1 Units of Measurement for Physical and Chemical Change

Review Questions

1.1 The main goal of chemistry is to seek to understand the behaviour of matter by studying the behaviour of atoms and molecules.

1.2 In solid matter, atoms or molecules pack close to each other in fixed locations. Although the atoms and molecules in a solid vibrate, they do not move around or past each other. Consequently, a solid has a fixed volume and rigid shape.

In liquid matter, atoms or molecules pack about as closely as they do in solid matter, but they are free to move relative to each other, giving liquids a fixed volume but not a fixed shape. Liquids assume the shape of their container.

In gaseous matter, atoms or molecules have a lot of space between them and are free to move relative to one another, making gases compressible. Gases always assume the shape and volume of their container.

1.3 A physical property is one that a substance displays without changing its composition, whereas a chemical property is one that a substance displays only by changing its composition via a chemical change.

1.4 Changes that alter only state or appearance, but not composition, are called physical changes. The atoms or molecules that compose a substance *do not change* their identity during a physical change. For example, when water boils, it changes its state from a liquid to a gas, but the gas remains composed of water molecules, so this is a physical change. When sugar dissolves in water, the sugar molecules are separated from each other, but the molecules of sugar and water remain intact.

In contrast, changes that alter the composition of matter are called chemical changes. During a chemical change, atoms rearrange, transforming the original substances into different substances. For example, the rusting of iron, the combustion of natural gas to form carbon dioxide and water, and the denaturing of proteins when an egg is cooked are examples of chemical changes.

1.5 In chemical and physical changes, matter often exchanges energy with its surroundings. In these exchanges, the total energy is always conserved; energy is neither created nor destroyed. Systems with high potential energy tend to change in the direction of lower potential energy, releasing energy into the surroundings.

1.6 Chemical energy is potential energy. It is the energy that is contained in the bonds that hold the molecules together. This energy arises primarily from electrostatic forces between the electrically charged particles (protons and electrons) that compose atoms and molecules. Some of these arrangements—such as the one within the molecules that compose gasoline—have a much higher potential energy than others. When gasoline undergoes combustion the arrangement of these particles changes, creating molecules with much lower potential energy and transferring a great deal of energy (mostly in the form of heat) to the surroundings. A raised weight has a certain amount of potential energy (dependent on the height the weight is raised) that can be converted to kinetic energy when the weight is released.

1.7 The SI base units include the metre (m) for length, the kilogram (kg) for mass, the second (s) for time, and the Kelvin (K) for temperature.

1.8 The two different temperature scales are Kelvin (K) and Celsius (°C). The size of the degree is the same in the Kelvin and the Celsius scales.

1.9 Prefix multipliers are used with the standard units of measurement to change the value of the unit by powers of 10.

For example, the kilometre has the prefix "kilo," meaning 1000 or 10^3. Therefore:

1 kilometre = 1000 metres = 10^3 metres

Similarly, the millimetre has the prefix "milli," meaning 0.001 or 10^{-3}.

1 millimetre = 0.001 metres = 10^{-3} metres

1.10 A derived unit is a combination of other units. Examples of derived units include speed in metres per second (m s^{-1}), volume in metres cubed (m^3), and density in grams per cubic centimetre (g cm^{-3}).

1.11 The density (d) of a substance is the ratio of its mass (m) to its volume (V):

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \text{ or } d = \frac{m}{V}$$

The density of a substance is an example of an intensive property, one that is independent of the amount of the substance. Mass is one of the properties used to calculate the density of a substance. Mass, in contrast, is an extensive property, one that depends on the amount of the substance.

1.12 An intensive property is a property that is independent of the amount of the substance. An extensive property is a property that depends on the amount of the substance.

1.13 Measured quantities are reported so that the number of digits reflects the uncertainty in the measurement. The nonplaceholding digits in a reported number are called significant figures.

1.14 In multiplication or division, the result carries the same number of significant figures as the measurement with the fewest significant figures.

1.15 In addition or subtraction, the result carries the same number of decimal places as the quantity with the fewest decimal places.

1.16 When taking a logarithm, the number of decimal places (the mantissa) is determined by the number of the significant figures in the number whose logarithm is being calculated. The number before the decimal place is the magnitude of the logarithm. For example, $\log_{10}(103.55) = 2.01515$

1.17 When taking an antilogarithm of a number, the mantissa of the number whose antilogarithm is being calculated determines the final significant figures in the answer. For example, $10^{1.236} = 17.2$

1.18 Accuracy refers to how close the measured value is to the actual value. Precision refers to how close a series of measurements are to one another or how reproducible they are. A series of measurements can be precise (close to one another in value and reproducible) but not accurate (not close to the true value).

1.19 Random error is error that has equal probability of being too high or too low. Almost all measurements have some degree of random error. Random error can, with enough trials, average itself out. Systematic error is error that tends toward being either too high or too low. Systematic error does not average out with repeated trials.

1.20 Using units as a guide to solving problems is often called dimensional analysis. Units should always be included in calculations; they are multiplied, divided, and cancelled like any other algebraic quantity.

Problems by Topic

1.21 (a) chemical property (burning involves breaking and making bonds, so bonds must be broken and made to observe this property)

(b) physical property (shininess is a physical property and so can be observed without making or breaking chemical bonds)

(c) physical property (odour can be observed without making or breaking chemical bonds)

(d) chemical property (burning involves breaking and making bonds, so bonds must be broken and made to observe this property)

1.22 (a) physical property (vaporization is a phase change and so can be observed without making or breaking chemical bonds)

(b) physical property (sublimation is a phase change and so can be observed without making or breaking chemical bonds)

(c) chemical property (rusting involves the reaction of iron with oxygen to form iron oxide; observing this process involves making and breaking chemical bonds)

(d) physical property (colour can be observed without making or breaking chemical bonds)

1.23 (a) chemical change (new compounds are formed as methane and oxygen react to form carbon dioxide and water)

(b) physical change (vaporization is a phase change and does not involve the making or breaking of chemical bonds)

(c) chemical change (new compounds are formed as propane and oxygen react to form carbon dioxide and water)

(d) chemical change (new compounds are formed as the metal in the frame is converted to oxides)

1.24 (a) chemical change (new compounds are formed as the sugar burns)

(b) physical change (dissolution is a phase change and does not involve the making or breaking of chemical bonds)

(c) physical change (this is simply the rearrangement of the atoms)

(d) chemical change (new compounds are formed as the silver converts to an oxide)

1.25 (a) physical change (vaporization is a phase change and does not involve the making or breaking of chemical bonds)

(b) chemical change (new compounds are formed)

(c) physical change (vaporization is a phase change and does not involve the making or breaking of chemical bonds)

1.26 (a) physical change (vaporization of butane is a phase change and does not involve changing its chemical composition)

(b) chemical change (new compounds are formed as the butane combusts)

(c) physical change (vaporization of water is a phase change and does not involve changing its chemical composition)

Units in Measurement

1.27 (a) **Given:** T = 0.00 °C **Find:** T in kelvins

Conceptual Plan: Use the relationship $\frac{\mathbf{T}}{\mathbf{K}} = \frac{\mathbf{T_C}}{\mathbf{°C}} + \mathbf{273.15}$

Solution: $\frac{T}{K} = \frac{0.00\ °C}{°C} + 273.15$; T = 273.15 K

Check: Temperature in kelvins is 273.15 units larger than °C. The answer has two decimal places.

(b) **Given:** T = 77 K **Find:** T in °C

Conceptual Plan: $\frac{\mathbf{T_C}}{\mathbf{°C}} = \frac{\mathbf{T}}{\mathbf{K}} - \mathbf{273.15}$

Solution: $\frac{T_C}{°C} = \frac{77\ K}{K} - 273.15$; T = –196 °C

Check: Temperature in °C is 273.15 units smaller than in kelvins. The answer has no decimal places.

(c) **Given:** T = 37.0 °C **Find:** T in kelvins

Conceptual Plan: $\frac{\mathbf{T}}{\mathbf{K}} = \frac{\mathbf{T_C}}{\mathbf{°C}} + \mathbf{273.15}$

Solution: $\frac{T}{K} = \frac{37.0\ °C}{°C} + 273.15$; T = 310.2 K

Check: Temperature in kelvins is 273.15 units larger than °C. The answer has one decimal place.

1.28 (a) **Given:** T = 100.0 °C **Find:** T in K

Conceptual Plan: $\frac{\mathbf{T}}{\mathbf{K}} = \frac{\mathbf{T_C}}{\mathbf{°C}} + \mathbf{273.15}$

Solution: $\frac{T}{K} = \frac{100.0\ °C}{°C} + 273.15$; T = 373.2 K

Check: Temperature in kelvins is 273.15 units larger than °C. The answer has one decimal place.

(b) **Given:** T = 2.735 K **Find:** T in °C

Conceptual Plan: $\frac{\mathbf{T_C}}{\mathbf{°C}} = \frac{\mathbf{T}}{\mathbf{K}} - \mathbf{273.15}$

Solution: $\frac{T}{°C} = \frac{2.735\ K}{K} - 273.15$; T = –270.415 °C

Check: Temperature in °C is 273.15 units smaller than in kelvins. The answer has three decimal places.

(c) **Given:** T = 22 °C **Find:** T in K

Conceptual Plan: $\frac{\mathbf{T}}{\mathbf{K}} = \frac{\mathbf{T_C}}{\mathbf{°C}} + \mathbf{273.15}$

Solution: $\frac{T}{K} = \frac{22\ °C}{°C} + 273.15$; T = 293 K

Check: Temperature in kelvins is 273.15 units larger than °C. The answer has no decimal places.

1.29 **Given:** T = –77.5 °C **Find:** T in K

Conceptual Plan: $\frac{\mathbf{T}}{\mathbf{K}} = \frac{\mathbf{T_C}}{\mathbf{°C}} + \mathbf{273.15}$

Solution: $\frac{T}{K} = \frac{-77.5\ °C}{°C} + 273.15$; T = 195.7 K

Check: Temperature in kelvins is 273.15 units larger than °C. The answer has one decimal place.

1.30 **Given:** T = 56.7 °C **Find:** T in K

Conceptual Plan: $\frac{\mathbf{T}}{\mathbf{K}} = \frac{\mathbf{T_C}}{\mathbf{°C}} + \mathbf{273.15}$

Solution: $\frac{T}{K} = \frac{56.7\ °C}{°C} + 273.15;\ T = 329.9\ K$

Check: Temperature in kelvins is 273.15 units larger than °C. The answer has one decimal place.

1.31 Use Table 1.2 to determine the appropriate prefix multiplier and substitute the meaning into the expressions.

(a) 10^{-9} is equivalent to "nano" so 1.2×10^{-9} m = 1.2 nanometres = 1.2 nm

(b) 10^{-15} is equivalent to "femto" so 22×10^{-15} s = 22 femtoseconds = 22 fs

(c) 10^{9} is equivalent to "giga" so 1.5×10^{9} g = 1.5 gigagrams = 1.5 Gg

(d) 10^{6} is equivalent to "mega" so 3.5×10^{6} L = 3.5 megalitres = 3.5 ML

1.32 Use Table 1.2 to determine the appropriate prefix multiplier and substitute the meaning into the expressions.

(a) 38.8×10^{5} g = 3.88×10^{6} g; 10^{6} is equivalent to "mega" so 3.88×10^{6} g = 3.88 megagrams = 3.88 Mg

(b) 55.2×10^{-10} s = 5.52×10^{-9} s; 10^{-9} is equivalent to "nano" so 5.52×10^{-9} s = 5.52 nanoseconds = 5.52 ns

(c) 23.4×10^{11} m = 2.34×10^{12} m; 10^{12} is equivalent to "tera" so 2.34×10^{12} m = 2.34 terametres = 2.34 Tm

(d) 87.9×10^{-7} L = 8.79×10^{-6} L; 10^{-6} is equivalent to "micro" so 8.79×10^{-6} L = 8.79 microlitres = 8.79 μL

1.33 Use Table 1.2 to determine the appropriate prefix multiplier and substitute the meaning into the expressions.

(a) 10^{-9} is equivalent to "nano" so 4.5 ns = 4.5 nanoseconds = 4.5×10^{-9} s

(b) 10^{-15} is equivalent to "femto" so 18 fs = 18 femtoseconds = 18×10^{-15} s = 1.8×10^{-14} s
Remember that in scientific notation the first number should be smaller than 10.

(c) 10^{-12} is equivalent to "pico" so 128 pm = 128×10^{-12} m = 1.28×10^{-10} m
Remember that in scientific notation the first number should be smaller than 10.

(d) 10^{-6} is equivalent to "micro" so 35 μm = 35 micrograms = 35×10^{-6} g = 3.5×10^{-5} m
Remember that in scientific notation the first number should be smaller than 10.

1.34 Use Table 1.2 to determine the appropriate prefix multiplier and substitute the meaning into the expressions.

(a) "μ" is equivalent to "micro" or 10^{-6} so 35 μL = 35 microlitres = 35×10^{-6} L = 3.5×10^{-5} L
Remember that in scientific notation the first number should be smaller than 10.

(b) "M" is equivalent to "mega" or 10^{6} so 225 Mm = 225 megametres = 225×10^{6} m = 2.25×10^{8} m
Remember that in scientific notation the first number should be smaller than 10.

(c) "T" is equivalent to "tera" or 10^{12} so 133 Tg = 133 teragrams = 133×10^{12} g = 1.33×10^{14} g
Remember that in scientific notation the first number should be smaller than 10.

(d) "c" is equivalent to "centi" or 10^{-2} so 1.5 cg = 1.5 centigrams = 1.5×10^{-2} g

1.35 (b) **Given:** 515 km **Find:** dm

Conceptual Plan: $\mathbf{km} \xrightarrow{\frac{1000\ m}{1\ km}} \mathbf{m} \xrightarrow{\frac{10\ dm}{1\ m}} \mathbf{dm}$

Solution: $515\ \cancel{km} \times \frac{1000\ \cancel{m}}{1\ \cancel{km}} \times \frac{10\ dm}{1\ \cancel{m}} = 5.15 \times 10^{6}\ dm$

Check: The units (dm) are correct. The magnitude of the answer (10^{6}) makes physical sense because a decimetre is a much smaller unit than a kilometre.

Given: 515 km **Find:** cm

Conceptual Plan: $\mathbf{km} \xrightarrow{\frac{1000\ m}{1\ km}} \mathbf{m} \xrightarrow{\frac{100\ cm}{1\ m}} \mathbf{cm}$

Solution: $515\ \cancel{\text{km}} \times \dfrac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}} \times \dfrac{100\ \text{cm}}{1\ \cancel{\text{m}}} = 5.15 \times 10^{7}\ \text{cm}$

Check: The units (cm) are correct. The magnitude of the answer (10^7) makes physical sense because a centimetre is a much smaller unit than either a kilometre or a decimetre.

(c) **Given:** 122.355 s **Find:** ms

Conceptual Plan: s $\rightarrow$ ms

$\dfrac{1000\ \text{ms}}{1\ \text{s}}$

Solution: $122.355\ \cancel{\text{s}} \times \dfrac{1000\ \text{ms}}{1\ \cancel{\text{s}}} = 1.22355 \times 10^{5}\ \text{ms}$

Check: The units (ms) are correct. The magnitude of the answer (10^5) makes physical sense because a millisecond is a much smaller unit than a second.

Given: 122.355 s **Find:** ks

Conceptual Plan: s $\rightarrow$ ks

$\dfrac{1\ \text{ks}}{1000\ \text{s}}$

Solution: $122.355\ \cancel{\text{s}} \times \dfrac{1\ \text{ks}}{1000\ \cancel{\text{s}}} = 1.22355 \times 10^{-1}\ \text{ks} = 0.122355\ \text{ks}$

Check: The units (ks) are correct. The magnitude of the answer (10^{-1}) makes physical sense because a kilosecond is a much larger unit than a second.

(d) **Given:** 3.345 kJ **Find:** J

Conceptual Plan: kJ $\rightarrow$ J

$\dfrac{1000\ \text{J}}{1\ \text{kJ}}$

Solution: $3.345\ \cancel{\text{kJ}} \times \dfrac{1000\ \text{J}}{1\ \cancel{\text{kJ}}} = 3.345 \times 10^{3}\ \text{J}$

Check: The units (J) are correct. The magnitude of the answer (10^3) makes physical sense because a joule is a much smaller unit than a kilojoule.

Given: 3.345×10^3 J (from above) **Find:** mJ

Conceptual Plan: J $\rightarrow$ mJ

$\dfrac{1000\ \text{mJ}}{1\ \text{J}}$

Solution: $3.345 \times 10^{3}\ \cancel{\text{J}} \times \dfrac{1000\ \text{mJ}}{1\ \cancel{\text{J}}} = 3.345 \times 10^{6}\ \text{mJ}$

Check: The units (mJ) are correct. The magnitude of the answer (10^6) makes physical sense because a millijoule is a much smaller unit than a joule.

1.36 (a) **Given:** 355 km s^{-1} **Find:** cm s^{-1}

Conceptual Plan: km s^{-1} $\rightarrow$ m s^{-1} $\rightarrow$ cm s^{-1}

$\dfrac{1000\ \text{m}}{1\ \text{km}}$ $\dfrac{100\ \text{cm}}{1\ \text{m}}$

Solution: $\dfrac{355\ \cancel{\text{km}}}{1\ \text{s}} \times \dfrac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}} \times \dfrac{100\ \text{cm}}{1\ \cancel{\text{m}}} = 3.55 \times 10^{7}\ \text{cm s}^{-1}$

Check: The units (cm s^{-1}) are correct. The magnitude of the answer (10^7) makes physical sense because a centimetre is a much smaller unit than a kilometre.

Given: 355 km s^{-1} **Find:** m ms^{-1}

Conceptual Plan: km s^{-1} $\rightarrow$ m s^{-1} $\rightarrow$ m ms^{-1}

$\dfrac{1000\ \text{m}}{1\ \text{km}}$ $\dfrac{1\ \text{s}}{1000\ \text{ms}}$

Solution: $\dfrac{355\ \cancel{\text{km}}}{1\ \cancel{\text{s}}} \times \dfrac{\cancel{1000}\ \text{m}}{1\ \cancel{\text{km}}} \times \dfrac{1\ \cancel{\text{s}}}{\cancel{1000}\ \text{ms}} = 355\ \text{m ms}^{-1}$

Check: The units (m ms^{-1}) are correct. The magnitude of the answer (10^2) makes physical sense because the conversion to metres increases the magnitude by a factor of 1000 and the conversion from seconds to milliseconds decreases the magnitude by a factor of 1000.

(b) **Given:** 1228 g L^{-1} **Find:** g mL^{-1}

Conceptual Plan: g L^{-1} $\rightarrow$ g mL^{-1}

$$\frac{1\ \text{L}}{1000\ \text{mL}}$$

Solution: $\dfrac{1228\ \text{g}}{1\ \cancel{\text{L}}} \times \dfrac{1\ \cancel{\text{L}}}{1000\ \text{mL}} = 1.228\ \text{g mL}^{-1}$

Check: The units (g mL^{-1}) are correct. The magnitude of the answer (1) makes physical sense because a millilitre is a much smaller unit than a litre.

Given: 1228 g L^{-1} **Find:** kg ML^{-1}

Conceptual Plan: g L^{-1} $\rightarrow$ kg L^{-1} $\rightarrow$ kg ML^{-1}

$$\frac{1\ \text{kg}}{1000\ \text{g}} \qquad \frac{10^6\ \text{L}}{1\ \text{ML}}$$

Solution: $\dfrac{1228\ \cancel{\text{g}}}{1\ \cancel{\text{L}}} \times \dfrac{1\ \text{kg}}{1000\ \cancel{\text{g}}} \times \dfrac{10^6\ \cancel{\text{L}}}{1\ \text{ML}} = 1.228 \times 10^6\ \text{kg ML}^{-1}$

Check: The units (g ML^{-1}) are correct. The magnitude of the answer (10^6) makes physical sense because the conversion to kilograms decreases the magnitude by a factor of 1000 and the conversion from litres to megalitres increases the magnitude by a factor of 10^6.

(c) **Given:** 556 mK s^{-1} **Find:** K s^{-1}

Conceptual Plan: mK s^{-1} $\rightarrow$ K s^{-1}

$$\frac{1\ \text{K}}{1000\ \text{mK}}$$

Solution: $\dfrac{556\ \cancel{\text{mK}}}{1\ \text{s}} \times \dfrac{1\ \text{K}}{1000\ \cancel{\text{mK}}} = 0.556\ \text{K s}^{-1}$

Check: The units (K s^{-1}) are correct. The magnitude of the answer (10^{-1}) makes physical sense because a millikelvin is a much smaller unit than a kelvin.

Given: 556 mK s^{-1} **Find:** μK ms^{-1}

Conceptual Plan: mK s^{-1} $\rightarrow$ K s^{-1} $\rightarrow$ μK s^{-1} $\rightarrow$ μK ms^{-1}

$$\frac{1\ \text{K}}{1000\ \text{mK}} \qquad \frac{10^{-6}\ \mu\text{K}}{1\ \text{K}} \qquad \frac{1\ \text{s}}{1000\ \text{ms}}$$

Solution: $\dfrac{556\ \cancel{\text{mK}}}{1\ \text{s}} \times \dfrac{1\ \cancel{\text{K}}}{1000\ \cancel{\text{mK}}} \times \dfrac{10^6\ \mu\text{K}}{1\ \cancel{\text{K}}} \times \dfrac{1\ \cancel{\text{s}}}{1000\ \text{ms}} = 556\ \mu\text{K ms}^{-1}$

Check: The units (μK ms^{-1}) are correct. The magnitude of the answer (10^2) makes physical sense because the conversion to microkelvins increases the magnitude by a factor of 1000 and the conversion from seconds to milliseconds decreases the magnitude by a factor of 1000.

(d) **Given:** 2.554 mg mL^{-1} **Find:** g L^{-1}

Conceptual Plan: mg mL^{-1} $\rightarrow$ g mL^{-1} $\rightarrow$ g L^{-1}

$$\frac{1\ \text{g}}{1000\ \text{mg}} \qquad \frac{1000\ \text{mL}}{1\ \text{L}}$$

Solution: $\dfrac{2.554\ \cancel{\text{mg}}}{1\ \text{mL}} \times \dfrac{1\ \text{g}}{1000\ \cancel{\text{mg}}} \times \dfrac{1000\ \cancel{\text{mL}}}{1\ \text{L}} = 2.554\ \text{g L}^{-1}$

Check: The units (g L^{-1}) are correct. The magnitude of the answer (3) makes physical sense because the conversion to grams decreases the magnitude by a factor of 1000 and the conversion from millilitres to litres increases the magnitude by a factor of 1000.

Given: 2.544 mg mL^{-1} **Find:** μg mL^{-1}

Conceptual Plan: mg mL^{-1} $\rightarrow$ g mL^{-1} $\rightarrow$ μg mL^{-1}

$$\frac{1\ \text{g}}{1000\ \text{mg}} \qquad \frac{10^6\ \mu\text{g}}{1\ \text{g}}$$

Solution: $\dfrac{2.554\ \cancel{\text{mg}}}{1\ \text{mL}} \times \dfrac{1\ \cancel{\text{g}}}{1000\ \cancel{\text{mg}}} \times \dfrac{10^6\ \mu\text{g}}{1\ \cancel{\text{g}}} = 2.554 \times 10^3\ \mu\text{g mL}^{-1}$

Check: The units (μg mL^{-1}) are correct. The magnitude of the answer (10^3) makes physical sense because a microgram is a much smaller unit than a milligram.

1.37 (a) **Given:** 254 998 m **Find:** km

Conceptual Plan: **m → km**

$\dfrac{1\ \text{km}}{1000\ \text{m}}$

Solution: $254\,998\ \cancel{\text{m}} \times \dfrac{1\ \text{km}}{1000\ \cancel{\text{m}}} = 2.54998 \times 10^2\ \text{km} = 254.998\ \text{km}$

Check: The units (km) are correct. The magnitude of the answer (10^2) makes physical sense because a kilometre is a much larger unit than a metre.

(b) **Given:** 254 998 m **Find:** Mm

Conceptual Plan: **m → Mm**

$\dfrac{1\ \text{Mm}}{10^6\ \text{m}}$

Solution: $254\,998\ \cancel{\text{m}} \times \dfrac{1\ \text{Mm}}{10^6\ \cancel{\text{m}}} = 2.54998 \times 10^{-1}\ \text{Mm} = 0.254998\ \text{Mm}$

Check: The units (Mm) are correct. The magnitude of the answer (10^{-1}) makes physical sense because a megametre is a much larger unit than a metre or kilometre.

(c) **Given:** 254 998 m **Find:** mm

Conceptual Plan: **m → mm**

$\dfrac{1\,000\ \text{mm}}{1\ \text{m}}$

Solution: $254\,998\ \cancel{\text{m}} \times \dfrac{1000\ \text{mm}}{1\ \cancel{\text{m}}} = 2.54998 \times 10^8\ \text{mm}$

Check: The units (mm) are correct. The magnitude of the answer (10^8) makes physical sense because a millimetre is a much smaller unit than a metre.

(d) **Given:** 254 998 m **Find:** cm

Conceptual Plan: **m → cm**

$\dfrac{100\ \text{cm}}{1\ \text{m}}$

Solution: $254\,998\ \cancel{\text{m}} \times \dfrac{100\ \text{cm}}{1\ \cancel{\text{m}}} = 2.54998 \times 10^7\ \text{cm}$

Check: The units (cm) are correct. The magnitude of the answer (10^7) makes physical sense because a centimetre is a much smaller unit than a metre, but larger than a millimetre.

1.38 (a) **Given:** 556.2×10^{-12} s **Find:** ms

Conceptual Plan: **s → ms**

$\dfrac{1000\ \text{ms}}{1\ \text{s}}$

Solution: $556.2 \times 10^{-12}\ \cancel{\text{s}} \times \dfrac{1000\ \text{ms}}{1\ \cancel{\text{s}}} = 5.562 \times 10^{-7}\ \text{ms}$

Check: The units (ms) are correct. The magnitude of the answer (10^{-7}) makes physical sense because a millisecond is a much smaller unit than a second.

(b) **Given:** 556.2×10^{-12} s **Find:** ns

Conceptual Plan: **s → ns**

$\dfrac{10^9\ \text{ns}}{1\ \text{s}}$

Solution: $556.2 \times 10^{-12} \cancel{\text{s}} \times \dfrac{10^9 \text{ ns}}{1 \cancel{\text{s}}} = 0.5562 \text{ ns}$

Check: The units (ns) are correct. The magnitude of the answer (0.6) makes physical sense because a nanosecond is a much smaller unit than a second.

(c) **Given:** 556.2×10^{-12} s **Find:** ps

Conceptual Plan: s $\rightarrow$ ps

$\dfrac{10^{12} \text{ ps}}{1 \text{ s}}$

Solution: $556.2 \times 10^{-12} \cancel{\text{s}} \times \dfrac{10^{12} \text{ ps}}{1 \cancel{\text{s}}} = 556.2 \text{ ps}$

Check: The units (ps) are correct. The magnitude of the answer (10^2) makes physical sense because a picosecond is a much smaller unit than a second.

(d) **Given:** 556.2×10^{-12} s **Find:** fs

Conceptual Plan: s $\rightarrow$ fs

$\dfrac{10^{15} \text{ fs}}{1 \text{ s}}$

Solution: $556.2 \times 10^{-12} \cancel{\text{s}} \times \dfrac{10^{15} \text{ fs}}{1 \cancel{\text{s}}} = 5.562 \times 10^5 \text{ fs}$

Check: The units (fs) are correct. The magnitude of the answer (10^5) makes physical sense because a femtosecond is a much smaller unit than a second.

1.39 **Given:** 1 m square 1 m^2 **Find:** number of 1 cm squares

Conceptual Plan: 1 m^2 $\rightarrow$ cm^2

$\dfrac{100 \text{ cm}}{1 \text{ m}}$

Notice that for squared units, the conversion factors must be squared.

Solution: 100 cm × 100 cm = 10,000 cm^2 = 10,000 1 cm squares

Check: The units of the answer are correct and the magnitude makes sense. The unit centimetre is smaller than a metre, so the value in square centimetres should be larger than in square metres.

1.40 **Given:** 4 cm on each edge cube **Find:** number of 1 cm cubes

Conceptual Plan: Read the information given carefully. The cube is 4 cm on each side.

$$l, w, h \rightarrow V$$
$$V = l\,w\,h$$
$$\text{in a cube } l = w = h$$

Solution: 4 cm × 4 cm × 4 cm = $(4 \text{ cm})^3$ = $\underline{6}4$ cm^3 = 64 × 1 cm cubes

Check: The units of the answer are correct and the magnitude makes sense. The unit 4 centimetres is larger than 1 centimetre, so the value in cubic centimetres should be larger.

Density

1.41 **Given:** mass and volume of penny, 2.35 g and 0.302 cm^3 respectively.

Find: Density of penny.

Conceptual Plan: $d_{penny} = m / V$, **compare to** d_{Cu}

Solution: $d_{penny} = \dfrac{m}{V} = \dfrac{2.35 \text{ g}}{0.302 \text{ cm}^3} = 7.78 \text{ g cm}^{-3}$

The density of pure copper is 8.96 g cm^{-3}. Therefore, this penny is not pure copper (and more likely, it is simply copper plated).

Check: The answer makes sense because the coin would have to be heavier to be copper. No pennies have been made of pure copper in several decades. A copper coin weighing 2.35 g contains roughly 2.5¢ of Cu. This penny is more likely copper plated.

1.42 **Given:** m = 1.41 kg, V = 0.314 L **Find:** d in g cm^{-3} and compare to pure titanium.

Conceptual Plan: $m, V \rightarrow d$ **then kg** $\rightarrow$ **g then L** $\rightarrow$ **cm³**

$$d = m/V \qquad \frac{1000\text{ g}}{1\text{ kg}} \qquad \frac{1000\text{ cm}^3}{1\text{ L}}$$

Solution: $d = \dfrac{1.41\ \cancel{\text{kg}}}{0.314\text{ L}} \times \dfrac{\cancel{1000}\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \times \dfrac{1\text{ L}}{\cancel{1000}\ \cancel{\text{cm}}^3} = 4.49\text{ g cm}^{-3}$

Check: The units (g cm^{-3}) are correct. The magnitude of the answer seems correct. The density of the frame is almost exactly the density of pure titanium (4.49 g cm^{-3} versus 4.51 g cm^{-3}).

1.43 **Given:** $m = 4.10 \times 10^3$ g, V = 3.25 L **Find:** d in g cm^{-3}

Conceptual Plan: $m, V \rightarrow d$ **then L** $\rightarrow$ **cm³**

$$d = m/V \qquad \frac{1000\text{ cm}^3}{1\text{ L}}$$

Solution: $d = \dfrac{4.10 \times 10^3\text{ g}}{3.25\ \cancel{\text{L}}} \times \dfrac{1\ \cancel{\text{L}}}{1000\text{ cm}^3} = 1.26\text{ g cm}^{-3}$

Check: The units (g cm^{-3}) are correct. The magnitude of the answer seems correct.

1.44 **Given:** m = 371 grams, V = 19.3 mL **Find:** d in g cm^{-3} and compare to pure gold.

Conceptual Plan: $m, V \rightarrow d$ $\quad d = m/V$

Compare to the published value. d (pure gold) = 19.3 g mL^{-1} (This value is in Table 1.4.)

Solution: $d = \dfrac{371\text{ g}}{19.3\text{ mL}} = 19.2\text{ g cm}^{-3}$

The density of the nugget is essentially the same as the density of pure gold (19.2 g mL^{-1} versus 19.3 g mL^{-1}) so the nugget could be gold.

Check: The units (g cm^{-3}) are correct. The magnitude of the answer seems correct and is essentially the same as the density of pure gold.

1.45 (a) **Given:** d = 1.11 g cm^{-3}, V = 417 mL **Find:** m

Conceptual Plan: $d, V \rightarrow m$ **then cm³** $\rightarrow$ **mL**

$$d = m/V \qquad \frac{1\text{ mL}}{1\text{ cm}^3}$$

Solution: $d = m / V$ Rearrange by multiplying both sides of equation by V. $\quad m = d \times V$

$$m = 1.11 \frac{\text{g}}{\cancel{\text{cm}}^3} \times \frac{1\ \cancel{\text{cm}}^3}{1\ \cancel{\text{mL}}} \times 417\ \cancel{\text{mL}} = 4.63 \times 10^2\text{ g}$$

Check: The units (g) are correct. The magnitude of the answer seems correct considering the value of the density is about 1 g cm^{-3}.

(b) **Given:** d = 1.11 g cm^{-3}, m = 4.1 kg **Find:** V in L

Conceptual Plan: $d, V \rightarrow m$ **then kg** $\rightarrow$ **g and cm³** $\rightarrow$ **L**

$$d = m/V \qquad \frac{1000\text{ g}}{1\text{ kg}} \qquad \frac{1\text{ L}}{1000\text{ cm}^3}$$

Solution: $d = m/V$ Rearrange by multiplying both sides of equation by V and dividing both sides of the equation by d.

$$V = \frac{m}{d} = \frac{4.1\ \cancel{\text{kg}}}{1.11\ \cancel{\text{g}}\text{ cm}^{-3}} \times \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} = 3.7 \times \cancel{10}^3\ \cancel{\text{cm}}^3 \times \frac{1\text{ L}}{\cancel{1000}\ \cancel{\text{cm}}^3} = 3.7\text{ L}$$

Check: The units (L) are correct. The magnitude of the answer seems correct considering the value of the density is about 1 g cm^{-3}.

1.46 (a) **Given:** d = 0.7857 g cm^{-3}, V = 28.56 mL **Find:** m

Conceptual Plan: $d, V \rightarrow m$

$$d = m/V$$

Solution: $d = m/V$ Rearrange by multiplying both sides of equation by V. $m = d \times V$

$$m = (0.7857\ \text{g}\ \cancel{\text{cm}^{-3}}) \times \frac{1\ \cancel{\text{cm}^3}}{1\ \cancel{\text{mL}}} \times (28.56\ \cancel{\text{mL}}) = 22.44\ \text{g}$$

Check: The units (g) are correct. The magnitude of the answer seems correct considering the value of the density is less than 1 g cm^{-3}.

(b) **Given:** d = 0.7857 g cm^{-3}, m = 6.54 g **Find:** V

Conceptual Plan: $d, m \rightarrow V$ **then cm^3 $\rightarrow$ mL**

$$d = m/V \qquad \frac{1\ \text{mL}}{1\ \text{cm}^3}$$

Solution: $d = m/V$ Rearrange by multiplying both sides of equation by V and dividing both sides of the equation by d.

$$V = \frac{m}{d} = \frac{6.54\ \cancel{\text{g}}}{0.7857\ \cancel{\text{g}}\ \text{cm}^{-3}} = 8.32\ \cancel{\text{cm}^3} \times \frac{1\ \text{mL}}{1\ \cancel{\text{cm}^3}} = 8.32\ \text{mL}$$

Check: The units (mL) are correct. The magnitude of the answer seems correct considering the value of the density is less than 1 g cm^{-3}.

1.47 **Given:** V = 245 L, d = 0.803 g mL^{-1} **Find:** m

Conceptual Plan: g mL^{-1} $\rightarrow$ g L^{-1} then $d, V \rightarrow m$

$$\frac{1000\ \text{mL}}{1\ \text{L}} \qquad d = m/V$$

Solution: $d = m/V$ Rearrange by multiplying both sides of equation by V. $m = d \times V$

$$m = 245\ \cancel{\text{L}} \times \frac{1000\ \cancel{\text{mL}}}{1\ \cancel{\text{L}}} \times (0.803\ \text{g}\ \cancel{\text{mL}^{-1}}) = 1.97 \times 10^5\ \text{g} = 1.97 \times 10^2\ \text{kg}$$

Check: The units (g) are correct. The magnitude of the answer seems correct considering the value of the density is less than 1 g mL^{-1} and the volume is very large.

1.48 **Given:** d_{fat} = 0.918 g cm^{-3}, m_{fat} = 5.00 kg **Find:** V_{fat}

Conceptual Plan: kg $\rightarrow$ g, $V = m/d$

Solution: $m_{fat} = 5.00\ \text{kg} \times \dfrac{1000\ \text{g}}{1\ \text{kg}} = 5.00 \times 10^3\ \text{g}$

$$V = \frac{m}{d} = \frac{5.00 \times 10^3\ \text{g}}{0.918\ \text{g cm}^{-3}} = 5450\ \text{cm}^3$$

Check: The answer is in correct units. Because fat density is slightly below 1.00, the expected volume for 5000 g would be slightly more than 5000 cm^3, which is what is obtained.

The Reliability of a Measurement and Significant Figures

1.49 In order to obtain the readings, look to see where the bottom of the meniscus lies. Estimate the distance between two markings on the device.

(a) 73.5 mL – the meniscus appears to be sitting between the 73 mL mark and the 74 mL mark.

(b) 88.2 °C – the mercury is between the 88 °C mark and the 89 °C mark, but it is closer to the lower number.

(c) 645 mL – the meniscus appears to be just above the 640 mL mark.

1.50 In order to obtain the readings, look to see where the bottom of the meniscus lies. Estimate the distance between two markings on the device. Use all digits on a digital device.

(a) 4.50 mL – the meniscus appears to be on the 4.5 mL mark.

(b) 27.43 °C – the mercury is just above the 27.4 °C mark. Note that the 10 s digit is only labelled every 10 °C.

(c) 0.873 g – read all the places on the digital display.

1.51 Remember that

1. interior zeroes (zeroes between two numbers) are significant.
2. leading zeroes (zeroes to the left of the first nonzero number) are not significant. They only serve to locate the decimal point.
3. trailing zeroes (zeroes at the end of a number) are categorized as follows:
   - Trailing zeroes after a decimal point are always significant.
   - Trailing zeroes before an implied decimal point are ambiguous and should be avoided by using scientific notation or by inserting a decimal point at the end of the number.

(a) $1\,\underline{0}5\underline{0}\,5\underline{0}1$ km

(b) $\cancel{0}.\cancel{0}\cancel{0}2\underline{0}$ m

(c) $\cancel{0}.\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}\cancel{0}2$ s

(d) $\cancel{0}.\cancel{0}\cancel{0}1\underline{0}9\underline{0}$ cm

1.52 Remember that

1. interior zeroes (zeroes between two numbers) are significant.
2. leading zeroes (zeroes to the left of the first nonzero number) are not significant. They only serve to locate the decimal point.
3. trailing zeroes (zeroes at the end of a number) are categorized as follows:
   - Trailing zeroes after a decimal point are always significant.
   - Trailing zeroes before an implied decimal point are ambiguous and should be avoided by using scientific notation or by inserting a decimal point at the end of the number.

(a) $18\underline{0}\,7\underline{0}1$ mi

(b) $\cancel{0}.\cancel{0}\cancel{0}1\underline{0}4\underline{0}$ m

(c) $\cancel{0}.\cancel{0}\cancel{0}571\underline{0}$ km

(d) $9\underline{0}\,2\underline{0}1$ m

1.53 Remember all of the rules from Section 1.7.

(a) Three significant figures. The 3, 1, and the 2 are significant (rule 1). The leading zeroes only mark the decimal place and are therefore not significant (rule 3).

(b) Ambiguous. The 3, 1, and the 2 are significant (rule 1). The trailing zeroes occur before an implied decimal point and are therefore ambiguous (rule 4). Without more information, we would assume 3 significant figures. It is better to write this as 3.12×10^5 to indicate three significant figures or as 3.12000×10^5 to indicate six (rule 4).

(c) Three significant figures. The 3, 1, and the 2 are significant (rule 1).

(d) Five significant figures. The 1s, 3, 2, and 7 are significant (rule 1).

(e) Ambiguous. The 2 is significant (rule 1). The trailing zeroes occur before an implied decimal point and are therefore ambiguous (rule 4). Without more information, we would assume one significant figure. It is better to write this as 2×10^3 to indicate one significant figure or as 2.000×10^3 to indicate four (rule 4).

1.54 Remember all of the rules from Section 1.7.

(a) Four significant figures. The 1s are significant (rule 1). The leading zeroes only mark the decimal place and are therefore not significant (rule 3).

(b) One significant figure. The 7 is significant (rule 1). The leading zeroes only mark the decimal place and are therefore not significant (rule 3).

(c) Ambiguous. The 1, 8, and the 7 are significant (rule 1). The first 0 is significant, since it is an interior 0 (rule 2). The trailing zeroes occur before an implied decimal point and are therefore ambiguous (rule 4). Without more information, we would assume four significant figures. It is better to write this as 1.087×10^5 to indicate four significant figures or as 1.08700×10^5 to indicate six (rule 4).

(d) Seven significant figures. The 1, 5, 6, and 3s are significant (rule 1). The trailing zeroes are significant because they are to the right of the decimal point and nonzero numbers (rule 4).

(e) Ambiguous. The 3 and 8 are significant (rule 1). The first 0 is significant because the first one is an interior zero. The trailing zeroes occur before an implied decimal point and are therefore ambiguous (rule 4). Without more information, we would assume three significant figures. It is better to write this as 3.08×10^4 to indicate three significant figures or as 3.0800×10^4 to indicate five (rule 4).

1.55 (a) $\pi = 3.14$: 3 significant figures

(b) 1 m^3 = 1000 dm^3: exact number (by definition), unlimited number of significant figures

(c) 5683.91 km^2: 6 significant figures

(d) 3.0×10^8 m s^{-1}: 2 significant figures

1.56 (a) 12 = 1 dozen: exact number, unlimited number of significant figures

(b) 11.4 g cm^{-3} (density of lead): 3 significant figures

(c) 2.42×10^{19} km: 3 significant figures

(d) 1.64×10^3 km^3: 3 significant figures

1.57 (a) 156.9 – The 8 is rounded up since the next digit is a 5.

(b) 156.8 – The last two digits are dropped since 4 is less than 5.

(c) 156.8 – The last two digits are dropped since 4 is less than 5.

(d) 156.9 – The 8 is rounded up since the next digit is a 9, which is greater than 5.

1.58 (a) 7.98×10^4 – The last digits are dropped since 4 is less than 5.

(b) 1.55×10^7 – The 8 is rounded up since the next digit is a 9, which is greater than 5.

(c) 2.35 – The 4 is rounded up since the next digit is a 9, which is greater than 5.

(d) 4.54×10^{-5} – The 3 is rounded up since the next digit is an 8, which is greater than 5.

Significant Figures in Calculations

1.59 (a) $9.15 \div 4.970 = 1.84$ – Three significant figures are allowed to reflect the three significant figures in the least precisely known quantity (9.15).

(b) $1.54 \times 0.03060 \times 0.69 = 0.033$ – Two significant figures are allowed to reflect the two significant figures in the least precisely known quantity (0.69). The intermediate answer (0.03251556) is rounded up since the first nonsignificant digit is a 5.

(c) $27.5 \times 1.82 \div 100.04 = 0.500$ – Three significant figures are allowed to reflect the three significant figures in the least precisely known quantity (27.5 and 1.82). The intermediate answer (0.50029988) is truncated since the first nonsignificant digit is a 2, which is less than 5.

(d) $(2.290 \times 10^6) \div (6.7 \times 10^4) = 34$ – Two significant figures are allowed to reflect the two significant figures in the least precisely known quantity (6.7×10^4). The intermediate answer (34.17910448) is truncated since the first nonsignificant digit is a 1, which is less than 5.

1.60 (a) $89.3 \times 77.0 \times 0.08 = 6 \times 10^2$ – One significant figure is allowed to reflect the one significant figure in the least precisely known quantity (0.08). The intermediate answer (5.50088×10^2) is rounded up since the first nonsignificant digit is a 5.

(b) $(5.01 \times 10^5) \div (7.8 \times 10^2) = 6.4 \times 10^2$ – Two significant figures are allowed to reflect the two significant figures in the least precisely known quantity (7.8×10^2). The intermediate answer (6.423076923×10^2) is truncated since the first nonsignificant digit is a 2, which is less than 5.

(c) $4.005 \times 74 \times 0.007 = 2$ – One significant figure is allowed to reflect the one significant figure in the least precisely known quantity (0.007). The intermediate answer (2.07459) is truncated since the first nonsignificant digit is a 0, which is less than 5.

(d) $453 \div 2.031 = 223$ – Three significant figures are allowed to reflect the three significant figures in the least precisely known quantity (453). The intermediate answer (223.042836) is truncated since the first nonsignificant digit is a 0, which is less than 5.

1.61 (a)

$$\begin{array}{r} 43.7 \\ \underline{-\,2.341} \\ 41.359 = 41.4 \end{array}$$

Round the intermediate answer to one decimal place to reflect the quantity with the fewest decimal places (43.7). Round the last digit up since the first nonsignificant digit is 5.

(b)

$$\begin{array}{l} \;\;17.6 \\ \;+2.838 \\ \;+2.3 \\ \underline{+110.77} \\ \;133.508 = 133.5 \end{array}$$

Round the intermediate answer to one decimal place to reflect the quantity with the fewest decimal places (2.3). Truncate nonsignificant digits since the first nonsignificant digit is 0.

(c)

$$\begin{array}{l} \;19.6 \\ +\,58.33 \\ \underline{\;-\,4.974} \\ \;72.956 = 73.0 \end{array}$$

Round the intermediate answer to one decimal place to reflect the quantity with the fewest decimal places (19.6). Round the last digit up since the first nonsignificant digit is 5.

(d)

$$\begin{array}{l} \;5.99 \\ \underline{-\,5.572} \\ \;0.418 = 0.42 \end{array}$$

Round the intermediate answer to two decimal places to reflect the quantity with the fewest decimal places (5.99). Round the last digit up since the first nonsignificant digit is 8.

1.62 (a)
$$\begin{array}{r} 0.004 \\ +\,0.09879 \\ \hline 0.10279 = 0.103 \end{array}$$

Round the intermediate answer to three decimal places to reflect the quantity with the fewest decimal places (0.004). Round the last digit up since the first nonsignificant digit is 9.

(b)
$$\begin{array}{r} 1239.3 \\ +9.73 \\ +3.42 \\ \hline 1252.45 = 1252.5 \end{array}$$

Round the intermediate answer to one decimal place to reflect the quantity with the fewest decimal places (1239.3). Round the last digit up since the first nonsignificant digit is 5.

(c)
$$\begin{array}{r} 2.4 \\ -1.777 \\ \hline 0.623 = 0.6 \end{array}$$

Round the intermediate answer to one decimal place to reflect the quantity with the fewest decimal places (2.4). Truncate nonsignificant digits since the first nonsignificant digit is 2.

(d)
$$\begin{array}{r} 532 \\ +7.3 \\ -48.523 \\ \hline 490.777 = 491 \end{array}$$

Round the intermediate answer to zero decimal places to reflect the quantity with the fewest decimal places (532). Round the last digit up since the first nonsignificant digit is 7.

1.63 The product of mass, m, and acceleration, a, is Force, F. Work (W) is when force is multiplied by distance, d, $W = m(\text{kg}) \times a(\text{m s}^{-2}) \times d(\text{m})$
The unit for Work, therefore, is kg mass^2 s^{-2}, or Joules (J).

1.64 Force, in units of Newtons, is when mass is the product of mass by acceleration:
$F = m(\text{kg}) \times a(\text{m s}^{-2})$, so Newtons has SI units of kg m s^{-2}.
Dividing force in newtons by area, we obtain the following:
N m^{-2} = kg m s^{-2} m^{-2} = kg s^{-2} m^{-1}
The force exerted on a unit area is pressure. The unit for pressure in SI units derived above is Pascals (Pa).

1.65 Perform operations in parentheses first. Keep track of significant figures in each step by noting which is the last significant digit in an intermediate result.

(a)
$$\begin{array}{r} (24.6681 \times 2.38) + 332.58 = 58.\underline{7}10078 \\ +332.58 \\ \hline 391.290078 = 391.3 \end{array}$$

The first intermediate answer has one significant digit to the right of the decimal, because it is allowed three significant figures (reflecting the quantity with the fewest significant figures (2.38)). Underline the most significant digit in this answer. Round the next intermediate answer to one decimal place to reflect the quantity with the fewest decimal places (58.7). Round the last digit up since the first nonsignificant digit is 9.

(b) $$\frac{(85.3-21.489)}{0.0059}=\frac{63.\underline{8}11}{0.0059}=1.\underline{0}81542\times10^{4}=1.1\times10^{4}$$

The first intermediate answer has one significant digit to the right of the decimal, to reflect the quantity with the fewest decimal places (85.3). Underline the most significant digit in this answer. Round the next intermediate answer to two significant figures to reflect the quantity with the fewest significant figures (0.0059). Round the last digit up since the first nonsignificant digit is 8.

(c) $$\begin{aligned}(512\div 986.7)+5.44 &= 0.51\underline{8}9014\\ &\underline{+5.44}\\ &5.9589014=5.96\end{aligned}$$

The first intermediate answer has three significant figures and three significant digits to the right of the decimal, reflecting the quantity with the fewest significant figures (512). Underline the most significant digit in this answer. Round the next intermediate answer to two decimal places to reflect the quantity with the fewest decimal places (5.44). Round the last digit up since the first nonsignificant digit is 8.

(d) $$\begin{aligned}[(28.7\times10^{5})\div 48.533]+144.99 &= 59\underline{1}35.01\\ &\underline{+\quad 144.99}\\ &59280.01=59300=5.93\times10^{4}\end{aligned}$$

The first intermediate answer has three significant figures, reflecting the quantity with the fewest significant figures (28.7×10^5). Underline the most significant digit in this answer. Since the number is so large this means that when the addition is performed, the most significant digit is the 100's place. Round the next intermediate answer to the 100's places and put in scientific notation to remove any ambiguity. Note that the last digit is rounded up since the first nonsignificant digit is 8.

1.66 Perform operations in parentheses first. Keep track of significant figures in each step, by noting which is the last significant digit in an intermediate result.

(a) $$\begin{aligned}[(1.7\times10^{6})\div(2.63\times10^{5})]+7.33 &= 6.\underline{4}63878\\ &\underline{+7.33}\\ &13.793878=13.8\end{aligned}$$

The first intermediate answer has one significant digit to the right of the decimal, because it is allowed two significant figures (reflecting the quantity with the fewest significant figures (1.7×10^6)). Underline the most significant digit in this answer. Round the final answer to one decimal place because the number with the fewest significant figures has one decimal place. Round the last digit up since the first nonsignificant digit is 9.

(b) $(568.99 - 232.1) \div 5.3 = 336.\underline{8}9 \div 5.3 = 63.564151 = 64$

The first intermediate answer has one significant digit to the right of the decimal, to reflect the quantity with the fewest decimal places (232.1). Underline the most significant digit in this answer. Round the next intermediate answer to two significant figures to reflect the quantity with the fewest significant figures (5.3). Round the last digit up since the first nonsignificant digit is 5.

(c) $(9443+45-9.9)\times8.1\times10^{6}=947\underline{8}.1\times8.1\times10^{6}=7.67726\times10^{10}=7.7\times10^{10}$

The first intermediate answer only has significant digits to the left of the decimal, reflecting the quantity with the fewest significant figures (9443 and 45). Underline the most significant digit in this answer. Round the next intermediate answer to two significant figures to reflect the quantity with the fewest significant figures (8.1×10^6). Round the last digit up since the first nonsignificant digit is 7.

(d) $(3.14 \times 2.4367) - 2.34 = 7.6\underline{5}1238$

$$\underline{-2.34}$$

$$5.311238 = 5.31$$

The first intermediate answer has three significant figures, reflecting the quantity with the fewest significant figures (3.14). Underline the most significant digit in this answer. This number has two significant digits to the right of the decimal point. Round the next intermediate answer to two significant digits to the right of the decimal point, since both numbers have two significant digits to the right of the decimal point. Note that the last digit is truncated since the first nonsignificant digit is 1.

Unit Conversions

1.67 (a) **Given:** 3.25 kg **Find:** g

Conceptual Plan: kg $\rightarrow$ g

Solution: $3.25\ \cancel{\text{kg}} \times \dfrac{1000\ \text{g}}{1\ \cancel{\text{kg}}} = 3.25 \times 10^3\ \text{g}$

Check: Unit of g is correct, and the magnitude makes sense because g is much smaller than kg.

(b) **Given:** 250 μs **Find:** s

Conceptual Plan: μs $\rightarrow$ s

Solution: $250\ \cancel{\mu\text{s}} \times \dfrac{1\ \text{s}}{1 \times 10^6\ \cancel{\mu\text{s}}} = 2.5 \times 10^{-4}\ \text{s}$

Check: Unit of s is correct, and the magnitude makes sense because μs is much smaller than s.

(c) **Given:** 0.345 L **Find:** cm^3

Conceptual Plan: L $\rightarrow$ cm^3

Solution: $0.345\ \cancel{\text{L}} \times \dfrac{1000\ \text{cm}^3}{1\ \cancel{\text{L}}} = 345\ \text{cm}^3$

Check: Unit of cm^3 is correct, and the magnitude makes sense because cm^3 is a thousand-fold smaller than L.

(d) **Given:** 257 dm **Find:** km

Conceptual Plan: 257 dm $\rightarrow$ m $\rightarrow$ km

Solution: $257\ \cancel{\text{dm}} \times \dfrac{1\ \cancel{\text{m}}}{10\ \cancel{\text{dm}}} \times \dfrac{1\ \text{km}}{1000\ \cancel{\text{m}}} = 0.0257\ \text{km}$

Check: The unit of km is correct. Consulting Table 1.2 of the text, deci is four orders of magnitude smaller than kilo, so the magnitude of the answer also makes sense.

1.68 (a) **Given:** 1405 μg **Find:** mg

Conceptual Plan: μg $\rightarrow$ g $\rightarrow$ mg

Solution: $1405\ \cancel{\mu\text{g}} \times \dfrac{1\ \cancel{\text{g}}}{1 \times 10^6\ \cancel{\mu\text{g}}} \times \dfrac{1000\ \text{mg}}{1\ \cancel{\text{g}}} = 1.405\ \text{mg}$

Check: Unit of mg is correct. Consulting Table 1.2, mg is larger than μg by three orders of magnitude, so the magnitude of the answer also makes sense.

(b) **Given:** 62 500 fs **Find:** ns

Conceptual Plan: fs $\rightarrow$ s $\rightarrow$ ns

Solution: $62\,500\ \cancel{\text{fs}} \times \dfrac{1\ \cancel{\text{s}}}{1 \times 10^{15}\ \cancel{\text{fs}}} \times \dfrac{1 \times 10^9\ \text{ns}}{1\ \cancel{\text{s}}} = 0.0625\ \text{ns}$

Check: The unit of ns is correct. Consulting Table 1.2 of the text, nano is six orders of magnitude larger than femto, so the magnitude of the answer also makes sense.

(c) **Given:** 1056 cm^2 **Find:** m^2
Conceptual Plan: cm^2 → m^2

Solution: $1056\ \cancel{cm}^2 \times \left(\frac{1\ m}{100\ \cancel{cm}}\right)^2 = 0.1056\ m^2$

Check: The unit of m^2 is correct. Since m is two orders of magnitude larger than cm, then m^2 must be four orders larger. Therefore, the magnitude of the answer also makes sense.

(d) **Given:** 25.0 mL **Find:** L
Conceptual Plan: mL → L

Solution: $25.0\ \cancel{mL} \times \frac{1\ L}{1000\ \cancel{mL}} = 0.0250\ L$

Check: The unit of litre is correct. A litre is three orders of magnitude larger than a mL, so the magnitude of the answer is correct.

1.69 **Given:** d = 10.0 km, v = 3.5 m s^{-1} **Find:** time in minutes
Conceptual Plan: (1) km → m, (2) Find time using $t = d/v$, (3) t(s) → t(min).

Solution: $d = 10.0\ km \times \frac{1000\ m}{1\ km} = 10\,000\ m$ (two significant figures)

$$t = \frac{d}{v} = \frac{10\,000\ m}{3.5\ m\ s^{-1}} = 2857.14\ s$$

$$t = 2857.14\ s \times \frac{1\ min}{60\ s} = 48\ min$$

Check: The unit is correct. A speed of 3.5 m s^{-1} is 210 m min^{-1}. To travel 10 000 m, the runner has to run roughly 50 minutes, which is very close to the answer. The final answer has two significant figures because the speed has two significant figures also.

1.70 **Given:** v = 8.1 m s^{-1}, d = 212 km **Find:** time in hours
Conceptual Plan: (1) km → m, (2) Find time using $t = d/v$, (3) t(s) → t(hour)

Solution: $d = 212\ km \times \frac{1000\ m}{1\ km} = 212\,000\ m$ (three significant figures)

$$t = \frac{d}{v} = \frac{212\,000\ m}{8.1\ m\ s^{-1}} = 26172.8\ s$$

$$t = 26172.8\ s \times \frac{1\ hr}{3600\ s} = 7.3\ hr$$

Check: The unit is correct. A speed of 8.1 m s^{-1} is roughly 29 000 m hr^{-1}. To travel 212 000 m takes roughly seven hours. The final answer has two significant figures.

1.71 (a) **Given:** 195 m^2 **Find:** km^2
Conceptual Plan: m^2 → km^2

$$\frac{(1\ km)^2}{(1000\ m)^2}$$

Notice that for squared units, the conversion factors must be squared.

Solution: $195\ \cancel{m}^2 \times \frac{(1\ km)^2}{(1000\ \cancel{m})^2} = 1.95 \times 10^{-4}\ km^2$

Check: The units (km^2) are correct. The magnitude of the answer (10^{-4}) makes physical sense because a kilometre is a much larger unit than a metre.

(b) **Given:** 195 m^2 **Find:** dm^2
Conceptual Plan: m^2 → dm^2

$$\frac{(10\ dm)^2}{(1\ m)^2}$$

Notice that for squared units, the conversion factors must be squared.

Solution: $195\ \cancel{m}^2 \times \frac{(10\ dm)^2}{(1\ \cancel{m})^2} = 1.95 \times 10^4\ dm^2$

Check: The units (dm^2) are correct. The magnitude of the answer (10^4) makes physical sense because a decimetre is a much smaller unit than a metre.

(c) **Given:** 195 m^2 **Find:** cm^2

Conceptual Plan: $\mathbf{m^2} \rightarrow \mathbf{cm^2}$

$$\frac{(100\ cm)^2}{(1\ m)^2}$$

Notice that for squared units, the conversion factors must be squared.

Solution: $195\ \cancel{m}^2 \times \frac{(100\ cm)^2}{(1\ \cancel{m})^2} = 1.95 \times 10^6\ cm^2$

Check: The units (cm^2) are correct. The magnitude of the answer (10^6) makes physical sense because a centimetre is a much smaller unit than a metre.

1.72 (a) **Given:** 115 m^3 **Find:** km^3

Conceptual Plan: $\mathbf{m^3} \rightarrow \mathbf{km^3}$

$$\frac{(1\ km)^3}{(1000\ m)^3}$$

Notice that for cubed units, the conversion factors must be cubed.

Solution: $115\ \cancel{m}^3 \times \frac{(1\ km)^3}{(1000\ \cancel{m})^3} = 1.15 \times 10^{-7}\ km^3$

Check: The units (km^3) are correct. The magnitude of the answer (10^{-7}) makes physical sense because a kilometre is a much larger unit than a metre.

(b) **Given:** 115 m^3 **Find:** dm^3

Conceptual Plan: $\mathbf{m^3} \rightarrow \mathbf{mm^3}$

$$\frac{(10\ dm)^3}{(1\ m)^3}$$

Notice that for cubed units, the conversion factors must be cubed.

Solution: $115\ \cancel{m}^3 \times \frac{(10\ dm)^3}{(1\ \cancel{m})^3} = 1.15 \times 10^5\ dm^3$

Check: The units (dm^3) are correct. The magnitude of the answer (10^5) makes physical sense because a decimetre is a much smaller unit than a metre.

(c) **Given:** 115 m^3 **Find:** cm^3

Conceptual Plan: $\mathbf{m^3} \rightarrow \mathbf{cm^3}$

$$\frac{(100\ cm)^3}{(1\ m)^3}$$

Notice that for cubed units, the conversion factors must be cubed.

Solution: $115\ \cancel{m}^3 \times \frac{(100\ cm)^3}{(1\ \cancel{m})^3} = 1.15 \times 10^8\ cm^3$

Check: The units (cm^3) are correct. The magnitude of the answer (10^8) makes physical sense because a centimetre is a much smaller unit than a metre.

1.73 **Given:** Area = 2.5×10^6 hm^2 **Find:** m^2 and km^2

Conceptual Plan: $\mathbf{hm^2 \rightarrow m^2 \rightarrow km^2}$

Solution: $2.5 \times 10^6 \text{ hm}^2 \times \left(\dfrac{100 \text{ m}}{1 \text{ hm}}\right)^2 = 2.5 \times 10^{10} \text{ m}^2$

$$2.5 \times 10^{10} \text{ m}^2 \times \left(\dfrac{1 \text{ km}}{1000 \text{ m}}\right)^2 = 2.5 \times 10^4 \text{ km}^2$$

Check: The units (m^2 and km^2) are correct. A hectametre is 100 times larger than a metre, so a hm^2 is 10^4 times larger, so the magnitude of the first answer makes sense. Similarly, a km^2 is 10^6 fold larger than m^2, which is what is expected for the second part of the answer.

1.74 To convert among the units of area, it is easiest to square the units of length.

$$1.8 \text{ km}^2 \times \left(\dfrac{10 \text{ hm}}{1 \text{ km}}\right)^2 = 180 \text{ hm}^2$$

$$1.8 \text{ km}^2 \times \left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right)^2 = 1.8 \times 10^6 \text{ m}^2$$

1.75 **Given:** 6.5 kg **Find:** mL **Other:** 80 mg/0.80 mL and 15 mg kg^{-1} body

Conceptual Plan: **kg body $\rightarrow$ mg $\rightarrow$ mL**

$$\frac{15 \text{ mg}}{1 \text{ kg body}} \qquad \frac{0.80 \text{ mL}}{80 \text{ mg}}$$

Solution: $6.5 \cancel{\text{ kg}} \times \dfrac{15 \cancel{\text{ mg}}}{1 \cancel{\text{ kg body}}} \times \dfrac{0.80 \text{ mL}}{80 \cancel{\text{ mg}}} = 0.975 \text{ mL} = 0.98 \text{ mL}$

Check: The units are correct. The magnitude of the answer (1 mL) makes physical sense because it is reasonable amount of liquid to give to a baby. Two significant figures are allowed because of the statement in the problem. Truncate the last digit because the first nonsignificant digit is a 2.

1.76 **Given:** 8.2 kg **Find:** mL **Other:** 100 mg/5.0 mL and 10 mg kg^{-1} body

Conceptual Plan: **kg body $\rightarrow$ mg $\rightarrow$ mL**

$$\frac{10 \text{ mg}}{1 \text{ kg body}} \qquad \frac{5.0 \text{ mL}}{100 \text{ mg}}$$

Solution: $8.2 \cancel{\text{ kg}} \times \dfrac{10 \cancel{\text{ mg}}}{1 \cancel{\text{ kg body}}} \times \dfrac{5.0 \text{ mL}}{100 \cancel{\text{ mg}}} = 4.\underline{0}81632653 \text{ mL} = 4.1 \text{ mL}$

Check: The units are correct. The magnitude of the answer (4 mL) makes physical sense because it is reasonable amount of liquid to give to a baby. Two significant figures are allowed because of the statement in the problem. Round up the last digit because the first nonsignificant digit is an 8.

Cumulative Problems

1.77 **Given:** solar year **Find:** seconds

Other: 60 seconds/minute; 60 minutes/hour; 24 hours/solar day; and 365.24 solar days/solar year

Conceptual Plan: **yr $\rightarrow$ day $\rightarrow$ hr $\rightarrow$ min $\rightarrow$ sec**

$$\frac{365.24 \text{ day}}{1 \text{ solar yr}} \quad \frac{24 \text{ hr}}{1 \text{ day}} \quad \frac{60 \text{ min}}{1 \text{ hr}} \quad \frac{60 \text{ sec}}{1 \text{ min}}$$

Solution: $1 \cancel{\text{ solar yr}} \times \dfrac{365.24 \cancel{\text{ day}}}{1 \cancel{\text{ solar yr}}} \times \dfrac{24 \cancel{\text{ hr}}}{1 \cancel{\text{ day}}} \times \dfrac{60 \cancel{\text{ min}}}{1 \cancel{\text{ hr}}} \times \dfrac{60 \text{ sec}}{1 \cancel{\text{ min}}} = 3.155\underline{6}736 \times 10^7 \text{ sec} = 3.1557 \times 10^7 \text{ sec}$

Check: The units (seconds) are correct. The magnitude of the answer (10^7) makes physical sense because each conversion factor increases the value of the answer—a second is many orders of magnitude smaller than a year. Five significant figures are allowed because all conversion factors are assumed to be exact, except for the 365.24 days/solar year (five significant figures). Round up the last digit because the first nonsignificant digit is a 7.

1.78 (a) "Million" translates to 10^6 in Table 1.2. Substitute this quantity into the expression and move the decimal point to be in proper scientific notation. Fifty million Frenchmen = 50×10^6 Frenchmen = 5×10^7 Frenchmen (assuming one significant figure in fifty).

(b) This can be expressed as two different ratios: 10 jokes / 100 enemies = 1.0×10^{-1} jokes per enemy or 100 enemies / 10 jokes = 1×10^1 enemies per joke.

(c) "Hundred" translates as 10^{-2} (since this modifies millionth, something less than 1) and "millionth" translates to 10^{-6} in Table 1.2. Substitute this quantity into the expression and move the decimal point to be in proper scientific notation. $1.8 \times 10^{-2} \times 10^{-6}$ cm = 1.8×10^{-8} cm

(d) "Thousand" translates to 10^3 in Table 1.2. Substitute this quantity into the expression and move the decimal point to be in proper scientific notation. Sixty thousand dollars = 60×10^3 dollars = 6×10^4 dollars (assuming one significant figure in sixty).

(e) The density of platinum (Table 1.4) = 21.4 g cm^{-3} = 2.14×10^1 g cm^{-3} moving the decimal point to be in proper scientific notation.

1.79 (a) Extensive – The volume of a material depends on how much there is present.

(b) Intensive – The boiling point of a material is independent of how much material you have, so these values can be published in reference tables.

(c) Intensive – The temperature of a material does not depend on how much there is present.

(d) Intensive – The electrical conductivity of a material is independent of how much material you have, so these values can be published in reference tables.

(e) Extensive – The energy contained in material depends on how much there is present. Many times energy is expressed in terms of Joules/mole, which then turns this quantity into an intensive property.

1.80 **Given:** 25 °C and –196 °C **Find:** why significant figures are 3 and 2, respectively.
Conceptual Plan: The problem is stated in units of °C, so it must be converted to another temperature unit to see if the significant figures can change. Try K. Begin by finding the equation that relates the quantity that is given (°C) and the quantity you are trying to find (K). $K = °C + 273.15$

K = 25 °C + 273.15 = 298 K $\rightarrow$ K = −196 °C + 273.15 = 77 K

A small positive temperature in °C gains significant figures because of the rules of addition for significant figures—going from 2 to 3. A very negative temperature in °C loses significant figures because of the rules of addition for significant figures—going from 3 to 2.

1.81 (a) $1.76 \times 10^{-3}/8.0 \times 10^2 = 2.2 \times 10^{-6}$. Two significant figures are allowed to reflect the quantity with the fewest significant figures (8.0×10^2).

(b) Write all figures so that the decimal points can be aligned:

  0.0187
+ 0.0002 All quantities are known to four places to the right of the decimal place,
− 0.0030 so the answer should be reported to four places to the right of the
  0.0159 decimal place or three significant figures.

(c) [(136000)(0.000322)/0.082)](129.2) = $6.899910244 \times 10^4 = 6.9 \times 10^4$. Round the intermediate answer to two significant figures to reflect the quantity with the fewest significant figures (0.082). Round up the last digit since the first nonsignificant digit is 9.

1.82 **Given:** € 1.67/L of gas, C$1.37/€ **Find:** price of gas in C$

Conceptual Plan: € /L × C$/€ → C$/L

Solution: $\text{Price of gasoline in C\$} = \frac{1.67 \text{ euro}}{1 \text{ L gasoline}} \times \frac{\text{C\$}1.37}{1 \text{ euro}} = \text{C\$}2.29 \text{ L}^{-1}$ of gasoline

Check: The answer is in the correct unit, and it makes logical sense because the euro to C$ exchange rate would place the per litre price of gasoline higher than the euro price.

1.83 **Given:** $r_{Au} = 3.8\text{ cm}^2$, $h_{Au} = 22$ cm, $d_{Au} = 19.3\text{ g cm}^{-3}$, $d_{sand} = 3.00\text{ g cm}^{-3}$ **Find:** V_{sand}

Conceptual Plan: (1) V_{Au} bar, (2) $m_{Au} = d_{Au} \times V_{Au} = m_{sand}$, (3) $V_{sand} = d_{sand}/m_{sand}$

Solution: $V_{cyl} = (\pi r^2) \times h = (\pi \times (3.8\text{ cm})^2) \times 22\text{ cm} = 998.0211\text{ cm}^3$ (two significant figures)

$$m_{Au} = d \times V = 19.3 \text{ g cm}^{-3} \times 998.021 \text{ cm}^3 = 19261.808 \text{ g}$$

Therefore, the volume of sand is:

$$V_{sand} = \frac{m}{d} = \frac{19261.808 \text{ g}}{3.00 \text{ g cm}^{-3}} = 6.4 \times 10^3 \text{ cm}^3$$

Check: The answer is in the correct units. Since density of gold is over six times greater than sand, the volume of sand needed would be six times larger than gold.

1.84 **Given:** $r = 1.0 \times 10^{-13}$ cm, $m = 1.7 \times 10^{-24}$ g **Find:** density **Other:** $V = (4/3)\ \pi r^3$

Conceptual Plan: $r \xrightarrow{V = (4/3)\pi r^3} V$ **then** $m, V \xrightarrow{d = m/V} d$

Solution: $V = (4/3)\pi r^3 = (4/3)(\pi)(1.0 \times 10^{-13}\text{cm})^3 = 4.\underline{1}88790205 \times 10^{-39} \text{ cm}^3$

$$d = \frac{m}{V} = \frac{1.7 \times 10^{-24} \text{ g}}{4.\underline{1}88790205 \times 10^{-39} \text{ cm}^3} = 4.\underline{0}58451049 \times 10^{14} \text{ g cm}^{-3} = 4.1 \times 10^{14} \text{ g cm}^{-3}$$

Check: The units (g cm^{-3}) are correct. The magnitude of the answer seems correct considering how small a nucleus is compared to an atom. Two significant figures are allowed to reflect the significant figures in 1.0×10^{-13} cm. Round the last digit up because the first non-significant digit is a 5.

1.85 **Given:** $d_{Ti} = 4.51\text{ g cm}^{-3}$, $m_{Ti} = 3.5$ kg **Find:** V_{Ti} in litres

Conceptual Plan: d(g cm⁻³) → d(kg L⁻¹), *V* = *m*/*d*

Solution: $\frac{4.51 \text{ g}}{\text{cm}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{1000 \text{ cm}^3}{1 \text{ L}} = 4.51 \text{ kg L}^{-1}$

$$V = \frac{m}{d} = \frac{3.5 \text{ kg}}{4.51 \text{ kg L}^{-1}} = 0.78 \text{ L}$$

Check: The units are L. Since the density of Ti is 4.51, the calculated volume has to be approximately ~4.5 times smaller than the mass.

1.86 **Given:** $d_{Fe} = 7.86\text{ g cm}^{-3}$ **Find:** d_{Fe} in kg m^{-3}

Conceptual Plan: $\text{g cm}^{-3} \rightarrow \text{kg m}^{-3}$

Solution: $d = \frac{7.86 \text{ g}}{\text{cm}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{(100 \text{ cm})^3}{(1 \text{ m})^3} = 7.86 \times 10^3 \text{ kg m}^{-3}$

Check: The answer is in the correct units. There are 10^6 cm^3 in 1 m^3, but 1000 g in 1 kg, so the magnitude of 10^3 difference in the answer makes sense.

1.87 **Given:** $r_{steel} = 0.56\text{ cm}^2$, $h_{steel} = 5.49$ cm, $m_{steel} = 41$ g **Find:** d_{steel}

Conceptual Plan: (1) V_{steel}, (2) d = m/V

Solution: $V_{steel} = (\pi\ r^2) \times \text{h} = (\pi \times [0.56 \text{ cm}]^2) \times 5.49 \text{ cm} = 5.40877 \text{ cm}^3$

$$d_{steel} = \frac{m}{V} = \frac{41 \text{ g}}{5.40877 \text{ cm}^3} = 7.6 \text{ g cm}^{-3}$$

Check: The answer is in the correct units (cm³) and is very close to the density of pure iron, which makes physical sense.

1.88 **Given:** m = 85 g **Find:** radius of the sphere (inches)
Other: density (aluminum) = 2.7 g cm⁻³
Conceptual Plan: ***m, d*** $\rightarrow$ ***V*** **then** ***V*** $\rightarrow$ ***r*** **then cm** $\rightarrow$ **dm**

$d = m/V$ $\quad V = (4/3)\pi r^3$ $\quad \dfrac{10\text{ cm}}{1\text{ dm}}$

Solution: $d = m/V$ Rearrange by multiplying both sides of the equation by V and dividing both sides of the equation by d.

$$V = \frac{m}{d} = \frac{85\ \cancel{\text{g}}}{2.7\ \cancel{\text{g}}\ \text{cm}^{-3}} = 3\underline{1}.48148148\ \text{cm}^3$$

$V = (4/3)\pi r^3$ Rearrange by dividing both sides of the equation by $(4/3)\pi$. $r^3 = \dfrac{3V}{4\pi}$

Take the cube root of both sides of the equation.

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{(3)\ (3\underline{1}.48148148\ \text{cm}^3)}{4\pi}\right)^{1/3} = (7.\underline{5}1565009\ \text{cm}^3)^{1/3} = 1.\underline{9}58794386\ \text{cm}$$

$$1.\underline{9}58794386\ \cancel{\text{cm}} \times \frac{1\ \text{dm}}{10\ \cancel{\text{cm}}} = 0.1\underline{9}58794386\ \text{dm} = 0.20\ \text{dm}$$

Check: The units (dm) are correct. The magnitude of the answer seems correct. The magnitude of the volume is about a third of the mass (density is about 3 g cm⁻³). The radius in cm seems right considering the geometry involved. The magnitude goes down when we convert from cm to dm because a dm is bigger than a cm. Two significant figures are allowed to reflect the significant figures in 2.7 g cm⁻³ and 85 g. Truncate the nonsignificant digits because the first nonsignificant digit is a 1.

1.89 **Given:** 185 m³ of H_2O **Find:** mass of water in kg
Other: d(H_2O) = 1.00 g cm⁻³
Conceptual Plan: **m³** $\rightarrow$ **cm³,** $\boldsymbol{m = d \times V}$**, g** $\rightarrow$ **kg**

Solution: $V = 185\ \text{m}^3 \times \dfrac{(100\ \text{cm})^3}{(1\ \text{m})^3} = 1.85 \times 10^8\ \text{cm}^3$

$$m = d \times V = (1\ \text{g cm}^{-3}) \times (1.85 \times 10^8\ \text{cm}^3) = 1.85 \times 10^8\ \text{g}$$

$$1.85 \times 10^8\ \text{g} \times \frac{1\ \text{kg}}{1000\ \text{g}} = 1.85 \times 10^5\ \text{kg}$$

Check: The answer is in the correct unit. Density of water in SI units is 1000 kg m⁻³, so the magnitude of the answer is correct.

1.90 **Given:** V_{ice} = 8921 L **Find:** mass of ice
Other: 1.00 cm³ ice = 0.917 g ice
Conceptual Plan: **L** $\rightarrow$ **cm³,** $\boldsymbol{m = d \times V}$**, g** $\rightarrow$ **kg**

Solution: $V = 8921\ \text{L} \times \dfrac{1000\ \text{cm}^3}{1\ \text{L}} = 8.921 \times 10^6\ \text{cm}^3$

$$m = d \times V = (0.917\ \text{g cm}^{-3}) \times (8.921 \times 10^6\ \text{cm}^3) = 8.181 \times 10^6\ \text{g}$$

$$8.181 \times 10^6\ \text{g} \times \frac{1\ \text{kg}}{1000\ \text{g}} = 8.181 \times 10^3\ \text{kg}$$

Check: The answer is in the correct unit. Multiplying both g and cm³ gives kg and L, so the density of water is 1.00 kg L⁻¹, so the mass and volume would have the same numerical value in this problem.

1.91 **Given:** Usage = 3.8 L/100 km, V_{gas} = 15 L **Find:** distance (D)

Conceptual Plan: D = V_{gas}/Usage

Solution: $\text{Distance} = 15\text{ L} \times \frac{100\text{ km}}{3.8\text{ L}} = 3.9 \times 10^2\text{ km}$

Check: The answer is in the correct unit (km). The amount of gas available is roughly four times the consumption per 100 km, so the distance travelled should be close to 400 km.

1.92 **Given:** Usage (highway) = 7.6 L/100 km, Usage (city) = 12 L/100 km, V_{tank} = 61.0 L

Find: distance (D)

Conceptual Plan: D = V_{tank}/Usage

Solution: $\text{Distance} = \frac{61.0\cancel{\text{L}}}{\text{tank}} \times \frac{100\text{ km}}{7.6\cancel{\text{L}}} = 8.0 \times 10^2\text{ km}$ per tank on the highway

Likewise,

$\text{Distance} = \frac{61.0\cancel{\text{L}}}{\text{tank}} \times \frac{100\text{ km}}{12\cancel{\text{L}}} = 5.1 \times 10^2\text{ km}$ per tank in the city

Check: The answers are in the correct unit (km). The range for highway driving is greater than city driving by a factor of ~1.5, which is the same factor as the consumption rate.

1.93 **Given:** radius of nucleus of the hydrogen atom = 1.0×10^{-13} cm; radius of the hydrogen atom = 52.9 pm

Find: volume fraction occupied by nucleus

Conceptual Plan: cm → m then pm → m then *r* → *V* then V_{atom}, $V_{nucleus}$ → % $V_{nucleus}$

$\frac{1\text{ m}}{100\text{ cm}}$ $\qquad \frac{1\text{ m}}{10^{12}\text{ pm}}$ $\qquad V = (4/3)\pi r^3$ $\qquad \% V_{nucleus} = \frac{V_{nucleus}}{V_{atom}}$

Solution: $1.0 \times 10^{-13}\ \cancel{\text{cm}} \times \frac{1\text{ m}}{100\ \cancel{\text{cm}}} = 1.0 \times 10^{-15}\text{ m}$ and $52.9\ \cancel{\text{pm}} \times \frac{1\text{ m}}{10^{12}\ \cancel{\text{pm}}} = 5.29 \times 10^{-11}\text{ m}$

$V = (4/3)\pi r^3$ Substitute into $\%V$ equation.

$\%V_{nucleus} = \frac{V_{nucleus}}{V_{atom}} \times 100\%$ → $\% V_{nucleus} = \frac{(4/3)\ \pi r_{nucleus}^3}{(4/3)\ \pi r_{atom}^3} \times 100\%$ Simplify equation.

$\% V_{nucleus} = \frac{r_{nucleus}^3}{r_{atom}^3} \times 100\%$ Substitute numbers and calculate result.

$$\%V_{nucleus} = \frac{(1.0 \times 10^{-15}\text{ m})^3}{(5.29 \times 10^{-11}\text{ m})^3} \times 100\% = (1.\underline{8}90359168 \times 10^{-5})^3 \times 100\% = 6.\underline{7}55118685 \times 10^{-13}\% = 6.8 \times 10^{-13}\%$$

Check: The units (none) are correct. The magnitude of the answer seems correct (10^{-15}), since a proton is so small. Converting fractions to percent is more common: 6.8×10^{-13}% of the atom is occupied by the proton. Two significant figures are allowed to reflect the significant figures in 1.0×10^{-13} cm. Round up the last digits because the first nonsignificant digit is a 5.

1.94 **Given:** radius of neon = 69 pm; 2.69×10^{22} atoms per litre **Find:** percent of volume occupied by neon (%)

Conceptual Plan: Assume 1 L total volume.

pm → m → cm then *r* → *V* then cm³ → L then L/atom → L then V_{Ne},V_{Total} → %V_{Ne}

$\frac{1\text{ m}}{10^{12}\text{ pm}}$ $\quad \frac{100\text{ cm}}{1\text{ m}}$ $\qquad V = (4/3)\pi r^3 = 1\text{ atom}$ $\qquad \frac{1\text{ L}}{1000\text{ cm}^3}$ $\qquad 2.69 \times 10^{22}\text{ atoms}$ $\qquad \% V_{Ne} = \frac{V_{Ne}}{V_{Total}} \times 100\%$

Solution: $69\ \cancel{\text{pm}} \times \frac{1\ \cancel{\text{m}}}{10^{12}\ \cancel{\text{pm}}} \times \frac{100\text{ cm}}{1\ \cancel{\text{m}}} = 6.9 \times 10^{-9}\text{ cm}$

$V = (4/3)\pi r^3 = (4/3)\pi\ (6.9 \times 10^{-9}\text{ cm})^3 = 1.\underline{3}7605528 \times 10^{-24}\text{ cm}^3$ per atom

$1.\underline{3}7605528 \times 10^{-24}\ \cancel{\text{cm}}^3 \times \frac{1\text{ L}}{1000\ \cancel{\text{cm}}^3} = 1.\underline{3}7605528 \times 10^{-27}$ L per atom

$\frac{1.\underline{3}7605528 \times 10^{-27}\text{ L}}{\text{atom}} \times 2.69 \times 10^{22}\text{ atoms} = 3.\underline{7}01588707 \times 10^{-5}$ L Substitute into $\%V$ equation.

$$\% V_{Ne} = \frac{V_{Ne}}{V_{Total}} \times 100\% = \frac{3.\underline{7}01588707 \times 10^{-5}\ \cancel{L}}{1\ \cancel{L}} \times 100\% = 3.\underline{7}01588707 \times 10^{-3}\% = 3.7 \times 10^{-3}\%$$

Check: This says that the separation between atoms is very large in the gas phase.
The units (%) are correct. The magnitude of the answer seems correct (10^{-3}%); it is known that gases are primarily empty space. Two significant figures are allowed to reflect the significant figures in 69 pm. Truncate the nonsignificant digits because the first nonsignificant digit is a 0.

1.95 **Given:** length of hydrogen molecule = 212 pm; diameter of ping pong ball = 4.0 cm, 6.02×10^{23} atoms and balls in a row
Find: row length (km)
Conceptual Plan: molecules → pm → m → km and ball → cm → m → km

$$\frac{212\ \text{pm}}{1\ \text{molecule}} \quad \frac{1\ \text{m}}{10^{12}\ \text{pm}} \quad \frac{1\ \text{km}}{1000\ \text{m}} \qquad \frac{4.0\ \text{cm}}{1\ \text{ball}} \quad \frac{100\ \text{cm}}{1\ \text{m}} \quad \frac{1\ \text{km}}{1000\ \text{m}}$$

Solution: $6.02 \times 10^{23}\ \cancel{\text{molecule}} \times \frac{212\ \cancel{\text{pm}}}{1\ \cancel{\text{molecule}}} \times \frac{1\ \cancel{\text{m}}}{10^{12}\ \cancel{\text{pm}}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} = 1.28 \times 10^{11}\ \text{km}$

$$6.02 \times 10^{23}\ \cancel{\text{balls}} \times \frac{4.0\ \cancel{\text{cm}}}{1\ \cancel{\text{ball}}} \times \frac{1\ \cancel{\text{m}}}{100\ \cancel{\text{cm}}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} = 2.4 \times 10^{19}\ \text{km}$$

Check: The units (km) are correct. The magnitude of the answers seem correct (10^{11} and 10^{19}). The answers are driven by the large number of atoms or balls. The ping pong ball row is 10^{8} times longer. Three significant figures are allowed to reflect the significant figures in 212 pm. Two significant figures are allowed to reflect the significant figures in 4.0 cm.

1.96 **Given:** D = 100 m, t = 9.58 s **Find:** speed in km hr^{-1}
Conceptual Plan: m, s → v (m s^{-1}) → v (km hr^{-1})

Solution: $v = \frac{100\ \text{m}}{9.58\ \text{s}} \times \frac{1\ \text{km}}{1000\ \text{m}} \times \frac{3600\ \text{s}}{1\ \text{hr}} = 37.6\ \text{km hr}^{-1}$

Check: The answer is in the correct unit. The number makes sense, because generally humans can sprint at ~30 km hr^{-1}.

1.97 **Given:** 39.33 g sodium/100 g salt; 1.25 g salt/100 g snack mix; Health Canada maximum 2.40 g sodium/day
Find: g snack mix
Conceptual Plan: g sodium → g salt → g snack mix

$$\frac{100\ \text{g salt}}{39.33\ \text{g sodium}} \qquad \frac{100\ \text{g snack mix}}{1.25\ \text{g salt}}$$

Solution: $\frac{2.40\ \cancel{\text{g sodium}}}{1\ \text{day}} \times \frac{100\ \cancel{\text{g salt}}}{39.33\ \cancel{\text{g sodium}}} \times \frac{100\ \text{g snack mix}}{1.25\ \cancel{\text{g salt}}} = 48\underline{8}.1770\ \text{g snack mix/day}$

$= 488$ g snack mix/day

Check: The units (g) are correct. The magnitude of the answer seems correct (500) since salt is less than half sodium and there is a little over a gram of salt per 100 grams of snack mix. Three significant figures are allowed to reflect the significant figures in the Health Canada maximum and in the amount of salt in the snack mix.

1.98 **Given:** 86.6 g lead/100 g galena; 68.5 g galena/100 g ore; 92.5 g lead extracted/100 g lead available; 1.500 cm radius sphere **Other:** d(lead) = 11.4 g cm^{-3} **Find:** g ore
Conceptual Plan:
$r_{sphere} \rightarrow V_{sphere}$ **then** $V_{sphere}, d_{sphere} \rightarrow m_{sphere}$**(g lead) → g lead available → g galena → g ore**

$$V = (4/3)\pi r^3 \qquad d = m/V \qquad \frac{100\ \text{g lead available}}{92.5\ \text{g lead extracted}} \qquad \frac{100\ \text{g galena}}{86.6\ \text{g lead}} \qquad \frac{100\ \text{g ore}}{68.5\ \text{g galena}}$$

Solution: Calculate m_{sphere} $V_{\text{sphere}} = (4/3)\pi\, r^3_{\text{sphere}} = (4/3)\pi\ (5.00\text{ cm})^3 = 52\underline{3}.5988\text{ cm}^3$

$d = m/V$ Solve for m by multiplying both sides of the equation by V. $m = V \times d$

$$m = 52\underline{3}.5988\ \cancel{\text{cm}^3} \times \frac{11.4\text{ g lead}}{1\ \cancel{\text{cm}^3}} = 59\underline{6}9.026\text{ g lead}$$

$$59\underline{6}9.026\ \cancel{\text{g lead}} \times \frac{100\ \cancel{\text{g lead available}}}{92.5\ \cancel{\text{g lead extracted}}} \times \frac{100\ \cancel{\text{g galena}}}{86.6\ \cancel{\text{g lead available}}} \times \frac{100\text{ g ore}}{68.5\ \cancel{\text{g galena}}}$$

$$= 1.0\underline{8}7811 \times 10^4\text{ g ore} = 1.09 \times 10^4\text{ g ore}$$

Check: The units (g) are correct. The magnitude of the answer seems correct (10^4) since lead is so dense and the sphere is not small. Three significant figures are allowed to reflect the significant figures in all of the information given.

1.99 **Given:** d(liquid nitrogen) = 0.808 g mL^{-1}; d(gaseous nitrogen) = 1.15 g L^{-1}; 175 L liquid nitrogen; 10.00 m × 10.00 m × 2.50 m room **Find:** fraction of room displaced by nitrogen gas

Conceptual Plan: L → mL then V_{liquid}, d_{liquid} → m_{liquid} then set $m_{\text{liquid}} = m_{\text{gas}}$ then m_{gas}, d_{gas} → V_{gas} then

$\frac{1000\text{ mL}}{1\text{ L}}$ $d = m/V$ $d = m/V$

Calculate the V_{room} → cm^3 → L then calculate the fraction displaced

$V = l \times w \times h \frac{(100\text{ cm})^3}{(1\text{ m})^3} \frac{1\text{ L}}{1000\text{ cm}^3}$ $\frac{V_{\text{gas}}}{V_{\text{room}}}$

Solution: $175\ \cancel{\text{L}} \times \frac{1000\text{ mL}}{1\ \cancel{\text{L}}} = 1.75 \times 10^5$ mL. Solve for m by multiplying both sides of the equation by V.

$$m = V \times d = 1.75 \times 10^5\ \cancel{\text{mL}} \times \frac{0.808\text{ g}}{1\ \cancel{\text{mL}}} = 1.4\underline{1}4 \times 10^5\text{ g nitrogen liquid} = 1.4\underline{1}4 \times 10^5\text{ g nitrogen gas}$$

$d = m/V$. Rearrange by multiplying both sides of the equation by V and dividing both sides of the equation by d.

$$V = \frac{m}{d} = \frac{1.4\underline{1}4 \times 10^5\ \cancel{\text{g}}}{1.15\frac{\cancel{\text{g}}}{\text{L}}} = 1.2\underline{2}9565 \times 10^5\text{ L nitrogen gas}$$

$$V_{\text{room}} = l \times w \times h = 10.00\ \cancel{\text{m}} \times 10.00\ \cancel{\text{m}} \times 2.50\ \cancel{\text{m}} \times \frac{(100\ \cancel{\text{cm}})^3}{(1\ \cancel{\text{m}})^3} \times \frac{1\text{ L}}{1000\ \cancel{\text{cm}^3}} = 2.50 \times 10^5\text{ L}$$

$$\frac{V_{\text{gas}}}{V_{\text{room}}} = \frac{1.2\underline{2}9565 \times 10^5\ \cancel{\text{L}}}{2.50 \times 10^5\ \cancel{\text{L}}} = 0.49\underline{1}8272 = 0.492$$

Check: The units (none) are correct. The magnitude of the answer seems correct (0.5) since there is a large volume of liquid and the density of the gas is about a factor of 1000 less than the density of the liquid. Three significant figures are allowed to reflect the significant figures in the densities and the volume of the liquid given.

1.100 **Given:** d(mercury at 0.0 °C) = 13.596 g cm^{-3}; d(mercury at 25.0 °C) = 13.534 g cm^{-3}; 3.380 g; 0.200 mm diameter capillary **Find:** distance mercury rises

Conceptual Plan: at each temperature m, d → V then mm → m → cm then V, r → h

$d = m/V$ $\frac{1\text{ m}}{1000\text{ mm}}$ $\frac{100\text{ cm}}{1\text{ m}}$ $V = \pi r^2 h$

then calculate the difference between the two heights

Solution: $d = m/V$. Rearrange by multiplying both sides of the equation by V and dividing both sides of the equation by d. $V = \frac{m}{d}$

$$\text{at 0.0 °C: } V = \frac{m}{d} = \frac{3.380\ \cancel{\text{g}}}{13.596\ \cancel{\text{g}}\text{ cm}^{-3}} = 0.248\underline{6}0253\text{ cm}^3$$

and at 25.0 °C: $V = \frac{m}{d} = \frac{3.380\ \cancel{g}}{13.534\ \cancel{g}\ cm^{-3}} = 0.249\underline{7}41392\ cm^3$

$r = 0.100\ \cancel{mm} \times \frac{1\ \cancel{m}}{1000\ \cancel{mm}} \times \frac{100\ cm}{1\ \cancel{m}} = 0.0100\ cm$ then $V = \pi r^2 h$.

Rearrange by dividing both sides of the equation by πr^2. $h = \frac{V}{\pi r^2}$

at 0.0 °C: $h = \frac{V}{\pi r^2} = \frac{0.248\underline{6}0253\ cm^3}{\pi\ (0.0100\ cm)^2} = 791.327\ cm$

and at 25.0 °C: $h = \frac{V}{\pi r^2} = \frac{0.249\underline{7}41392\ cm^3}{\pi\ (0.0100\ cm)^2} = 794.952\ cm$

the difference in height is 794.952 cm − 791.327 cm = 0.$\underline{7}$48990 cm = 3.6 cm

Check: The units (cm) are correct. The magnitude of the answer seems correct (3.6) since there is a relatively small change in temperature and the two densities are very close to each other. Only one significant figure is allowed because the heights have four significant figures and so the error is in the tenths place.

Challenge Problems

1.101 **Given:** F = 2.31×10^4 N, A = 125 cm^2 **Find:** pressure

Conceptual Plan: A (cm^2) → A(m^2), $P = \frac{F}{A}$

Solution: $A = 125\ cm^2 \times \frac{1\ m^2}{(100\ cm)^2} = 0.0125\ m^2$

$$P = \frac{F}{A} = \frac{2.31 \times 10^4\ N}{0.0125\ m^2} = 1.85 \times 10^6\ Pa$$

$$1.85 \times 10^6\ Pa \times \frac{1\ bar}{1 \times 10^5\ Pa} = 18.5\ bar$$

Check: The answer is in units of barr. Barr is a convenient unit for atmospheric pressure, and normal atmospheric pressure is 1.01×10^5 Pa. The pressure experienced by the diver's mask is more than 20 times that of atmospheric pressure.

1.102 A Newton (N) has units of kg m s^{-2}. If a dyne has mass measured in units of grams, this will make a dyne a factor of 10^3 larger than a Newton (N). To get the additional factor of 10^2 needed to get to a factor of 10^5, the length must be in centimetres (cm) $1\ \text{dyne} = 1\frac{\cancel{g} \cdot \cancel{cm}}{s^2} \times \frac{1\,kg}{1000\ \cancel{g}} \times \frac{1\,m}{100\ \cancel{cm}} = 10^{-5}\frac{kg \cdot m}{s^2} = 10^{-5}N.$

1.103 Referring to the definition of energy in Chapter 6, 1 Joule = 1 J = kg m^2 s^{-2}. For kinetic energy, if the units of mass are the kilogram (kg) and the units of the velocity are metres/second (m s^{-1}) then

kinetic energy units $= mv^2 = kg\left(\frac{m}{s}\right)^2 = \frac{kg \cdot m^2}{s^2} = J.$ Since a Newton (N) is a unit of force and has units of kg m s^{-2}, pressure = force/area and has units of N m^{-2}, and force = (mass) × (acceleration) or $F = ma$, then 3/2 PV units $= \frac{N}{\cancel{m^2}} \cdot m^{\cancel{3}} = \frac{kg \cdot m}{s^2} \cdot m = \frac{kg \cdot m^2}{s^2} = J.$

1.104 **Given:** $m_{BH} = 1 \times 10^3$ suns. $r_{BH} = 0.5\ r$ (moon) **Find:** d_{BH} in g cm^{-3}

Other: $d_{sun} = 1.4 \times 10^3$ kg m^{-3}, $r_{sun} = 7.0 \times 10^5$ km, $d_{moon} = 3.48 \times 10^3$ km

Conceptual Plan: $d_{BH} = m_{BH}/V_{BH}$

m_{BH}: $r_{sun} \rightarrow V_{sun} \rightarrow$ ($V = 4/3\ \pi r^3$), V_{sun}, $d_{sun} \rightarrow m_{sun} \rightarrow$, $m_{sun} \rightarrow m_{BH}$, kg $\rightarrow$ g

V_{BH}: $d_{moon} \rightarrow r_{moon}$, $r_{BH} = 0.5\ d_{moon}$, $V_{BH} = 4/3\ \pi r^3$, km³ $\rightarrow$ m³ $\rightarrow$ cm³

Solution: $V_{sun} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7.0\times10^5\,\text{km})^3 = 1.43675504\times10^{18}\,\text{km}^3$

$$V_{sun} = 1.43675504\times10^{18}\ \text{km}^3 \times \frac{(1000\ \text{m})^3}{(1\ \text{km})^3} = 1.43675504\times10^{27}\ \text{m}^3$$
$$m_{sun} = d_{sun}\times V_{sun} = (1.4\times10^3\ \text{kg m}^{-3})(1.43675504\times10^{27}\ \text{m}^3) = 2.011457056\times10^{30}\ \text{kg}$$
$$m_{BH} = (1\times10^3)(2.011457056\times10^{30}\ \text{kg}) = 2.011457056\times10^{33}\ \text{kg}$$
$$m_{BH} = 2.011457056\times10^{33}\ \text{kg}\times\frac{1000\ \text{g}}{1\ \text{kg}} = 2.011457056\times10^{36}\ \text{g}$$
$$r_{BH} = \frac{1}{2}r_{moon} = \frac{1}{2}\left(\frac{1}{2}\times d_{moon}\right) = 0.5\times0.5\times3.48\times10^3\ \text{km} = 870\ \text{km}$$
$$V_{BH} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(870\ \text{km})^3 = 2.758330916\times10^9\ \text{km}^3$$
$$V_{BH} = 2.758330916\times10^9\ \text{km}^3\times\frac{(1000\ \text{m})^3}{(1\ \text{km})^3}\times\frac{(100\ \text{cm})^3}{(1\ \text{m})^3} = 2.758330916\times10^{24}\ \text{cm}^3$$
$$d_{BH} = \frac{2.011457056\times10^{36}\ \text{g}}{2.758330916\times10^{24}\ \text{cm}^3} = 7.292297832\times10^{11}\ \text{g cm}^{-3} = 7.3\times10^{11}\ \text{g cm}^{-3}$$

Check: The units (g cm^{-3}) are correct. The magnitude of the answer seems correct (10^{11}), since we expect extremely high numbers for black holes. Two significant figures are allowed to reflect the significant figures in the radius of our sun (7.0×10^5 km) and the average density of the sun (1.4×10^3 kg m^{-3}). Truncate the nonsignificant digits because the first nonsignificant digit is a 4.

1.105 **Given:** cubic nanocontainers with an edge length = 25 nanometres
Find: a) volume of one nanocontainer; b) grams of oxygen that could be contained by each nanocontainer; c) grams of oxygen inhaled per hour; d) minimum number of nanocontainers per hour; and e) minimum volume of nanocontainers

Other: (pressurized oxygen) = 85 g L^{-1}; 0.28 g of oxygen per litre; average human inhales about 0.50 L of air per breath and takes about 20 breaths per minute; adult total blood volume = ~5 L

Conceptual Plan:

(a) **nm $\rightarrow$ m $\rightarrow$ cm then $l \rightarrow V$ then cm³ $\rightarrow$ L**

$\frac{1\ \text{m}}{10^9\ \text{nm}}$ $\frac{100\ \text{cm}}{1\ \text{m}}$ $V = l^3$ $\frac{1\ \text{L}}{1000\ \text{cm}^3}$

(b) **L $\rightarrow$ g pressurized oxygen**

$\frac{85\ \text{g oxygen}}{1\ \text{L nanocontainers}}$

(c) **hr $\rightarrow$ min $\rightarrow$ breaths $\rightarrow$ L$_{air}$ $\rightarrow$ g_{O_2}**

$\frac{60\ \text{min}}{1\ \text{hr}}$ $\frac{20\ \text{breath}}{1\ \text{min}}$ $\frac{0.50\ \text{L}_{air}}{1\ \text{breath}}$ $\frac{0.28\ \text{g}_{CO}}{1\ \text{L}_{air}}$

(d) **grams oxygen $\rightarrow$ number nanocontainers**

$\frac{1\ \text{nanocontainer}}{\text{part (b) grams of oxygen}}$

(e) **number nanocontainers $\rightarrow$ volume nanocontainers**

$\frac{\text{part (a) volume}}{\text{of 1 nanocontainer}}$

Solution:

(a) $$25\ \cancel{\text{nm}} \times \frac{1\ \cancel{\text{m}}}{10^9\ \cancel{\text{nm}}} \times \frac{100\ \text{cm}}{1\ \cancel{\text{m}}} = 2.5\times10^{-6}\ \text{cm}$$

$$V = l^3 = (2.5\times10^{-6}\ \text{cm})^3 = 1.\underline{5}625\times10^{-17}\ \cancel{\text{cm}^3} \times \frac{1\ \text{L}}{1000\ \cancel{\text{cm}^3}} = 1.\underline{5}625\times10^{-20}\ \text{L} = 1.6\times10^{-20}\ \text{L}$$

(b) $$1.\underline{5}625\times10^{-20}\ \cancel{\text{L}} \times \frac{85\ \text{g oxygen}}{1\ \cancel{\text{L}}\ \text{nanocontainers}} = 1.\underline{3}28125\times10^{-18}\ \frac{\text{g pressurized } O_2}{\text{nanocontainer}}$$

$$= 1.3\times10^{-18}\ \frac{\text{g pressurized } O_2}{\text{nanocontainer}}$$

(c) $$1\ \cancel{\text{hr}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \frac{20\ \cancel{\text{breath}}}{1\ \cancel{\text{min}}} \times \frac{0.50\ \cancel{\text{L}_{\text{air}}}}{1\ \cancel{\text{breath}}} \times \frac{0.28\ \text{g}O_2}{1\ \cancel{\text{L}_{\text{air}}}} = 1.\underline{6}8\times10^2\ \text{g oxygen} = 1.7\times10^2\ \text{g oxygen}$$

(d) $$1.\underline{6}8\times10^2\ \cancel{\text{g oxygen}} \times \frac{1\ \text{nanocontainer}}{1.3\times10^{-18}\ \cancel{\text{g of oxygen}}} = 1.\underline{2}92307692\times10^{20}\ \text{nanocontainers}$$

$$= 1.3\times10^{20}\ \text{nanocontainers}$$

(e) $$1.\underline{2}92307692\times10^{20}\ \cancel{\text{nanocontainers}} \times \frac{1.5625\times10^{-20}\ \text{L}}{\cancel{\text{nanocontainer}}} = 2.\underline{0}19230769\ \text{L} = 2.0\ \text{L}$$

This volume is much too large to be feasible, since the volume of blood in the average human is 5 L.

Check:

(a) The units (L) are correct. The magnitude of the answer (10 – 20) makes physical sense because these are very, very tiny containers. Two significant figures are allowed, reflecting the significant figures in the starting dimension (25 nm − 2 significant figures). Round up the last digit because the first nonsignificant digit is a 6.

(b) The units (g) are correct. The magnitude of the answer (10^{-18}) makes physical sense because these are very, very tiny containers and very few molecules can fit inside. Two significant figures are allowed, reflecting the significant figures in the starting dimension (25 nm) and the given concentration (85 g L^{-1}) − 2 significant figures in each. Truncate the nonsignificant digits because the first nonsignificant digit is a 2.

(c) The units (g oxygen) are correct. The magnitude of the answer (10^2) makes physical sense because of the conversion factors involved and the fact that air is not very dense. Two significant figures are allowed because it is stated in the problem. Round up the last digit because the first nonsignificant digit is an 8.

(d) The units (nanocontainers) are correct. The magnitude of the answer (10^{20}) makes physical sense because these are very, very tiny containers and we need a macroscopic quantity of oxygen in these containers. Two significant figures are allowed, reflecting the significant figures in both of the quantities in the calculation – 2 significant figures. Round up the last digit because the first nonsignificant digit is a 9.

(e) The units (L) are correct. The magnitude of the answer (2) makes physical sense because of the magnitudes of the numbers in this step. Two significant figures are allowed reflecting the significant figures in both of the quantities in the calculation – 2 significant figures. Truncate the nonsignificant digits because the first nonsignificant digit is a 1.

1.106 Assume that all of the spheres are the same size. Let x = the percentage of spheres that are copper (expressed as a fraction) and so the volume of copper = (427 cm³)x and the volume of lead = (427 cm³)(1 − x).

Since the density of copper is 8.96 g cm⁻³, the mass of copper $= (427\ \cancel{\text{cm}^3})x \times \dfrac{8.96\ \text{g}}{\cancel{\text{cm}^3}} = 38\underline{2}5.92(x)$ g.

Since the density of lead is 11.4 g cm⁻³, the mass of lead $= (427\ \cancel{\text{cm}^3})(1-x) \times \dfrac{11.46\ \text{g}}{\cancel{\text{cm}^3}} = 48\underline{9}3.42\ (1-x)$ g.

Since the total mass is 4.36 kg = 4360 g, 4360 g = $38\underline{2}5.92(x)$ g + $48\underline{9}3.42\ (1 - x)$ g. Solving for x, $10\underline{6}7.5(x)$ g = $5\underline{3}3.6$ g $\rightarrow$ $x = 0.4\underline{9}9859$ or 50.% of the spheres are copper.

Check: This answer makes sense, since the average density of the spheres = 4360 g/427 cm³ = 10.2 g cm⁻³ and the average of the density of copper and the density of lead = (8.96 + 11.4)/ 2 g cm⁻³ = 10.2 g cm⁻³.

Conceptual Problems

1.107 No. Since the container is sealed, the atoms and molecules can move around, but they cannot leave. If no atoms or molecules can leave, the mass must be constant.

1.108 (c) is the best representation. When solid carbon dioxide (dry ice) sublimes, it changes phase from a solid to a gas. Phase changes are physical changes, so no molecular bonds are broken. This diagram shows molecules with one carbon atom and two oxygen atoms bonded together in every molecule. The other diagrams have no carbon dioxide molecules.

1.109 This problem is similar to Problem 42, only the dimension is changed to 7 cm on each edge.
Given: 7 cm on each edge cube **Find:** cm³
Conceptual Plan: Read the information given carefully. The cube is 7 cm on each side.
$l, w, h \rightarrow V$
$V = l\,w\,h$
in a cube $l = w = h$
Solution: 7 cm × 7 cm × 7 cm = (7 cm)³ = 343 cm³ or 343 cubes

1.110 In order to determine which number is large, the units need to be compared. There is a factor of 1000 between grams and kg, in the numerator. There is a factor of (100)³ or 1,000,000 between cm³ and m³. This second factor more than compensates for the first factor. Thus Substance A with a density of 1.7 g cm⁻³ is denser than Substance B with a density of 1.7 kg m⁻³.

1.111 Remember that density = mass/volume.

(a) The darker-coloured box has a heavier mass, but a smaller volume, so it is denser than the lighter-coloured box.

(b) The lighter-coloured box is heavier than the darker-coloured box and both boxes have the same volume, so the lighter-coloured box is denser.

(c) The larger box is the heavier box, so it cannot be determined with this information which box is denser.