Solution 1.25

The flow of air through a wind turbine is considered. Based on unit considerations, a proportionality relation is to be obtained for the mass flow rate of air through the blades.

Assumptions Wind approaches the turbine blades with a uniform velocity.

Analysis The mass flow rate depends on the air density, average wind velocity, and the cross-sectional area which depends on hose diameter. Also, the unit of mass flow rate $\dot{m}$ is kg/s. Therefore, the independent quantities should be arranged such that we end up with the proper unit. Putting the given information into perspective, we have

$\dot{m}$ [kg/s] is a function of ρ [kg/m^3], D [m], and V [m/s}

It is obvious that the only way to end up with the unit "kg/s" for mass flow rate is to multiply the quantities ρ and V with the square of D. Therefore, the desired proportionality relation is

$\dot{m}$ is proportional to $\rho D^2 V$

or,

$$\dot{m} = C\rho D^2 V$$

where the constant of proportionality is $C = \pi/4$ so that $\dot{m} = \rho(\pi D^2 / 4)V$

Discussion Note that the dimensionless constants of proportionality cannot be determined with this approach.