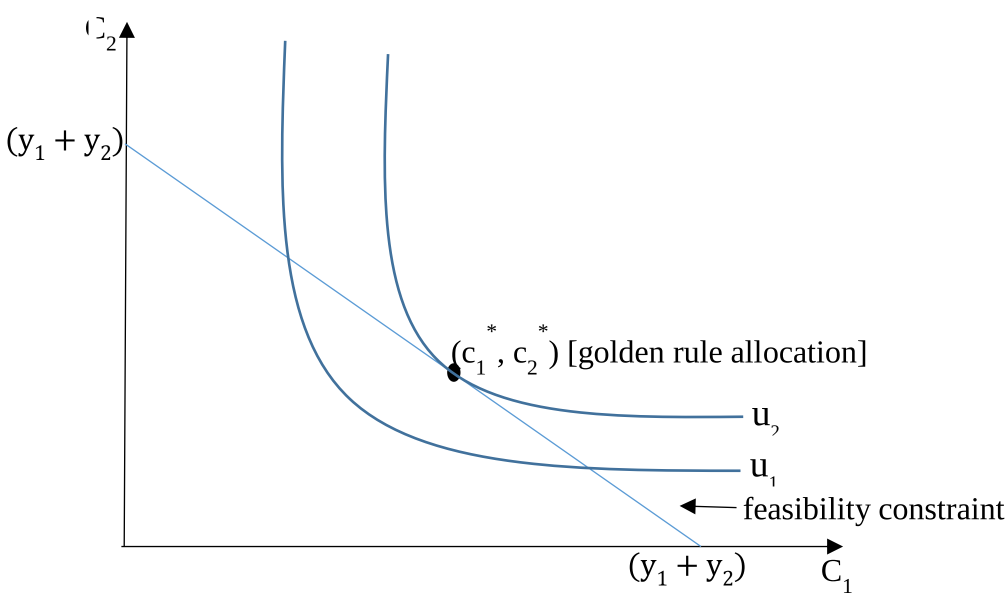
  Dividing throughout by N, we have

- Equation 1.5 $c_{1,t} + c_{2,t} \leq y_1 + y_2$

- Equation 1.6 Assuming a stationary allocation, the per capita constraint becomes

  $$c_1 + c_2 \leq y_1 + y_2.$$

(b) (c_1^*, c_2^*) → Golden Rule Allocation: allocation that maximizes the utility of future generations.



Exercise 1.2

- Bundle A: $c_1 = 6$, $c_2 = 12$
- Bundle B: $c_1 = 4$, $c_2 = 10$
- $c_1^A > c_1^B$ & $c_2^A > c_2^B$ $\Rightarrow$ Bundle A is strictly preferred to Bundle B by assumption **#3** of "More is preferred to less"

Exercise 1.3 $(Supply\ of\ transfers\ by\ the\ young)_t = N_t(y_t - c_{1,t})$

Since the young consume half their endowment in every period, ie $c_{1,t} = \frac{1}{2}y_t$, we can rewrite the above equation as:

$$(Supply\ of\ transfers\ by\ the\ young)_t = N_t\left(y_t - \frac{1}{2}y_t\right) = N_t\frac{y_t}{2}$$

$$(Demand\ for\ transfers\ by\ the\ old)_t = N_{t-1}x_t\emptyset_{t-1}$$

In equilibrium, we have,

$$N_{t-1}x_t\emptyset_{t-1} = N_t\frac{y_t}{2}$$

$$\Rightarrow x_t = \frac{1}{\emptyset_{t-1}}\frac{N_t}{N_{t-1}}\frac{y_t}{2}$$

Using, $N_t = 1.1N_{t-1}$ and $\emptyset_{t-1} = y_t - c_{1,t-1} = y_t - \frac{y_{t-1}}{2} = y_t - \frac{1}{2}\frac{y_t}{1.05} = \frac{0.55}{1.05}y_t$, we have,

$$x_t = \frac{1.05}{0.55y_t}\frac{1.1N_{t-1}}{N_{t-1}}\frac{y_t}{2} = 1.05$$

Exercise 1.4

Economy A	Economy B
$y_1 = 20$, $y_2 = 0$	$y_1 = 20$, $y_2 = 0$

c_1 , c_2= 10, 10	c_1, c_2 = 8, 12

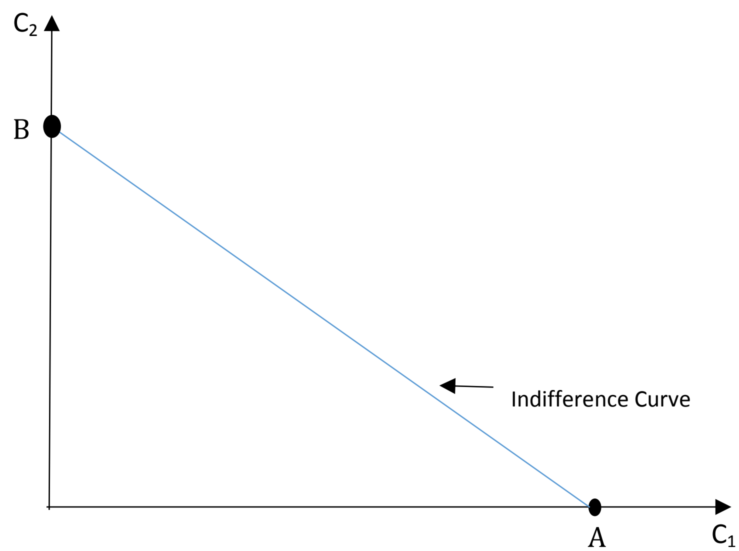
(a) Since these choices maximize "lifetime welfare" $\rightarrow$ it maximizes the utility of the current and future generations (Golden Rule Allocation) at the cost of the initial old's welfare. With a constant level of population and a stationary allocation, the feasibility set for both economies is the same,

$$c_1 + c_2 \leq 20$$

However, as the preferences of the individuals are different, it implies that we are going to be on two different sets of indifference curves for each of the two economies. As such this situation is Pareto incomparable.

(b) These choices maximize "lifetime welfare" [Golden Rule Allocation] $\Rightarrow$ it maximises the utility of current and future generations and <u>not</u> the utility of the initial old. If the utility of the initial old was to be maximized, we would end up with a corner solution, implying that people consume nothing when young and that would not maximise the utility of the future generations.

Exercise 1.5 (a) Constant marginal utility $\rightarrow$ Indifference Curves are linear



(b) $\frac{Mu_1}{Mu_2} = \frac{dc_2}{dc_1} \Rightarrow \frac{dc_2}{dc_1} = \frac{Mu_1}{Mu_2}$

If $Mu_1 > Mu_2 \rightarrow |slope| > 1$, we end up with a corner solution at A. Individuals only consume in the first period of their lives

(c) If $Mu_1 < Mu_2 \rightarrow |slope| < 1$, we end up with a corner solution at B. Individuals only consume in the second period of their lives.