# Chapter 9 Solutions

**9.1** $\vec{E}_1 \cdot \vec{E}_2 = (1/2)(\vec{E}_1 e^{-i\omega t} + \vec{E}_1^* e^{i\omega t}) \cdot (1/2)(\vec{E}_2 e^{-i\omega t} + \vec{E}_2^* e^{i\omega t})$, where

$$\text{Re}(z) = (1/2)(z + z^*).$$
$$\vec{E}_1 \cdot \vec{E}_2 = (1/4)[\vec{E}_1 \cdot \vec{E}_2 e^{-2i\omega t} + \vec{E}_1^* \cdot \vec{E}_2^* e^{2i\omega t} + \vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_1^* \cdot \vec{E}_2].$$

The last two terms are time independent, while $\langle \vec{E}_1 \cdot \vec{E}_2 e^{-2i\omega t} \rangle \to 0$ and $\langle \vec{E}_1^* \cdot \vec{E}_2^* e^{2i\omega t} \rangle \to 0$ because of the $1/T\omega$ coefficient. Thus

$$I_{12} = 2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = (1/2)(\vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_1^* \cdot \vec{E}_2).$$

**9.2** The largest value of $r_1 - r_2$ is equal to $a$. Thus if $\varepsilon_1 = \varepsilon_2$, $\delta = k(r_1 - r_2)$ varies from 0 to $ka$. If $a \gg \lambda$, $\cos\delta$ and therefore $I_{12}$ will have a great many maxima and minima and therefore average to zero over a large region of space. In contrast if $a \ll \lambda$, $\delta$ varies only slightly from 0 to $ka \ll 2\pi$. Hence $I_{12}$ does not average to zero, and from Eq. (9.17), $I$ deviates little from $4I_0$. The two sources effectively behave as a single source of double the original strength.

**9.3** Dropping the common time factor $E_1 = E_0 \exp(2\pi i z/\pi)$ and $E_2 = E_0 \exp[(2\pi i/\lambda)(z\cos\theta + y\sin\theta)]$, adding these at the $z = 0$ plane yields $E = E_0\{1 + \exp[(2\pi i/\lambda)(y\sin\theta)]\}$. The absolute square of this is the irradiance viz.

$$I(y) = 2E_0^2\left[1 + \cos\left(\frac{2\pi}{\lambda} y\sin\theta\right)\right]$$

and the rest follows from the identity $\cos 2\theta = 2\cos^2\theta - 1$. The cosine squared has zeros at $y = m\lambda/(2\sin\theta)$ where $m$ is an odd integer. The fringe separation is $\lambda/\sin\theta$. As $\theta$ increases, the separation decreases.

**9.4** A bulb at $S$ would produce fringes. We can imagine it as made up of a very large number of incoherent point sources. Each of these would generate an independent pattern, all of which would then overlap. Bulbs at $S_1$ and $S_2$ would be incoherent and could not generate detectable fringes.

**9.5** $y_m = sm\lambda/a \approx 14.5 \times 10^{-2}$ m and $\lambda = 0.0145\ \text{m} : \nu = \upsilon/\lambda = 23.7$ kHz. This is Young's Experiment with the sources out-of-phase.

**9.6** This is comparable to the "two-slit" configuration, (Figure 9.11), so we can use (9.29) $a\sin\theta_m = m\lambda$ ($\theta_m$ may not be "small"). Let $m = 1$, $\sin\theta = y/(s^2 + y^2)^{1/2}$, so,

$$ay = \lambda(s^2 + y^2)^{1/2}; \quad (a^2 - \lambda^2)y^2 = \lambda^2 s^2;$$
$$y = \lambda s/(a^2 - y^2)^{1/2}. \quad c = \nu\lambda,$$

so $\lambda = c/\nu = (3 \times 10^8\ \text{m/s})/(1.0 \times 10^6\ \text{Hz}) = 300$ m.

$y = (300\ \text{m})(2000\ \text{m})/((600\ \text{m})^2 - (300\ \text{m})^2)^{1/2} = 1.15 \times 10^3$ m

**9.7** $\Delta y = \frac{s}{a}\lambda$

$$s = \frac{a\Delta y}{\lambda}$$

Using $a = 1\times10^{-4}$ m, $\lambda = 589$ nm, $\Delta y = 3.00$ mm

$$s = \frac{(1\times10^{-4}\,\text{m})(3\times10^{-3}\,\text{m})}{5.89\times10^{-7}\,\text{m}} = 0.509\text{m}$$

**9.8** $\Delta y_{vac} = \frac{s}{a}\lambda_0$

Using $a = 1\times10^{-3}$m, $\lambda_0 = 589.3$ nm, $s = 5.000$ m

$$\Delta y_{vac} = \frac{5.000\text{ m}}{(1\times10^{-3}\text{ m})}(5.893\times10^{-7}\text{m}) = 2.9465\text{ mm}$$

$$n = \frac{c}{\upsilon} = \frac{\nu\lambda_0}{\nu\lambda} = \frac{\lambda_0}{\lambda}$$

$$\lambda = \frac{\lambda_0}{n}$$

$$\Delta y_{air} = \frac{s}{a}\lambda = \frac{s}{a}\frac{\lambda_0}{n}$$

$$\Delta y_{air} = \frac{5.000\text{ m}}{(1\times10^{-3}\text{ m})}\frac{5.893\times10^{-7}\text{ m}}{1.00029} = 2.9456\text{ mm}$$

Thus the pattern expands from 2.946 mm to 2.947 mm.

**9.9** (a) $r_1 - r_2 = \pm\lambda/2$, hence $a\sin\theta_1 = \pm\lambda/2$ and

$$\theta_1 \approx \pm\lambda/2a = \pm(1/2)(632.8\times10^{-9}\text{m})/(0.200\times10^{-3}\text{m})$$
$$= \pm1.58\times10^{-3}\text{ rad},$$

or since

$$y_1 = s\theta_1 = (1.00\text{ m})(\pm1.58\times10^{-3}\text{ rad}) = \pm1.58\text{ mm}.$$

(b) $y_5 = s5\lambda/a = (1.00\text{ m})5(632.8\times10^{-9})/(0.200\times10^{-3}\text{ m}) = 1.582\times10^{-2}$ m. (c) Since the fringes vary as cosine-squared and the answer to (a) is half a fringe width, the answer to (b) is 10 times larger.

**9.10** $y_1 = \frac{s}{a}m\lambda = \frac{s}{a}m\frac{\lambda_0}{n}$

Using $a = 1\times10^{-3}$ m, $\lambda_0 = 589.3$ nm, $s = 3.000$ m, $n = 1.33$

$$y_1 = \frac{3.000\text{ m}}{(1\times10^{-3}\text{ m})}\frac{5.893\times10^{-7}\text{ m}}{1.33} = \pm1.329\text{ mm}$$

**9.11** $\theta_m$ is "small," so we can use (9.28) $\theta_m = m\lambda/a$, $\theta_m$ is radian, $a = m\lambda/\theta_m = [4(6.943\times10^{-7}\text{ m})]/[1°(2\pi\text{ rad}/360°)] = 1.59\times10^{-4}$ m.

**9.12** $\Delta y \simeq (s/a)\lambda$, so,

$s = a\Delta y/\lambda = [(1.0\times 10^{-4}\text{ m})(10\times 10^{-3}\text{ m})]/(4.8799\times 10^{-7}\text{ m}) = 2.05\text{ m}.$

**9.13** (9.28) $\theta_m = m\lambda/a$. Want $\theta_{1,\text{red}} = \theta_{2,\text{violet}}$; $(1)\lambda_{\text{red}}/a = (2)\lambda_{\text{violet}}/a$; $\lambda_{\text{violet}} = 390\text{ nm}.$

**9.14**

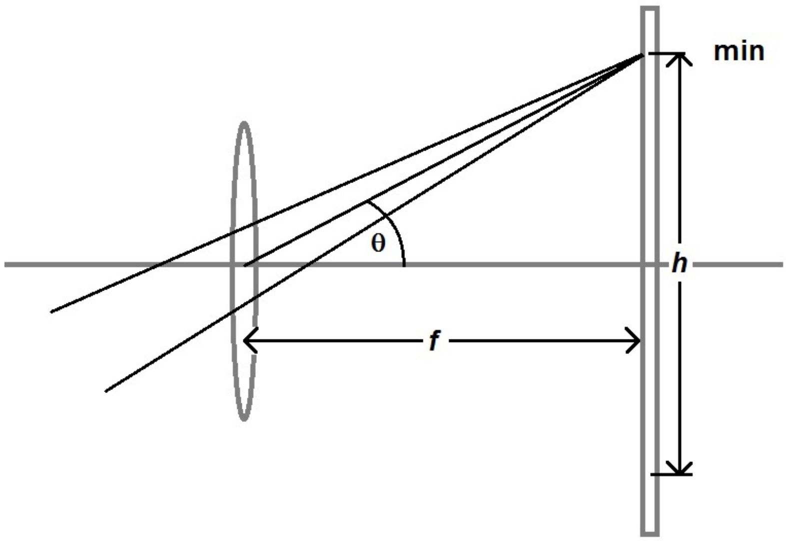
$$y_1 = \frac{s}{a}m\lambda$$

$$\theta_m \approx \frac{y_m}{s} = \frac{m\lambda}{a}$$

$$s = f$$

$$y_m = f\theta_m = \frac{mf\lambda}{a}$$

**9.15**




$$f\theta = \frac{h}{2}$$

$$r_1 - r_2 = \frac{\lambda}{2} = a\theta$$

$$\theta = \frac{\lambda}{2a}$$

$$\frac{h}{2} = f\theta = \frac{f\lambda}{2a}$$

$$h = \frac{f\lambda}{a}$$

**9.16** Follow section (9.3.1), except that (9.26) becomes $r_1 - r_2 = (2m' - 1)(\lambda/2)$ for destructive interference, where $m' = \pm 1, \pm 2, \ldots,$ so that $(2m' - 1)$ is an odd integer. This leads to an expression equivalent to (9.28), $\theta_{m'} = (2m-1)\lambda/2a$.

**9.17** $y_m = \frac{s}{a} m\lambda$

$\lambda = y_m \frac{a}{ms}$

Using $a = 2.7\times10^{-3}$ m, $s = 4.60$ m, $m = 5$:

$$\lambda = (5\times10^{-3}\ \text{m})\frac{2.7\times10^{-4}\ \text{m}}{5(4.60\ \text{m})} = 587\ \text{nm}$$

**9.18** Follow section (9.3.1), except that (9.26) becomes $r_1 - r_2 + \Lambda = m\lambda$, where $\Lambda$ = Optical path differences in beam. Following $r_1$, $\Lambda = nd$ (for $\theta_m$ "small").

$$(r_1 - r_2) = m\lambda - \Lambda;\ a\theta_m = m\lambda - nd;\ \theta_m = (m\lambda - nd)/a.$$

**9.19** As in section (9.3.1), we have constructive interference when $OPD = m\lambda$. There is an added OPD due to the angle, $\theta$, of the plane wave equal to $a \sin\theta$, so (9.26) becomes $r_1 - r_2 + a\sin\theta = m\lambda$. (9.24) $\theta_m \simeq y/s$ and (9.25) $r_1 - r_2 \simeq ay/s$ are unchanged, for small $\theta_m$ so $r_1 - r_2 = m\lambda - a\sin\theta = a(y/s) = a\theta_m$; $\theta_m = (m\lambda/a) - \sin\theta$.

**9.20** (9.27) $y_m = (s/a)m\lambda$; $y_{1,\text{ red}} = [(2.0\ \text{m})/(2.0\times10^{-4}\ \text{m})](1)(4\times10^{-7}\ \text{m})$

$= 4.0\times10^{-3}$ m.

$y_{1,\text{ violet}} = [(2.0\ \text{m})/(2.0\times10^{-4}\ \text{m})](2)(6\times10^{-7}\ \text{m}) = 12.0\times10^{-3}$ m.

Distance $= 8.0\times10^{-3}$ m.

**9.21** $r_2^2 = a^2 + r_1^2 - 2ar_1\cos(90° - \theta)$. The contribution to $\cos\delta/2$ from the third term in the Maclaurin expansion will be negligible if

$$(k/2)(a^2\cos^2\theta/2r_1) \ll \pi/2; \qquad \text{therefore} \qquad r_1 \ll a^2/\lambda.$$

**9.22** $E = mv^2/2$; $v = 0.42\times10^6$ m/s; $\lambda = h/mv = 1.73\times10^{-9}$ m; $\Delta y = s\lambda/a = 3.46$ mm.

**9.23** $\Delta\nu/\Delta\lambda = \nu/\lambda$; $\quad \delta\nu = \nu\Delta\lambda/\lambda = 1/\Delta t_c$;

$c = \nu\lambda$, so $\nu = c/\lambda$.

$\delta\nu = (c/\lambda)\Delta\lambda/\lambda = c\Delta\lambda/\lambda^2$;

$\Delta t_c = \lambda^2/c\Delta\lambda$; $\quad \Delta l_c = c\Delta t_c = (\lambda^2/\Delta\lambda)$

$= (500\ \text{nm})^2/(2.5\times10^{-3}\ \text{nm})$

$= 1\times10^8\ \text{nm} = 0.1\ \text{m} \simeq \Lambda$.

**9.24** $\bar{E} = E_o e^{i\omega t} + E_o e^{\omega t + \delta} + E_o e^{i(\omega t + 5\delta/2)}$. $I = \langle \bar{E}^2 \rangle_T = \langle \bar{E}\cdot\bar{E} \rangle_T$, so, as in section 9.1, $I = (3/2)E_o^2 + 2E_o^2\{\frac{1}{2}(\cos\delta + \cos(3\delta/2) + \cos(5\delta/2))\}$ (three terms of $\bar{E}_i\cdot\bar{E}_i$, 3 cross terms of $\bar{E}_i\cdot\bar{E}_j$). For each beam,

$$I_i = \langle \bar{E}_i^2 \rangle_T = \frac{1}{2}E_o^2,$$

at $\theta = 0$, so that for all three together $I(\theta = 0) = \frac{3}{2}E_j^2$. Note that $(r_2 - r_1) = a\sin\theta$ so that

$$\delta_2 = k(r_2 - r_1) = k(a\sin\theta);\ (r_3 - r_1) = (5a/2)\sin\theta$$

so that $\delta_3 = k(r_3 - r_1) = k(\frac{5}{2}a\sin\theta)$ where $\delta = ka\sin\theta$. So,

$$I(\theta) = I(0)/3 + (2I(0)/9)(\cos\delta + \cos(3\delta/2) + \cos(5\delta/2))$$

When $\theta = 0$, the second term is zero.

**9.25** $\Delta y = \dfrac{(R+d)\lambda}{2R\theta}$

$\theta = \dfrac{(R+d)\lambda}{2R\Delta y}$

Using $\lambda = 6.000\times10^{-7}\,\text{m}$, $R = 1.000$ m, $d = 3.900$ m, $\Delta y = 2\times10^{-3}\,\text{m}$:

$$\theta = \frac{(1.000\text{ m} + 3.900\text{ m})(6.000\times10^{-7}\text{ m})}{2(1.000\text{ m})(2\times10^{-3}\text{ m})} = 0.000735\text{ rad} = 0.0421°$$

**9.26** $S = Z = R + d = 1 + d$

$d = 1$

$\Delta y = \dfrac{(R+d)\lambda}{2R\theta}$

$\theta = \dfrac{\lambda}{\Delta y} = \dfrac{5.89\times10^{-7}\text{ m}}{5\times10^{-4}\text{ m}} = 0.00118$ rad

**9.27** A ray form $S$ hits the biprism at an angle $\theta_i$ (w.r.t normal), is refracted at angle $\theta_t$, and hits the second face at angle $(\theta_t + \alpha)$.

(4.4) (1) $\sin\theta_i = (n)\sin\theta_t$. $(n)\sin(\theta_t + \alpha) = (1)\sin(\theta/2 + \alpha)$, where angle $\theta$ is defined in Figure 9.24.

As $\theta_i \to 0$, $\theta_t \to 0$; $\alpha, \theta$ are both "small."

$n\sin\alpha = \sin(\theta/2 + \alpha)$, so $n\alpha \simeq (\theta/2) + \alpha$, $\theta = 2(n-1)\alpha$. From the figure $\tan(\theta/2) = (a/2)/d$, so

$$\theta/2 \simeq (a/2)/d, \qquad \theta = a/d. \qquad a/d = 2(n-1)\alpha, \qquad a = 2d(n-1)\alpha.$$

**9.28** From Problem 9.19, $a = 2d(n-1)\alpha$; $s = 2d$, so $d = 1m$.

$$\Delta y = (s/a)\lambda = s\lambda/2d(n-1)\alpha; \quad \alpha = s\lambda/2d(n-1)\Delta y$$
$$= [(2m)(5.00\times10^{-7}\text{ m})]/[2(1\text{ m})(1.5-1)(5\times10^{-4}\text{ m})] = 0.002\text{ rad.}$$

**9.29** $\Delta y = s\lambda_0/2d\alpha(n-n')$.

**9.30** Using $\lambda = 5.893\times10^{-7}$ m, $s = 5.00$ m, $a = 1\times10^{-2}$ m:

$$2\Delta y = 2\frac{s}{a}\lambda = 2\frac{(5.00\text{ m})(5.893\times10^{-7}\text{ m})}{2(1\times10^{-2}\text{ m})} = 0.295\text{ mm}$$

**9.31** $\Delta y = (s/a)\lambda$, $a = 10^{-2}$ cm, $a/2 = 5\times10^{-3}$ cm.

**9.32** $\delta = k(r_1 - r_2) + \pi$ Lloyd's mirror,

$$\delta = k((a/2)\sin\alpha - [\sin(90° - 2\alpha)]\,(a/2)\sin\alpha) + \pi,$$
$$\delta = ka(1-\cos2\alpha)/2\sin\alpha + \pi,$$

maximum occurs for $\delta = 2\pi$ when $\sin\alpha(\lambda/a) = (1-\cos2\alpha) = 2\sin^2\alpha$.

First maximum $\alpha = \sin^{-1}(\lambda/2a)$.

**9.33** $E_{1r}$ is reflected once. $E_{1r} = E_{oi}\ \ r_{\theta=0}$ (see 4.47)
$= E_{oi}(n-1)/(n+1) = E_{oi}(1.52-1)/(1.52+1) = 0.206E_{oi}$.
$E_{2r}$ is transmitted once, reflected once, then transmitted.
$E_{2r} = E_{oi}(t_{\theta=0})(r'_{\text{glass-air}})(t'_{\text{glass-air}}) = E_{oi}[2/(1+n)][(1-n)/(1+n)][2n/(n+1)]$
$= 4n(1-n)/(n+1)^3 = E_{oi}[4(1.52)(1-1.52)]/(1+1.52)^3 = -0.198E_{oi}$,
(see 4.48) (− indicates $\pi$ phase changed).

$E_{3r}$ is transmitted, reflected 3 times (internally), and then transmitted.

$$E_{3r} = E_{oi}t(r')^3 t' = E_{oi}[2/(1+n)][(1-n)/(1+n)]^3[(2n)/(n+1)]$$
$$= [4n(1-n)^3]/(n+1)^5 = E_{oi}[4(1.52)(1-1.52)^3]/(1.52+1)^5$$
$$= -0.008E_{oi}$$

for water in air.

$$E_{1r} = E_{oi}(1.333-1)/(1.333+1) = 0.143E_{oi}.$$
$$E_{2r} = E_{oi}[4(1.333)(1-1.333)]/(1+1.333)^3 = -0.140E_{oi}.$$
$$E_{3r} = E_{oi}[4(1.333)(1-1.333)^3]/(1.333+1)^5 = -0.003E_{oi}.$$

**9.34** Here $1.00 < 1.34 < 2.00$, hence from Eq. (9.36) with $m = 0$,
$d = (0 + 1/2)(633\text{ nm})/2(1.34) = 118$ nm.

**9.35** (9.36) $d\cos\theta_t = (2m+1)(\lambda_f)/4$ for a maximum at (near) normal incidence, and taking $m = D$ (lowest value)

$$d = \lambda_f/4 = \lambda_o/4n = (5.00\times10^{-7}\text{ m})/4(1.36) = 9.2\times10^{-6}\text{ m}.$$

**9.36** $d\cos\theta_t = 2\,m\left(\dfrac{\lambda_m}{4}\right)$ for minimum reflection $= 2\,m(\lambda_0/4\,n)$

at $\theta \simeq 0, \lambda_0 = \dfrac{2\,nd}{m} = \dfrac{[2(1.34)(550.0\text{ nm})]}{m} = \dfrac{(1474\text{ nm})}{m}$

for $m = 1,2,3,\ldots$; $\lambda_0 = 1474$ nm, 737 nm, 368.5 nm,…

**9.37** In this case, one drops the relative phase shift of $\pi$ from (9.34):

$$\delta = \frac{4\pi n_f}{\lambda_0} d\cos\theta_i$$
$$2\pi = \frac{4\pi n_f}{\lambda_0} d\cos\theta_i$$
$$\cos\theta_t = \frac{\lambda_0}{2n_f d} = \frac{(4.60\times10^{-7}\text{ m})\ 25\ nm = 2.5\times10^{-8}\,m}{2(1.333)(2.50\times10^{-8}\text{ m})}$$
$$\theta_t = 46.356°$$
$$\sin\theta_i = (1.333)\ \sin(46.356°) = 0.9646$$
$$\theta_i = 74.7°$$

**9.38** Eq. (9.37) $m = 2n_f d/\lambda_0 = 10{,}000$. A minimum, therefore central dark region.

**9.39** The fringes are generally a series of fine jagged bands, which are fixed with respect to the glass.

**9.40** $\Delta x = \lambda_f/2\alpha$, $\alpha = \lambda_0/2n_f\Delta x$, $\alpha = 5\times10^{-5}$ rad = 10.2 seconds.

**9.41** (9.40) $\Delta x = \lambda_f/2\alpha$ for fringe separation where $\alpha = d/x$.
$\Delta x = \lambda_f/2(d/x) = x\lambda_f/2d$. Number of fringes = (length)/(separation) = $x/\Delta x$ so,

$$x/\Delta x = 2d/\lambda_f = [2(7.618\times10^{-5}\text{ m})]/(5.00\times10^{-7}\text{ m}) = 304.72 = 304\text{ fringes.}$$

**9.42** $d_m = \left(m+\frac{1}{2}\right)\frac{\lambda_f}{2}$

$$d_{172} = \left(172+\frac{1}{2}\right)\frac{(5.893\times10^{-7}\text{ m})}{2} = 50.8\ \mu\text{m}$$

**9.43** $x^2 = d_1[(R_1-d_1)+R_1] = 2R_1d_1 - d_1^2$. Similarly $x^2 = 2R_2d_2 - d_2^2$.
$d = d_1 - d_2 = (x^2/2)(1/R_1 - 1/R_2)$, $d = m\lambda_f/2$. As $R_2 \to \infty$, $x_m$ approaches Eq. (9.43).

**9.44** (9.42) $x_m = [(m+1/2)\lambda_f R]^{1/2}$, air film, $n_f = 1$, so $\lambda_f = \lambda_o$.
$R = x_m^2/(m+1/2)\lambda_o = (0.01\text{ m})^2/(20.5)(5\times10^{-7}\text{ m}) = 9.76\text{ m}$.

**9.45**

$$x_m^2 - x_{m-1}^2 = \lambda_f R(m_m - m_{m-1})$$

$$R = \frac{x_m^2 - x_{m-1}^2}{\lambda_f R(m_m - m_{m-1})}$$

Use

$$2d_m = \left(m+\frac{1}{2}\right)\lambda_0$$

$$m = \frac{2d_m - \frac{1}{2}}{\lambda_0}$$

Since the offset is a constant $\Delta d$:

$$R = \frac{x_m^2 - x_{m-1}^2}{\frac{\lambda_f}{\lambda_0}R\left(2d_m + \Delta d - \frac{1}{2} - \left(2d_{m-1} + \Delta d - \frac{1}{2}\right)\right)}$$

$$R = \frac{n_f(x_m^2 - x_{m-1}^2)}{2R(d_m - d_{m-1})}$$

Thus the radius of curvature can be measured independent of $\Delta d$.

**9.46** $x_m = (m\lambda_f R)^{1/2}$

$$x_{m+1} - x_m = (\lambda_f R)^{1/2}(\sqrt{m+1} - \sqrt{m})$$

$$x_{m+2} - x_{m+1} = (\lambda_f R)^{1/2}(\sqrt{m+2} - \sqrt{m+1})$$

$$\frac{x_{m+1} - x_m}{x_{m+2} - x_{m+1}} = \frac{(\lambda_f R)^{1/2}(\sqrt{m+1} - \sqrt{m})}{(\lambda_f R)^{1/2}(\sqrt{m+2} - \sqrt{m+1})} = \frac{\sqrt{m+1} - \sqrt{m}}{\sqrt{m+2} - \sqrt{m+1}}$$

Expand the square roots for large *m* (keeping only the first few terms):

$$\sqrt{m} = m^{1/2}$$

$$\sqrt{m+1} = m^{1/2} + \frac{1}{2}m^{-1/2} - \frac{1}{8}m^{-3/2}$$

$$\sqrt{m+2} = m^{1/2} + m^{-1/2} - \frac{1}{2}m^{-3/2}$$

$$\frac{x_{m+1} - x_m}{x_{m+2} - x_{m+1}} = \frac{m^{1/2} + \frac{1}{2}m^{-1/2} - \frac{1}{8}m^{-3/2} - m^{1/2}}{m^{1/2} + m^{-1/2} - \frac{1}{2}m^{-3/2} - m^{1/2} - \frac{1}{2}m^{-1/2} + \frac{1}{8}m^{-3/2}}$$

$$= \frac{\frac{1}{2}m^{-1/2} - \frac{1}{8}m^{-3/2}}{\frac{1}{2}m^{-1/2} - \frac{3}{8}m^{-3/2}} = \frac{m^{-1/2} - \frac{1}{4}m^{-3/2}}{m^{-1/2} - \frac{3}{4}m^{-3/2}} = \frac{1 - \frac{1}{4}m^{-1}}{1 - \frac{3}{4}m^{-1}} = \frac{4m-1}{4m-3} =\sim 1 + \frac{1}{2m}$$

For m = 50

$$\frac{\sqrt{50+1} - \sqrt{50}}{\sqrt{50+2} - \sqrt{50+1}} = 1.0099$$

$$1 + \frac{1}{2(50)} = 1.01$$

**9.47** A motion of $\lambda/2$ causes a single fringe pair to shift past, hence $92(\lambda/2) = 2.53 \times 10^{-5}\,\text{m}$ and $\lambda = 550$ nm.

**9.48** $\Delta d = N(\lambda_o/2) = (1000)(5.00 \times 10^{-7}\ \text{m})/2 = 2.50 \times 10^{-4}\ \text{m}$.

**9.49** $\Delta d = N\dfrac{\lambda_0}{2}$

$$N = \frac{2\Delta d}{\lambda_0} = \frac{2(1 \times 10^{-4}\,\text{m})}{5 \times 10^{-7}\,\text{m}} = 400$$

**9.50** $\Lambda = \Delta d = N(\lambda_o/2)$; $\Lambda = (n_{\text{air}}x - n_{\text{vacuum}}x)$;
$N = 2\Lambda/\lambda_o = [2(1.00029 - 1.00000)(0.10\ \text{m})]/(6.00 \times 10^{-7}\ \text{m}) = 97$.

**9.51** Differentiating $\nu = \dfrac{\lambda}{c}$:

$$\Delta\lambda_0 = \frac{c}{\nu^2}\Delta\nu = \frac{\lambda_0^2}{c}\Delta\nu$$

$$\Delta\nu = \frac{1}{\Delta t}$$

$$\Delta l_c = c\Delta t$$

$$\Delta\lambda_0 = \frac{\lambda_0^2}{c}\frac{1}{\Delta t} = \frac{\lambda_0^2}{\Delta l_c}$$

$$2D = \Delta l_c$$

$$\Delta\lambda_0 = \frac{\lambda_0^2}{2D}$$

$$D = \frac{\lambda_0^2}{2\Delta\lambda_0} = \frac{(6.43847 \times 10^{-7}\,\text{m})^2}{2(0.0013\ \text{nm})} = 0.1594\ \text{m}$$

**9.52** Fringe pattern comes from the interference of two beams, one that passes through the lower medium ($n_1$), and is reflected off its mirror, one that passes through the top medium ($n_2$) and is reflected off its mirror. The two beams reflect off the front surface of the other medium.
It might be used to compare $n_1$ and $n_2$ (especially if one changes, such as due to pressure or temperature), or compare the flatness of one surface, to a known optically flat surface.

**9.53** $E_t^2 = E_t E_t^* = E_0^2 (tt')^2/(1-r^2 e^{-i\delta})(1-r^2 e^{i\delta}),$
$I_t = I_i (tt')^2/(1-r^2 e^{-i\delta} - r^2 e^{i\delta} + r^4).$

**9.54** (a) $R = 0.8944,$ therefore $F = 4R/(1-R)^2 = 321.$

(b) $\gamma = 4\sin^{-1}\left(1/\sqrt{F}\right) = 0.223.$ (c) $F = 2\pi/0.223.$ (d) $C = 1+F.$

**9.55** $$2/[1+F(\Delta\delta/4)^2] = 0.81[1+1/(1+F(\Delta\delta/2)^2)],$$
$F^2(\Delta\delta)^4 - 15.5F(\Delta\delta)^2 - 30 = 0.$

**9.56** $I = I_{\max}\cos^2\delta/2,\quad I = I_{\max}/2$ when $\delta = \pi/2,$ therefore $\gamma = \pi.$ Separation between maxima is $2\pi.$ $F = 2\pi/\gamma = 2.$

**9.57** (4.47) $r_{\theta i=0} = (n_t - n_i)/(n_t + n_i).$ Bare substrate: $r = (n_s - 1)/(n_s + 1).$ Substrate with film: $r' = t_{o-f} r_{f-s} t_{f-o}.$
(4.48) $t_{\theta i=0} = 2n_i/(n_i + n_t),$ so, $r' = [2/(1+n_f)][(n_s - n_f)/(n_s + n_f)][2n_f/(n_f+1)],$ where $n_f = n.$ Note that for $n_s > n_f > 1$, both $r$ and $r'$ are positive. But, with thickness $\lambda_f/4,$ a $\pi$ phase shift occurs due to the OPD in the $r'$ beam, so $r_{\text{net}} = r - r'$.
Thus, the $r'$ beam (partially) cancels the $r$ beam.

**9.58** At near normal incidence $(\theta_i \approx 0)$ the relative phase shift between an internally and externally reflected beam is $\pi$ rad. That means a total relative phase difference of $(2\pi/\lambda_f)[2(\lambda_f/4)]+\pi$ or $2\pi.$ The waves are in phase and interfere constructively.

**9.59** $n_0 = 1,\quad n_s = n_g,\quad n_1 = \sqrt{n_g}.$

$\sqrt{1.54} = 1.24,$ $d = \lambda_f/4 = \lambda_0/4n_1 = 540/4(1.24)\text{ nm} = 167\text{ nm}.$
No relative phase shift between two waves.

**9.60** The refracted wave will traverse the film twice, and there will be no relative phase shift on reflection. Hence $d = \lambda_0/4n_f = (550\text{ nm})/4(1.38) = 99.6\text{ nm}.$

**9.61** $d\cos\theta_t = (2m+1)\left(\frac{\lambda_m}{4}\right).$ Let $\theta_t = 0,$ $m = 0,$ (minimum thickness).

$$d = \frac{\lambda_0}{4n_f} = \frac{5.00\times 10^{-7}\,\text{m}}{4(1.30)} = 96\,\text{nm}$$

**9.62** Note that in the triangle including $\theta$ and $r_1$, the length of the side from $P_1$ to a plane, parallel to the surface, and containing point $z(x)$ is $r_1 \cos\theta$. So, from zero elevation,

$$h = r_1 \cos\theta + z(x) \text{ or } z(x) = h - r_1\cos\theta.$$

(9.108) can be demonstrated on the triangle $(a, r_1, r_2)$, where $a$ is the length of the boom:

$$r_2^2 = r_1^2 + a^2 - 2r_1 a\cos\left(\alpha + 90° - \theta\right) = \sin\left(\gamma\right) = -\cos\left(90° + \gamma\right)$$

and $\delta = k(r_2 - r_1) = (2\pi/\lambda)(r_2 - r_1).$